Introduction to Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 62
Stress-Strain Relations for Individual Layers of a Laminate

Hello, welcome to Introduction to Composites. We have started a new topic yesterday and specifically what we started developing were strain displacement relations for laminates composite laminates and what we had shown yesterday was that the displacement in composite laminate at any location x y z, it can be expressed in terms of mid plane displacements and mid plane curvatures. So, that is what we are going to re express that as.

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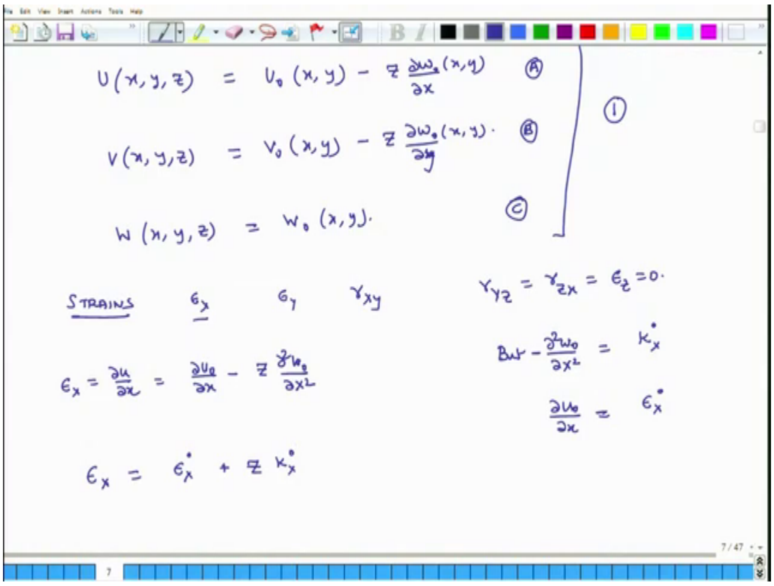

So, we had shown that u which is the displacement at any location in a composite. So, any location is specified by these coordinates, it can be written as mid plane displacements at location x y minus z times the slope of the plate at that particular point. So, here w naught is a function of x and y. Similarly the wa displacement in y is x v which is a function of x y z and that equals v naught x y minus z times del w naught over del x, so u del y and w naught is a function of x and y only and then the last relation is for mid plane displacement ok. So, mid plane displacement is w x y z and that is equal to w naught x y and the reason why we are saying that is because we have already assumed that the la displaced com compression or extension of the line in the z direction is assumed to be 0.

So, these are the 3 displacements for any point on a composite laminate and this is coming directly from geometry here material properties are not at all involved till. So, far so now what we will do is we will develop relations for strains using these displacement relations. So, what strains we are interested in? So, we are interested in epsilon x epsilon y and gamma x y because all other strains what are the other strains gamma y z equals gamma z x equals epsilon z, we have assumed that they are zero based on the assumptions which we discussed that earlier.

So, what we will do is these are equations 1 and what is epsilon x epsilon x is nothing but del u over del x ok, it is partial derivative of u with respect to x. So, if I use this relation in equation 1 A. So, this is A this is B this is C, I get del u naught over del x minus z times del 2 w naught over del x square and I am just omitting terms in parenthesis because this is getting repetitive, but u is a function of x y and z but u naught and w naught do not depend on z they are the values of displacement functions at the mid plane. So, they only vary with x and y they do not depend on z at all, so this is again I wanted to specify.

Now, we know that the second derivative of w naught with respect to x with x is what it is the curvature of the plate in the x direction. So, we call it k x and because it is the curvature of the mid plane because, the top surface will have a little different curvature the bottom surface will have different curvature. So, here we are interested in calculating the curvature of the mid plane. So if so that is why we have a subscript and we also say that del u naught over del x is what it is the mid plane strain mid plane strain. So, using these things we can say that epsilon x is equal to mid plane strain minus z. So, I am sorry this curvature is defined as negative of second derivative of w, so this becomes plus k x times this thing so this is one equation.

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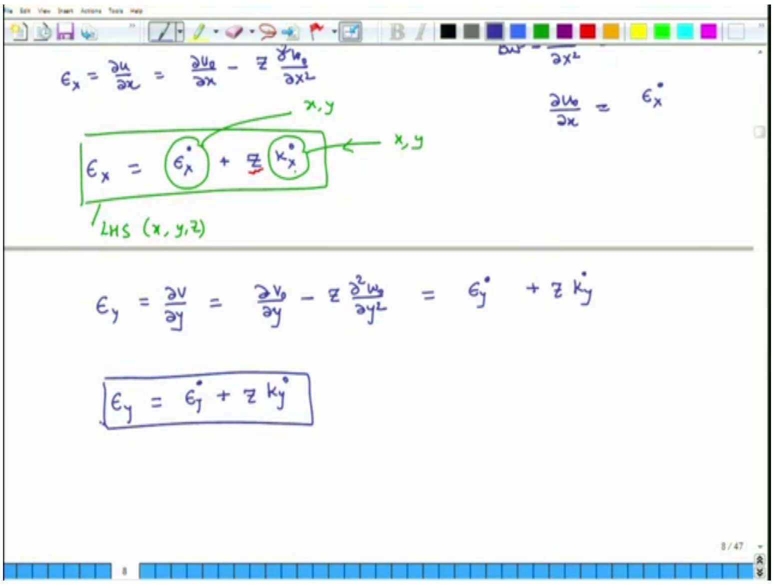

Now, once again this lhs is a function of x y and z, but this it depends only on x and y and mid plane strain also depends only on x and y. So, the reason I write left hand is function of x y and z is because, on the right side you have an explicit term involving z; if this z was not there then right left side would also be independent of z ok.

Next we look at the strain displacement relation epsilon y, so epsilon y is del v over del y and if we use the equation B if we use equation B and we differentiate this entire equation B with respect to y we get del v naught over del y minus z times del 2 w naught over del y square and because minus of second derivative of w with respect to y is curvature in y direction.

So, this I can write it as plus z times k y and it is a mid plane entity and partial derivative of v naught with respect to y is the mid plane strain epsilon y. So, I can say that epsilon y equals mid plane strain epsilon in the y direction plus z times k y this is the second equation and the third relation is Gamma x y is what it is del u over del y plus del v over del x ok. So, we do the same math again and we find so u is del u naught over del y minus z del 2 w naught over del x del y.

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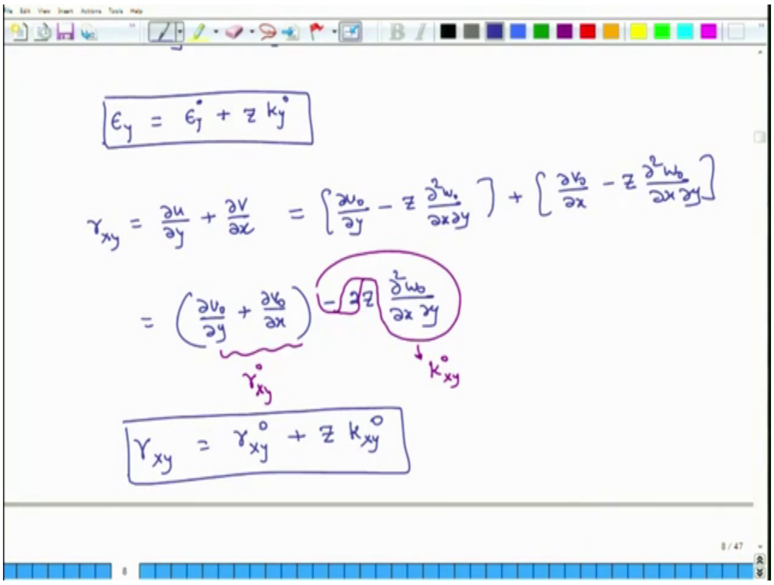

So, this is the first part plus del v naught over del x minus z del 2 w naught over w del x del y ok. So, this is equal to del u naught over del y plus del v naught over del x minus 2 z del 2 w naught over del x del y and this the mid plane strain gamma x y and this term negative of this is k x y, it is the twisting curvature it is not the bending curvature, but the twisting curvature actually the 2 also comes into this.

So, we can say that gamma x y equals gamma x y naught plus z times k x y naught. So, these are the strain displacement relations and for purpose of completeness we will rewrite them in a vector form.

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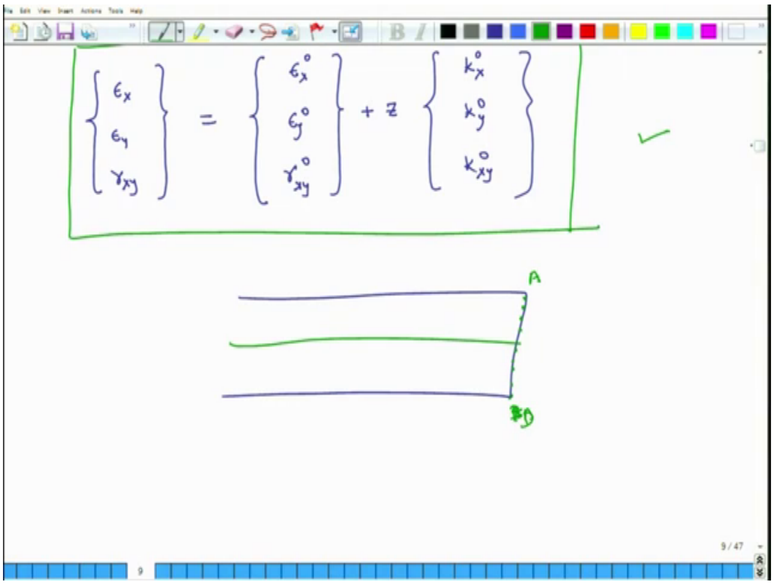

So, I have 3 strains epsilon x epsilon y gamma x y and these are equal to the sum of their mid plane strains, epsilon x naught epsilon y naught gamma x y naught plus z times and z is the coordinate of the point of interest. So, z times respective curvatures k x naught k y naught and k x y naught, so this is the overall system of strain displacement relations. So, this is concludes our discussion on strain displacement relations next what we will discuss are stresses in laminates stresses in laminates and how they vary. So, our next topic is stresses in laminates.

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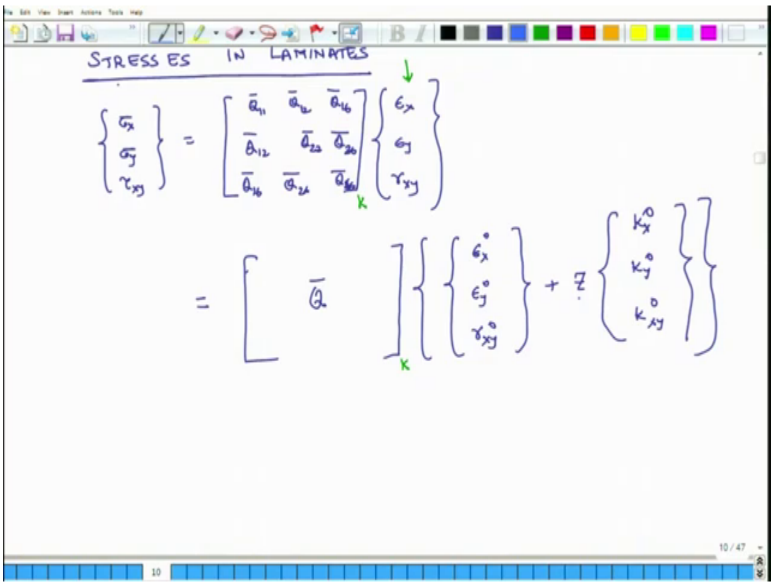

So, now the question is that if I know the mid plane strain and mid plane curvatures. So what does that mean that if this is a plate and if I know the strain at the mid plane epsilon x naught epsilon y naught and gamma y naught and I also know the mid plane curvatures, then using this relation I can calculate strains at all the positions on this entire axis on this entire line segment AB using this relation, because for each point all I have to do is measure the value of z and plug in the value to calculate epsilon x epsilon y and gamma x y, if I know epsilon x naught with mid plane strains and between curvatures ok.

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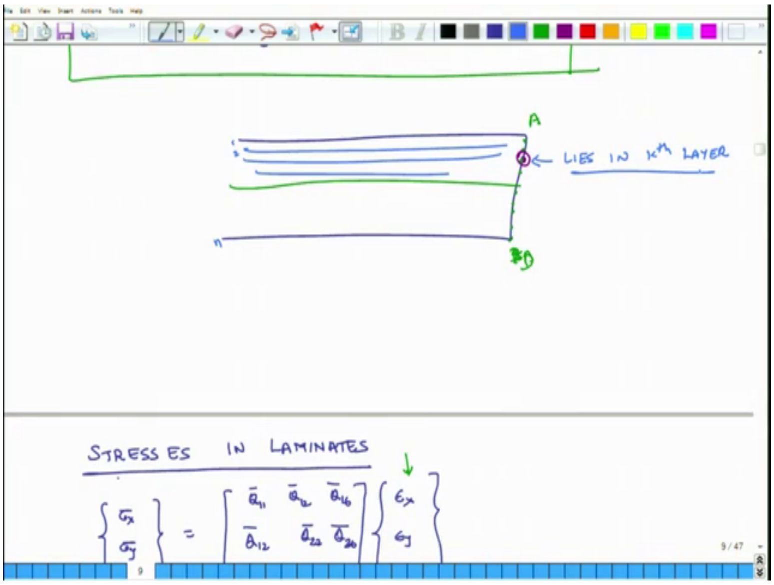

So, so the point is that I can for all the points on line AB or actually AD which because we used earlier AD I can calculate strains. Now if I have these strains the question is can I calculate stresses at every point on this line segment, so that is what we are going to do. So, we know and through our earlier lectures that for any composite for any composite, we had actually developed this relation that sigma x sigma y tau x y is equal to the cube r matrix times epsilon x epsilon y and gamma x y and what is this cube our matrix Q 1 1 bar Q 1 2 bar Q 1 6 bar Q 2 2 bar Q 2 6 bar and Q Q 6 6 bar right.

So, if I know the epsilon x values of epsilon x epsilon y and gamma x y and if I know the lamination sequence then for. So, I know that suppose I am interested in finding the strain stress at this point, if I know which particular layer exists at this particular point then for that particular layer I can calculate Q bar matrix and once I know the Q bar matrix and I also already know epsilon x epsilon y epsilon gamma x y. So, I can multiply this Q bar matrix with this vector and I can get the stresses ok, I can get stresses at the point of interest.

So, and then we have already shown that the strain vector epsilon x epsilon y gamma x y is equal to what the sum of mid plane strains times z times curvatures. So, in short we can write it as oh. So, I will so this is my Q bar matrix times epsilon x naught epsilon y naught gamma x y naught plus z times curvatures and here I will put a subscript k and I will also put a subscript k and what does k indicate it indicates the k th layer. So, suppose this point of interest is corresponds to the k th layer.

So, we suppose this composite has several layers, so well 1 2 3 you know 1 2 and so on so forth and these are total N layers and suppose this lies in k th layer and that is the point of our interest. So, it is for that k th layer we should umm identify the Q bar matrix, so that is why we put a subscript here and then we multiply that by the strain vector and then and of course, the coordinate of the z coordinate and then we will be able to get the overall stress in the system at that point.

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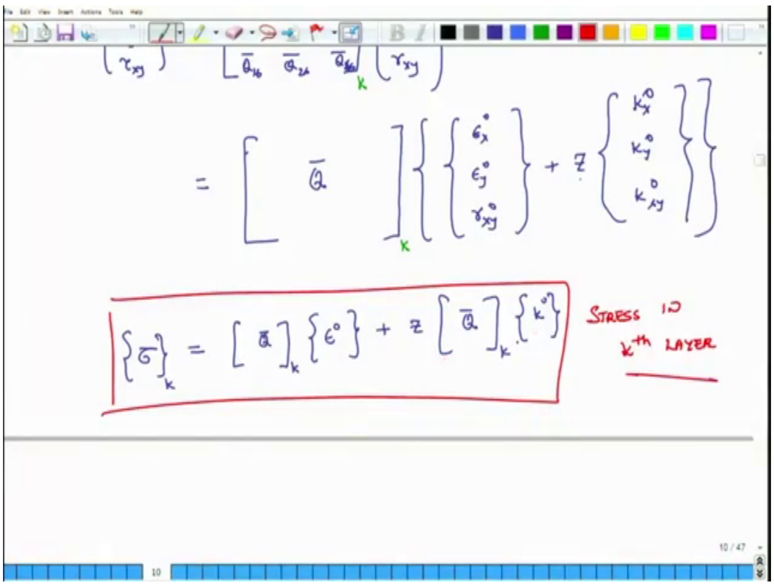

So, this is equal to so I will in short I can write it as Q bar for k th layer times mid plane strain vector plus z times Q bar for the k th layer times the curvature vector. So, this is my overall stress in the k th layer. So, this is the relation for the stress in k th layer stress.

So, this I think concludes our discussion for today, what we will do tomorrow is we will have another five ten minutes more of discussion on this and we will try to understand how stresses and strains vary in the z direction for a given value of x and y and once that is done. Then we will move on to the next level of this analysis, where we will start developing more sophisticated relationships between Q strains and stresses. So, with that we conclude our discussion and we will meet once again tomorrow.

Thank you.