Introduction to Composites
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Lecture – 60
Importance of Sign of Shear Stress in context of Strength of A Unidirectional Lamina

Hello, welcome to Introduction to Composites. Today is the last day of this week and what we will do today is we will discuss the work maximum work theory or the theory from Sahil, which is used to predict failure of unidirectional lamina. We under plane stress more accurately relative to other theories. And we will discuss some of it is final aspects and the most important thing we will discuss in that context is the direction of shear stress or the shear strain and how it influences the strength of the material.

So, once we go through this it will become very clear to us that, we have to be very careful while considering the direction of shear stress as we are calculating the parameter and trying to figure out whether the material is going to fail or not. So, first we will discuss the basic thinking as to what we are trying to state. So, let us write down the relation for failure or you know for the failure Criteria.

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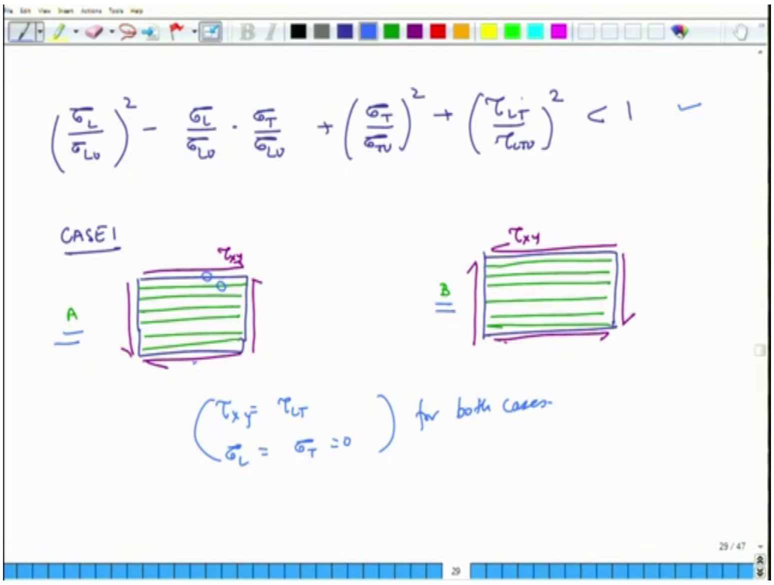

So, it is sigma L divided by sigma L u prime minus sigma L divided by sigma L u times sigma T divided by sigma L u plus sigma T divided by sigma T u whole square plus tau

L T divided by tau L T u and this number as long as it is less than 1, the material is not going to fail the material is not going to fail.

Now, let us consider 2 scenarios and these are simple scenarios, but that will make the things clear, case 1 in the first case we have 2 situations. So, this is situation A and there is another situation B. So, in the first situation the fibres are oriented like this and in the second situation also the fibres are oriented in the same way.

So, in both the cases fibres are oriented in the 0 direction, the only difference in these 2 situations is that in the first case I am applying my shear stress in a positive way, so this is positive shear stress. So, this is tau xy is positive and in the other case the shear stress is negative.

Now, in both these situations physically when you look at it what is the shear stress trying to do, the shear stress on the top surface is trying to remove the top layer with respect to the layer just below it right. This top layer it is trying to remove this top layer with respect to just the layer just below it that is what the shear stress is trying to do same thing this guy this guy is also trying to do.

So, we so this is what it is trying to do and the same thing is happening in the other situation and the resistance from the material because of it is stiffness is going to be the same. So, in so in case a in case in the first case this situation and this situation will not give us any different results ok, they will both fail at the same value of tau x y.

Look at the relation for Sahil, if I look at both these cases sigma L is 0 sigma T is 0, the only thing which is nonzero is tau L T because, tau x y is same as tau L T right tau x y is equal to tau L T and sigma L is equal to sigma T is equal to 0 for both cases. So, what that means, is whether the shear stress is positive or negative if I the failure shear stress will be same for both the situations, because I have a square term here so that right.

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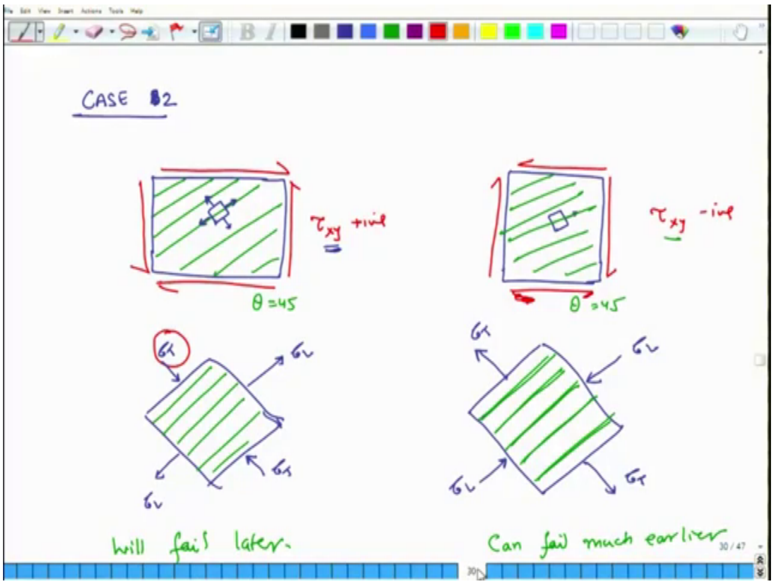

So, it does not matter, next look at case B and this is where you will see the difference case B. So, here also we have oh I am sorry it is not case B case 2 and here we have 2 situations in the first situation we have fibres oriented at 45 degrees and in other situation also fibres are oriented at 45 degrees, in both the situations fibres are oriented at 45 degrees.

So, theta is equal to 45 theta is equal to 45 in the first situation we are applying a positive shear stress, in the first situation we are applying a positive shear stress and in the second situation we are applying a negative shear stress. So, here tau xy is positive here tau xy is negative tau x y is positive and tau x y is negative.

Now, in this both these cases, I can also resolve these shear stresses in the L T plane and I can redraw these pictures in the L T plane. So, here the shear stress is in the y direction in the x y plane I can resolve this and I can make another picture where I depict instead of tau x y I depict sigma L sigma T and tau L T and I can actually calculate this values by the those as the transformation relations, but physically also we can see what is going to happen.

So, if I have to make an L T plane then the picture will look something like this, so this is my direction of fibre. So, essentially what I am doing is I am taking a small cross section of this and I am cutting it like this and I am measuring trying to calculate; what are the stresses in these directions this is. So, if is the same sample, but it is just a different way of looking at things.

So, if that is the case then what I see is that the component of this force tau x y, on this plane it will be in this direction and the component and of course on the negative side it will be on this direction similarly the component of the shear on the top, it will be what in this direction it will be in the compression direction and it will be like this.

So, this will be sigma L and this will be sigma T, in the second case when tau x y is negative we do the same thing. So, again we cut a small portion and see what kind of stresses are in the L and T directions and let us see how they look like just based on and we can actually calculate using the relations we have developed, but we do not need those relations.

So, so this is the direction of fibre and when you look at the tau shear stress on the positive for surface, shear stress on the positive surface is going in the negative x in the in the minus y. So, it is component is going to be compressive in nature and on the opposite side on me same thing will happen right. Similarly the top surface it will generate a tensile stress on these 2 surfaces. So, here sigma L is compressive and sigma T is tensile.

Now, when you look at these 2 pictures what is what is it that you see which 1 is going to fail earlier here because, I am pulling in the tensile direction the chances see here I am compressing in the in the transverse direction. So, the chances that it will fail very early is very less, but here because, the fibres are like this and I am in the transverse direction I am pulling and we know that failure in the transverse direction in the tension because of tension is very less.

So, this can fail much earlier and the first situation will fail later because, of the role of sigma T, because typically sigma T u is much much smaller than sigma T u prime. Let us look at some of the numbers which we had cited earlier for instance we and 1 example we did we said that For instance in this composite look at this sigma sigma T u prime is 600 and 10 MPA, but sigma T u is extremely small 72.

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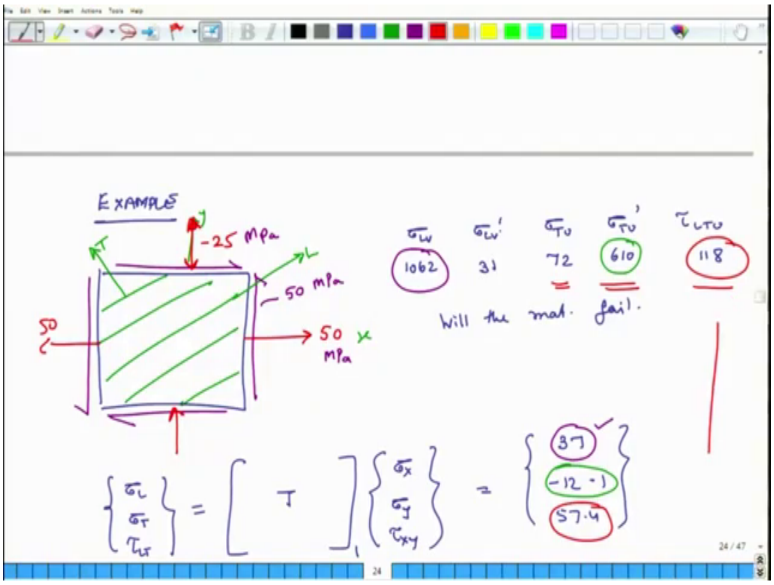

So, the point what I am trying to make is that in the transverse direction the tensile strength is extremely small; so the strengths if the shear if the direction of shear changes from positive to negative, it can have a very significant impact on the strength of the material. So, we will be make this thing a little more clear by doing an example and these effects of the direction of shear become very important, especially when theta is not equal to 0 degrees for theta equals 0 as we discussed earlier it does not really matter.

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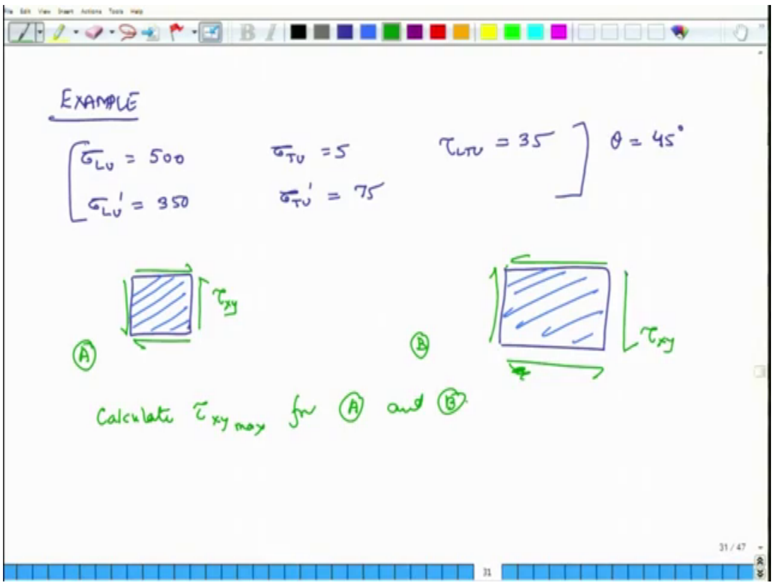

But when theta is nonzero then it starts having a very significant influence. So, the example we are going to say is that sigma L u is 500 sigma T u is 5 tau L T u is 35 sigma L u prime is 30 350 and sigma T u prime is 75.

So, these are the material properties and the question is and theta for purposes of simplicity is 45 degrees and so we have a composite specimen like this the fibres are oriented at 45 degrees angle to the x axis and all the only load on the material is tau x y tau x y is the only load. So, this is case a and in the other case.

So here the shear stress is negative, so the question is calculate tau x y max for cases A and case B, what is the maximum shear stress this kind of a material can handle.

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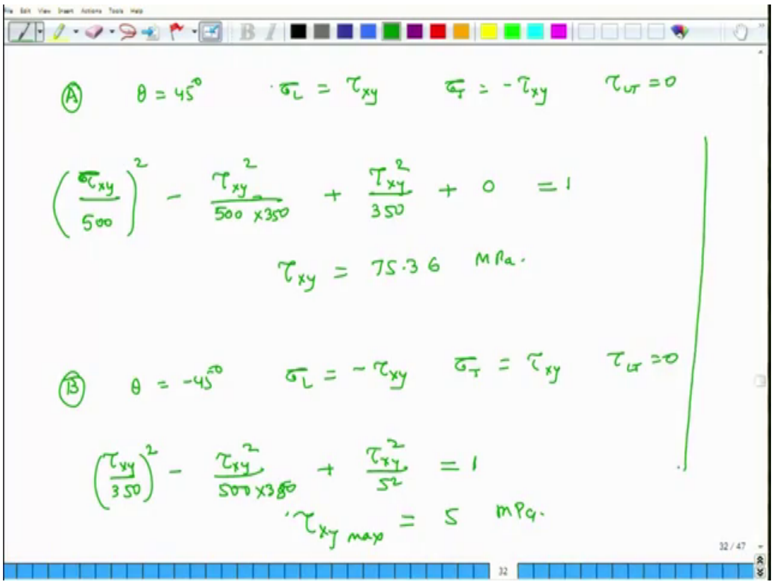

So, first we will do case a so what we have to do is we have to first calculate in terms of tau x y sigma L sigma T and tau L T. So, case a theta is equal to 45 degrees and; so with that sigma L is equal to tau x y right because, it is tau x y cosine square theta and this is sigma T is equal to minus tau x y and tau L T is 0.

So, you do the calculation sigma L sigma L is tau x y divided by 500 square minus tau x y square divided by 500 times 350 plus tau x y square divided by sigma T u square, which is 350 plus 0 and for failure this should be equal to 1, so tau x y works out to 75.36 MPA.

So, this is for case 1 now in case B theta is equal to minus 45 degrees. So, sigma L is equal to minus tau x y sigma T is equal to plus tau x y and tau L T is 0. So, we again do that thing, so tau x y by so instead of 500. Now I have 350 square minus tau x y square divided by 500 times 350 plus tau x y square and this 1 makes a really big difference divided by 5 square and this for failure should be 1. So, tau x y max is equal to 5 MPA.

Student: Theta (Refer Time: 18:03).

Theta is minus 45 degrees or you can say theta is same, but tau x y is negative of tau x y it is the same thing. So, you get 5 MPA so the point what I am trying to make is that just by change of the direction of shear it can have a very significant impact.

So, we have to be really careful in understanding; what is the exact direction of shear because, intuitively we do understand tension is positive comprehension is negative, but shear we do not differentiate the direction of shear a lot of times, but in this case the direction of shear is extremely important.

So, this concludes our discussion for this week and also whatever we have tried to cover we have covered it starting next week, we will develop an approach. So, as to how to analyze the strength and failure of composite materials with several layers, but till so far we have been dealing with only a single layer next week we will start dealing with several layers.

So, that concludes our discussion and I look forward to seeing you next week at the same time best of the weekend and have a great weekend. Bye.