Introduction to Composites
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Lecture – 61
Strain Displacement Relations for a Laminate

Hello, welcome to Introduction to Composites. Today is the start of the second last week of this course and what we plan to do over this week and also the next week is try to develop a method through which we can predict the stiffness of laminates not just individual lamina, but entire laminates which are stabbed the sequences of different layers of unidirectional composites and then also figure out how to predict failure in these laminates.

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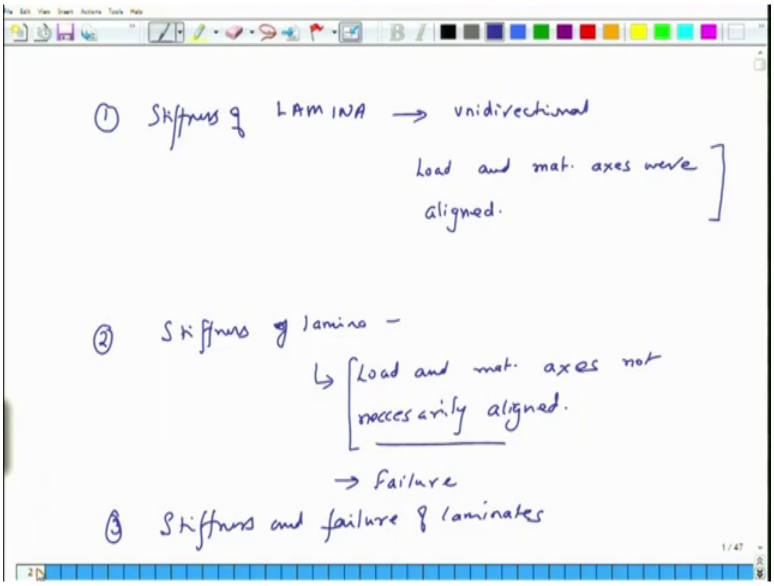

So, till. So, far in this course what we have learnt is that the first thing we learnt was how to predict stiffness of lamina, this is what we learnt and specifically in that too we learned when it was a unidirectional lamina and the orientation of the load and the material axis they were aligned.

So, this was the first theme of this course and we were able to predict the stiffness of the lamina and also the strength of the lamina. And then as a next step and this first step we did we expressed stiffness of the lamina in terms of first material constituents, which are fiber properties and matrix properties and then once we were confident of predicting the stiffness of lamina when the load and material axis both are aligned then we made it a little more general, so we in the next step we still focused on a lamina.

But then our situation was that load and material axis not necessarily aligned. So, this is what we learned over last couple of weeks and so we are at this stage and once we did that we also evaluated how to predict failure in such situations ok, so these are the 2 broad things we have learnt. So, the next step is that the stiffness and failure of laminates of laminates this is what we will focus on. So, what is a laminate it is a series of lamina individual layers of composites and they are all glued together.

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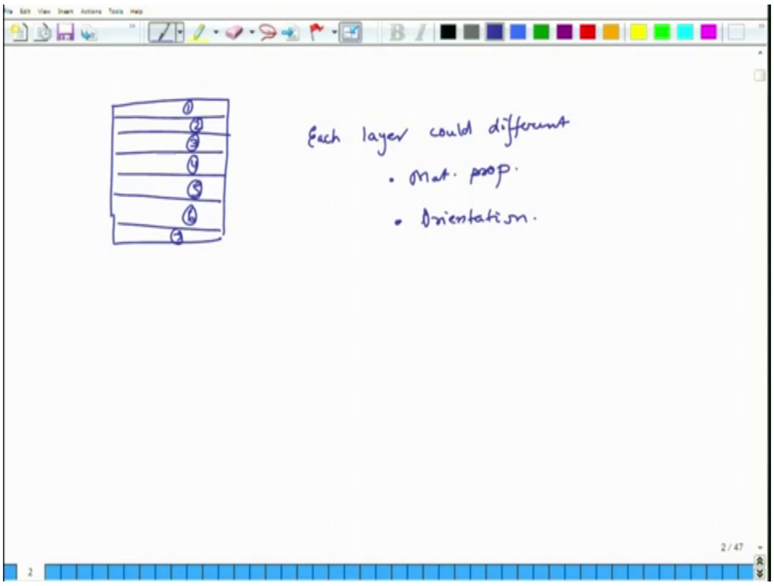

So, a laminate if you look at it is cross section and you zoom in then it may have several layers of composites. So, this could be 1 2 3 4 5 6 and whatever number of layers could be there and each layer it could be different, it could be different in terms of material properties and it also could be different in terms of orientation. So, what we will learn is a method to predict the stiffness of a thick laminate and also later we will learn how to predict failure in such laminates. So, we will make certain assumptions before we start developing this theory. So, we will make some assumptions.

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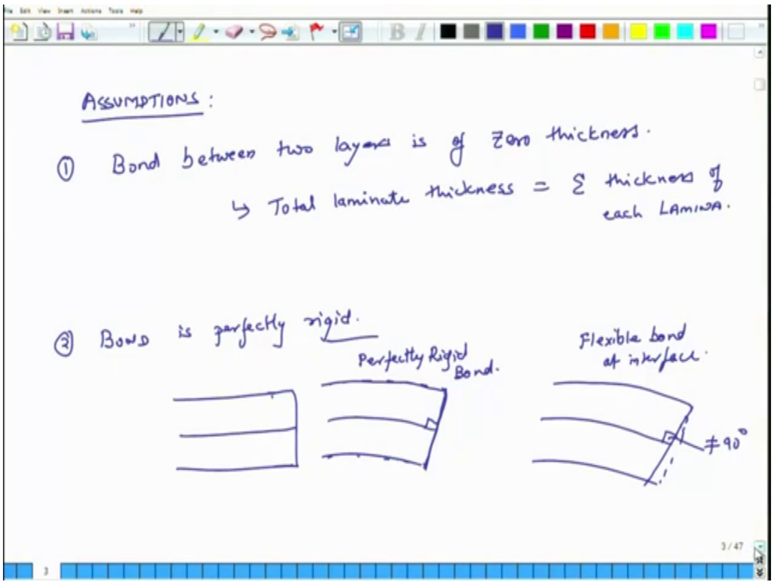

So, the first assumption we will make is that each layer the bond between 2 layers is of 0 thickness this is 1 assumption we will make the bond between 2 layers is of 0 thickness; what that means, is that total laminate thickness, so this equals sum of thickness of each layer each lamina. So, we do not have to worry about the thickness of the bond between 2 layers that is one thing.

The second assumption we will make is bond is perfectly rigid perfectly rigid. So, what that means is so the second assumption as I mentioned is that bond between 2 adjacent layers is perfectly rigid, now that has a very significant implication. So, consider this book and just assume that each of these pages is an individual layer or a lamina ok. Now if the bond was not rigid or rather the bond was not existence, if the bond was nonexistent then as I bend this book right as I bend this book one layer slides on top of each layer the adjacent layer. So, as I bend it page number one slides on page number to page number 2 slides on page number 3 and so on and so forth. So, this is this is how the edge will look like, when it is not bent the edges straight when it is bent then edge becomes longer and also each layer slides on top of or ba or below adjacent layers.

But when the bond is perfectly rigid which means, when I bend it there is no sliding there is no sliding going to happen and when there is no sliding going to happen then, so I will I will actually draw a picture. So, suppose there is a laminate which has 2 layers only and if the bond is perfectly rigid then when it bends, then this edge will remain straight ok. So, this is the deformed surface laminate suppose I want to bend this laminate then this edge it remains straight and this angle it also remains at 90 degrees, so this is when you have perfectly rigid bond ok.

If the bond was not perfectly rigid and they were sliding which was being permitted, then the shape may not look like this. So, this would have been the ideal shape, but it may look something like this something like this angle, this may not be equal to 90 degrees. So, this is when flexible bond is there at interface, so this is a very important presumption ok. So, a bond imply is perfectly rigid mathematically it means several things perfect rigid bond.

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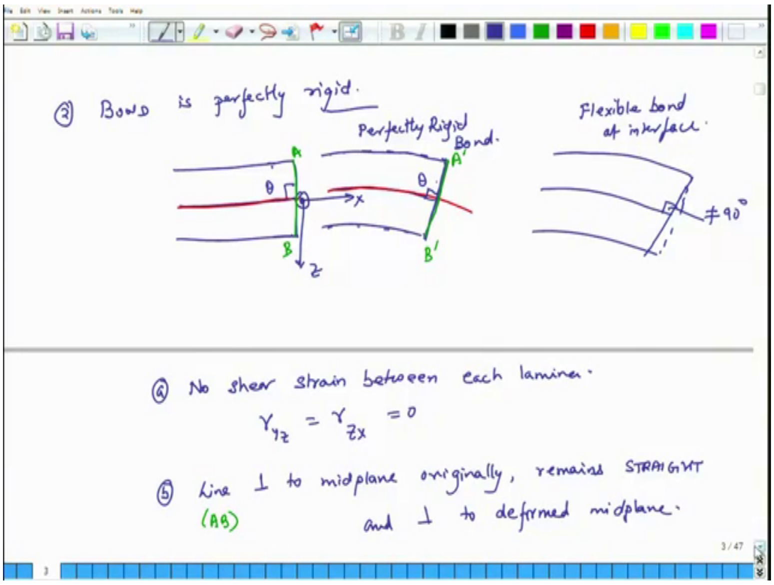

It implies a no shear strain between the layers each lamina and if this is my z axis, if this is my if this is my x axis and the plane vertical to the computer screen is y axis, if there is no shear strain between each lamina essentially what that means, is that gamma y z equals gamma xz equals 0.

I am sorry gamma y z gamma equals gamma y z z x z x this is equal to 0, this is another implication. So, first implication is that if the bond is perfectly rigid it does not permit any shear strain gamma y z or gamma z x, the other thing is if the bond is perfectly rigid then the other thing it means is that this angle. So see this angle was theta. So, theta was initially 90 degrees and when there is bending of the plate theta is still remains 90 degrees, so this is the normality assumption.

So, what in wa words what we say is that line perpendicular to mid plane. So, this is the line which is per this is the mid plane this is the mid plane and this is the line which is perpendicular to the mid plane. So, this is line let us say AB and once the thing has bent this is A prime B prime our mid plane is this and the line A prime B prime is this.

So, line which was so which line we are talking about we are talking about line AB, so line which was perpendicular to the mid plane originally it was originally this also remains a straight. So, it does not develop any curvature it still remains street and perpendicular to deformed mid plane. So, this theta still remains ninety degrees theta is still domains 90 degrees. So, this is the second thing and the third assumption we make is that whenever we have a composite plate.

So this is it is thickness and thickness is represented by line AB and then I put loads on it suppose I put some external load. The thing may bend and twist and do all sorts of things, but 1 thing we know that will not happen. So, this is the band configuration is that the length of line AB the length of line AB will not change because, all the external loads are only causing the plate to band nothing is getting the there is no force which is actually compressing the compressing the plate from both sides.

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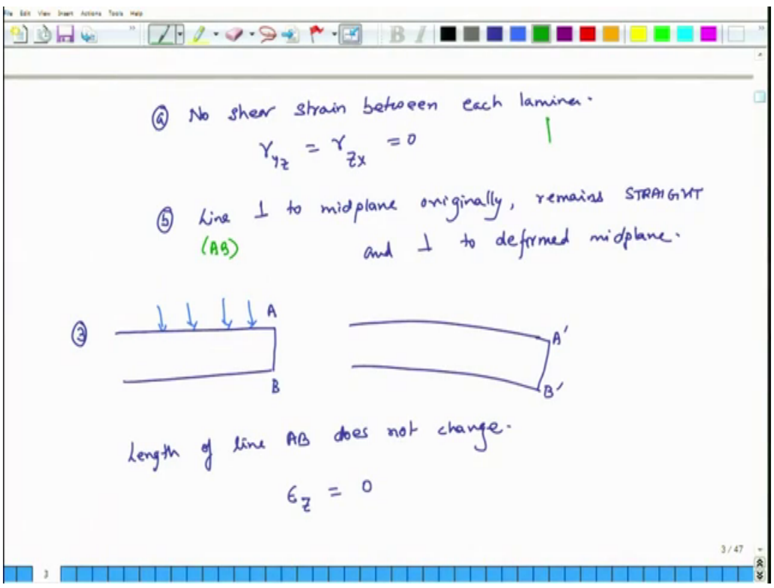

So, length of line may be does not change this is another assumption we make and essentially mathematically what it means is that epsilon z is equal to 0. So, these are the

3 assumptions which we make and based on these assumptions now what we will do is we will develop a relation for strains in such a plate.

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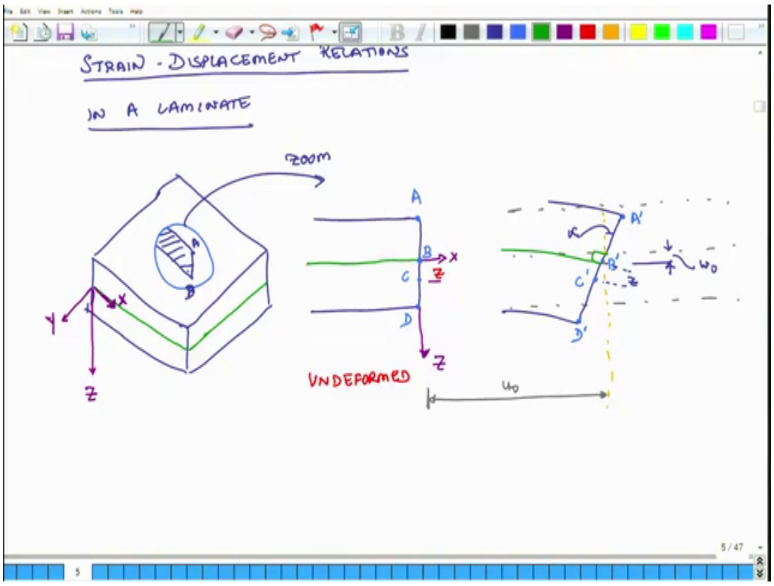

So, we will develop strain displacement relations in a laminate. So, this is will develop relations between strains and displacements. So, now what we are going to do is based on these assumptions will develop relations between displacements in the 3 directions u v and w and strains and we have already said that 3 strains are already 0 epsilon z gamma y z and gamma z x. So, the 3 strains we will really worried about are epsilon x epsilon y and the shear stress, I am sorry the shear strain gamma xy, so what we will do is we will start by drawing a picture.

So, so consider this so this is a small flat plate, so what I will draw is I will also draw the mid plane. So, plate has uniform thickness and the green line represents the mid plane of the thing and I now draw the coordinate system; so from the coordinate system, when we go down towards the bottom side of the plate that is the positive z axis. So, this is the convention we are going to use and this is my this is the x axis and the third axis is y axis. So, A 3 axes x y and z as depicted here the x axis runs on the mid plane of the plate in this direction on the mid plane also we have y axis and going down is positive z axis and so let us look at a small portion of this plate.

So, what we are going to see is so this is a small portion and in this portion we have this small portion and let us call this top point as A and the bottom point is D. So, this is A so

I am cutting a small portion and I am looking at it and then I zoom it and this is the deformed un-deformed position. So, when I look at the un deformed position it looks like this. So, this is a significantly zoomed view so the top point is point A the lower point is point B and the mid plane in this view appears as a just as line because, we are looking at it from the end. So, this is my mid plane and the plane the point where this mid plane cuts the edge AB that is point B.

So, this is my x axis and my excuse me this is point D. So, the lower point is point D this is point B this is point A this is point B and the axis going normal to the plane of this board that is my y axis and I now identify another point on this line A B or A D and I will call that. So, this additional point is point C and this particular point. So, this is z axis vertical axis going down is positive z axis. So, this distance is x no z so the location of point C is z millimeters down in the z direction online, so this is un deformed situation this is un deformed situation.

Now, when I put some load on this composite plate, so I can put some extra mass or weight then this composite plate is going to bend. So, what I will do is I will draw the band configuration also. So, these are my original lines, now 2 things can happen 1 is I can I have a plate and I can pull it like this and as I pull it this line ad it will just move horizontally in the x direction that is one thing it will happen.

So, that is a translation a line ad will move and it will translate then and if I also put some extra loads on top of that, this line ad which is this vertical line it will not only move because of the stretching thing, but it can will also rotate it will also rotate. So, both these things can happen because of extensional things it can stretch and it can also rotate because of vertical loads. So, I will depict both of these things in the revised updated graph or a picture.

So, so let us say it stretches by a distance u naught. So, first what I am doing is I am just stretching or pulling this line ad in the positive x direction by a distance u naught and when I do that then this is my new location and now this is my mid plane and then I rotate it and then I rotate it. So, when I rotate it the new configuration looks like this ok. So, this distance this distance is u naught and this rotation is by a certain distance.

So, we will again mark these points here so the revised positions are A prime this is D prime the mid plane is there. So, this is B prime and a distance below that is C prime and the overall configuration looks something like this ok. So, 3 things have happened actually 3 things have happened not just the 2 things 3 things have happened, I have pulled the plate in u direction in x direction I have rotated it and I have also pushed it downwards I have also pushed it downwards, otherwise this B prime would lie on the original mid plane line, but now it has gone down.

So, how much I have pushed it down the mid plane has gone down by a distance w naught. So, this is w naught and the rotation has happened by an angle let us call this angle alpha alpha and the distance B to C is still distance B to C is still z and it does not change because we have said that the line AD it does not stretch or contract in the z direction.

So, B to C also does not change ok, so B to C distance still remains z the line has the midpoint has shifted down by w naught, it has gone out by u naught and line ad prime has rotated by an angle alpha and the new mid plane line is this thing and this angle is still ninety degrees because, we assumed that earlier that there is no shear happening and when there is no shear happening then this angle remains at 90 degrees.

So, we will write down a couple of important points: point B prime it moves by u naught in x direction.

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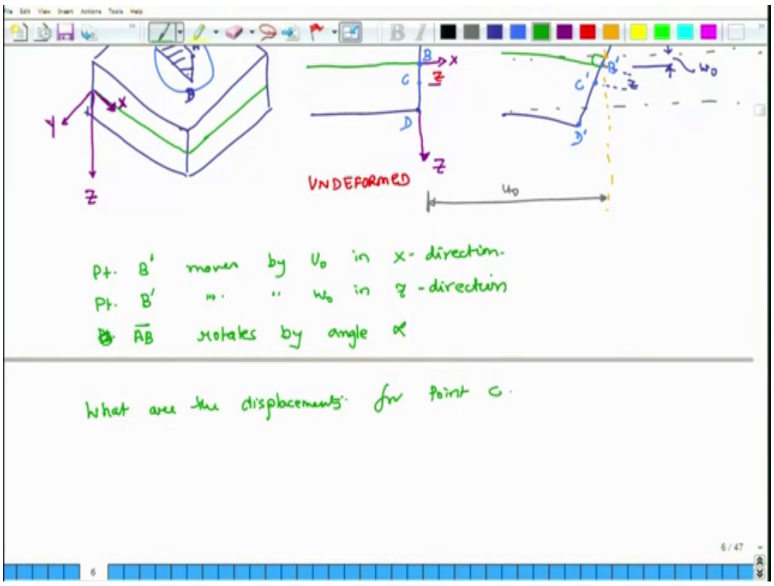

So, we are talking about the midpoint right now we are just talking about the midpoint of the system, point B prime moves by distance w naught w naught in the positive z direction in z direction and AB, this line AB it rotates by angle alpha and it is rotating because the plate is bending, so it will rotate.

So, then the question is what are the coordinates for point C this is the question, we want to calculate the coordinates of point C or actually I will not call it what are the displacements for point C not coordinates because what we are really interested in displacements and points C is any point lying on line AB and it is it is z millimeters or meters or from the midpoint mid plane ok.

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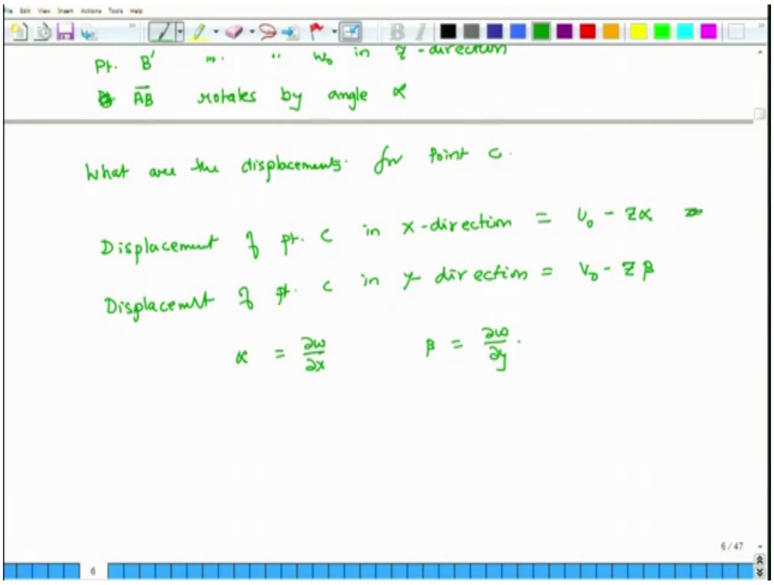

So, we I want to figure that out, so displacement of point C in x direction is equal to what C it is equal to this distance. So, what is the displacement of point C, first thing is point B moves out by u naught and with respect to the new position of B C is moving inwards by some distance this distance and what is this distance this is nothing but z times alpha because. This angle alpha is extremely small. So, if you compute it will be actually z times sin of alpha and sin of alpha is same as alpha, so it works out to z times alpha.

So the new, so the total displacement of point C is u naught minus z or times alpha now this is the situation when the plate is, so suppose this is my x direction. So, this is my displacement in this direction I call it u. So, when the plate bends like this and if there is a point C on this vertical edge, then it is motion will be u naught which is the mid plane displacement minus z which is the coordinate of that point times rotation alpha and that is when the plate rotates bends like this ok.

Now the plate there can also bend like this in the y direction because, when I am low putting load it will bend like this and it will also bend like this it will also bend like this. So, point C will experience not only displacement in x direction, but will also experience displacement in y direction and we can create a similar figure and we can using similar figures, but for the y direction we can say that displacement of point C in y direction is equal to u naught minus z times. Now in that direction let us say the rotation of that vertical line a B is beta then I can call it z times beta, whatever is the angle in the other plane rotation in the other plane ok.

Next thing we know is that alpha is equal to del w over del x, so it is v naught minus z times beta ok. So, alpha is del w over del x right basically it is the slope of this line slope of this line and slope of this line is del w over del x and similarly we can say that beta equals del w over del y ok, so this displacement is.

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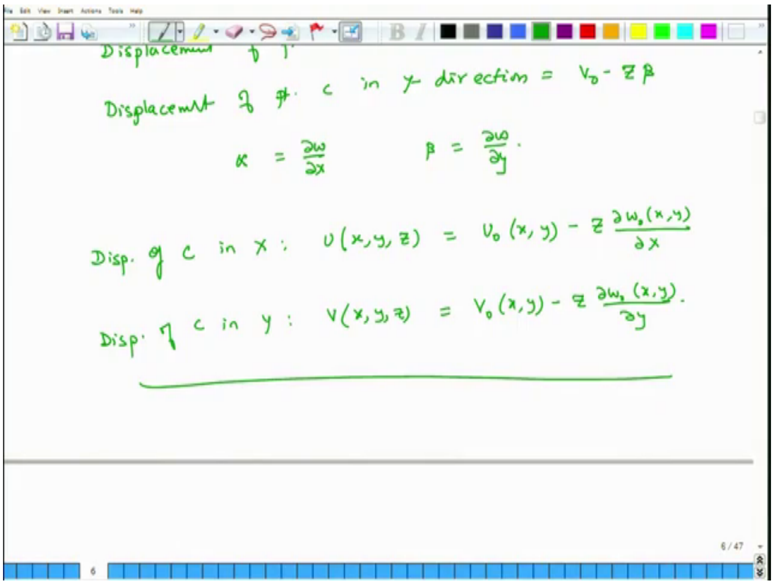

So, we can say displacement of C in x, what is it is u it is not u naught and u changes from it can vary from point to point. So, it is u is a function of x y and it also depends on z because z is here ok.

So, u x y z is equal to u naught and what is u naught, u naught is the displacement of point B and point B always lies on the mid plane. So, when it is lying on the mid plane z is immaterial, so u naught only changes from point to point. So, u naught is a function of x and y minus z times del del w naught x y divided by del x ok.

Similarly, displacement of C in y direction is v and it changes from point to point and it also depends on z and this is equal to v naught which is the mid lane displacement at that coordinate system x y, but it does not depend on z because it is already at mid plane. So, del z is already prescribed minus z del w xy over del y ok.

So, this concludes our discussion for today, what we will do is we will use these relations to develop now more formal strain displacement relations for a composite laminates and that will form one important basis over which we will develop a more detailed theory. Thank you and we will meet tomorrow with the same topic.

Thanks a lot.