Introduction to Composites
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Lecture – 58
Stress Strain Relations for A Lamina With Arbitrary Orientation Part II

Hello welcome to introduction to composites, in this week which is the tenth week of the course today is the fourth day in particular and what we plan to do today is continue the discussion which we had yesterday. Close that discussion and then move on to a new topic which is about failure of composite materials specifically when individual layers of composites especially orthotropic sheets, when they are loaded in a plane stress state and when they fail then how can we predict their failure, but before that we want to complete the point of discussion which we had earlier.

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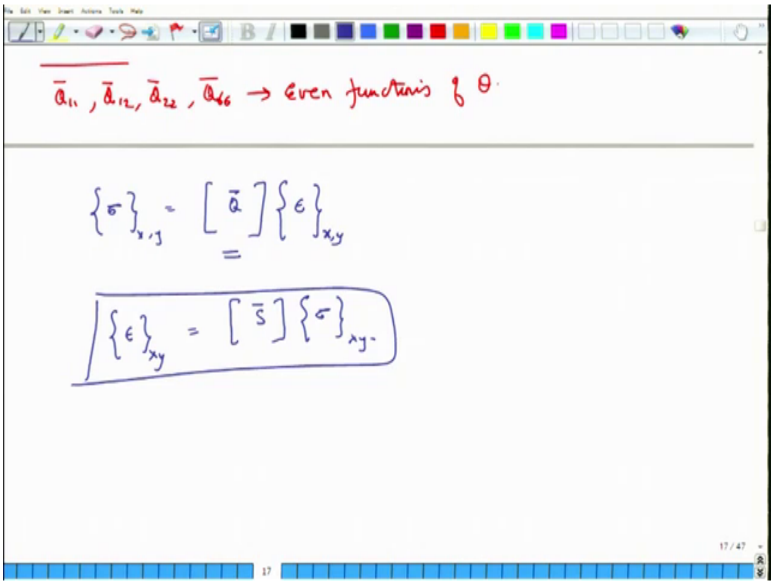

So, what we have shown is that we can now calculate sigma x you know when you measure stress in with respect to x and y plane. So, that is equal to a matrix Q bar times a strain vector measured with respect to x axis and we know how to calculate all the elements of Q bar matrix.

Similarly, we can also calculate strain in terms of a S bar matrix and the stress vector sigma x y and how do we do that we actually just to exactly all do all the steps which we have discussed earlier you know.

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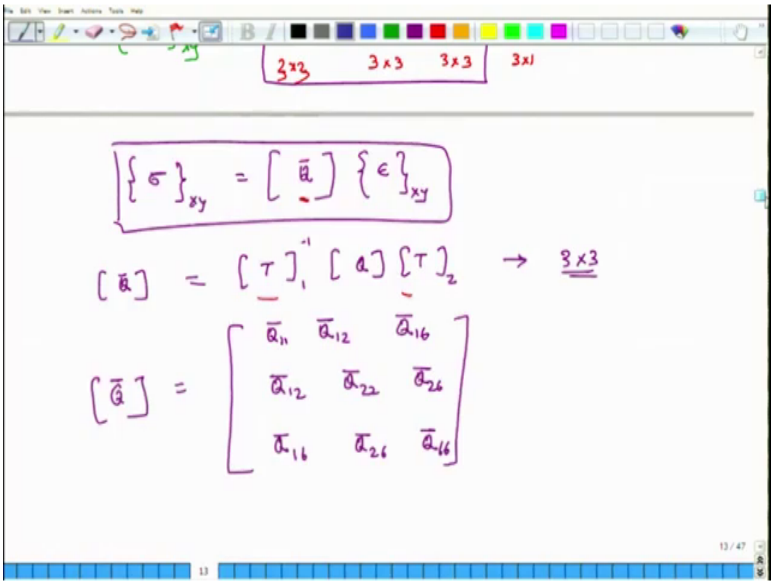

So, we convert all these in terms of L and T and then we figure out and using that we can calculate the elements of this S bar matrix also the process is exactly the same. So, my suggestion would be that you please go and actually do all these steps because it will be useful for you to understand. What I will do is I will now give you the relations of all the elements of S bar matrix in terms of S matrix.

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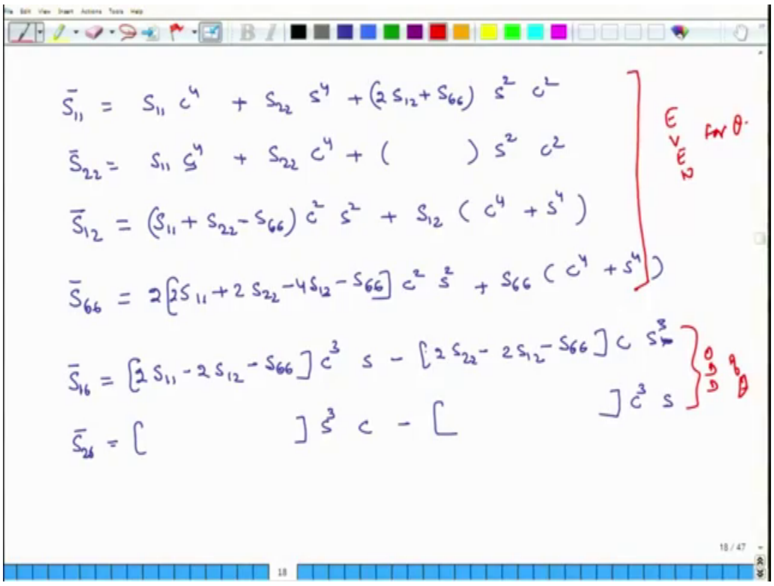

So, S 1 1 bar and I am just going to give this directly. So, please go back and do this so that you are able to show for yourself that these relations are indeed correct and you are convinced of the results. So, this is equal to S 1 1 times cosine 4 plus S 2 2 times sin 4 plus 2 S 1 2, plus S 6 6 times sin square theta cosine square theta ok. S 2 2 bar equals S 1 1 times sin 4 theta plus S 2 2 times cos 4 theta plus the same term times sin square times cosine square. Then we have S 1 2 bar this is equal to S 1 1 plus S 2 2 minus S 6 6 times cosine square theta, sin square theta plus S 1 2 cosine 4 plus sin 4 theta and then S 6 6 bar equals 2 S 1 1 plus 2 S 2 2 minus 4 S 1 2 minus S 6 6, so this is like this.

So, it is twice of 2 S one one plus 2 S 2 2 minus 4 S 1 2 minus S 6 6 times cosine square theta, sin square theta plus S 6 6 cosine 4 theta plus sin 4 theta and then S 1 6 bar equals 2 S 1 1 minus 2 S 1 2 minus S 6 6, cos cube times sin minus 2 S 2 2 minus 2 S 1 2 minus S 6 6 times cos times sin cube. And S 2 6 bar equals this entire thing, but this is sin cube times cosine minus this entire thing, times cosine cubed times sin.

So, once again we see that the first 4 elements are even functions for theta and these 2 guys are odd functions of theta ok. So, S 1 1 S 2 2, S 6 6 and S 1 2 all these bars if I change from theta to negative theta the value does not change. But S 1 6 bar and S 2 6 bar their values swaps from positive to negative if I make theta to negative theta. So, this is very important to understand and we clear about.

Next what we will do is we will do an example.

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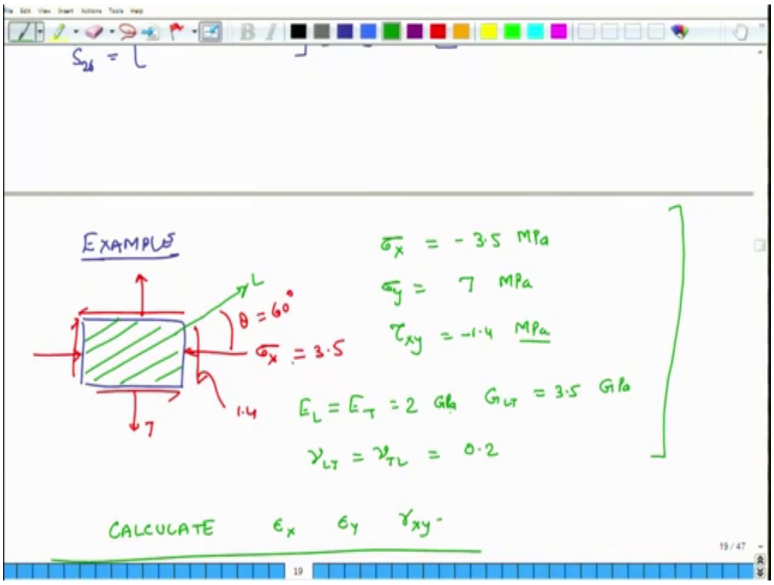

So, what do you want to do here. So, we have a unidirectional lamina and I am applying some so it is the fibers are in this direction ok. So, this is my L direction and I am applying sigma x. So, actually I am compressing it I am not pulling it in tension. So, I am applying a sigma x which is compressive and that equals sigma x is 3.5 m p a and then I am also pulling it in the transverse direction. So, that is the tensile load and that value is 7 m p a and then finally, there is also a shear stress and the shear value is 1.4 m p a and this angle theta is 60 degrees.

So, remember we always measure with respect to x axis going counter clockwise is positive theta. So, theta is 60 degrees ok. So, just to be clear sigma x is equal to minus 3.5 m p a, sigma y is tensile. So, it is 7 m p a and tau x y because it is in the negative direction positive direction we have defined earlier also. So, that its value is 1.4 m p a and now what we will do is we will also give you the material properties E L is equal to E T and that equals to G L T is 3.5. So, this is all G p a.

So, tau x y is negative here and the Poisson's ratio nu L T is same as nu T l. So, looks like this is a balanced lamina and this is equal to 0.2. So, these are the data and the question is calculate we have to find strains in the x direction. So, epsilon x, epsilon y and gamma x y these are the strains we have to calculate. So, there are several ways we can do this what we will do here is we will first compute sigma L sigma T and tau L T I mean we can directly do the computation.

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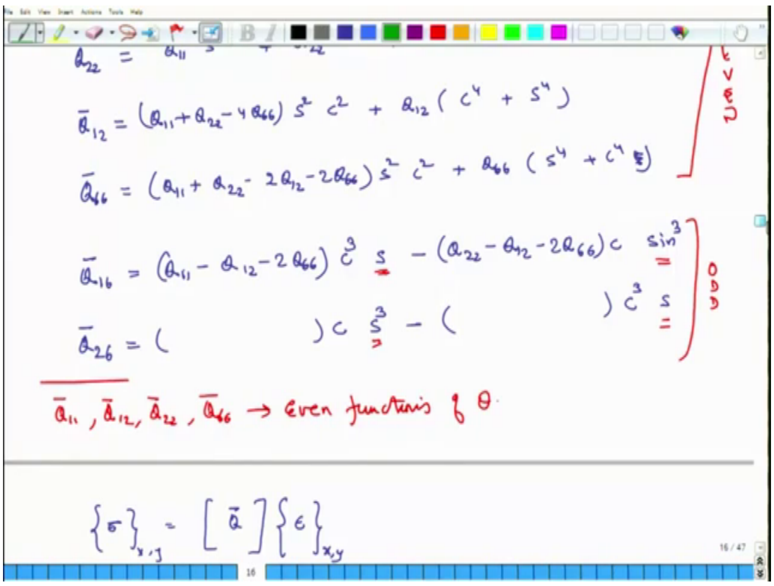

Because we know sigma x, sigma y and tau x y one way to do the calculation is we directly compute first the Qs using these relations. So, first we calculate Q 1 1, Q 2 2, Q 6 6 from those we calculate Q 1 1 bar, Q 2 2 bar and Q 6 all these bars and once we have all this.

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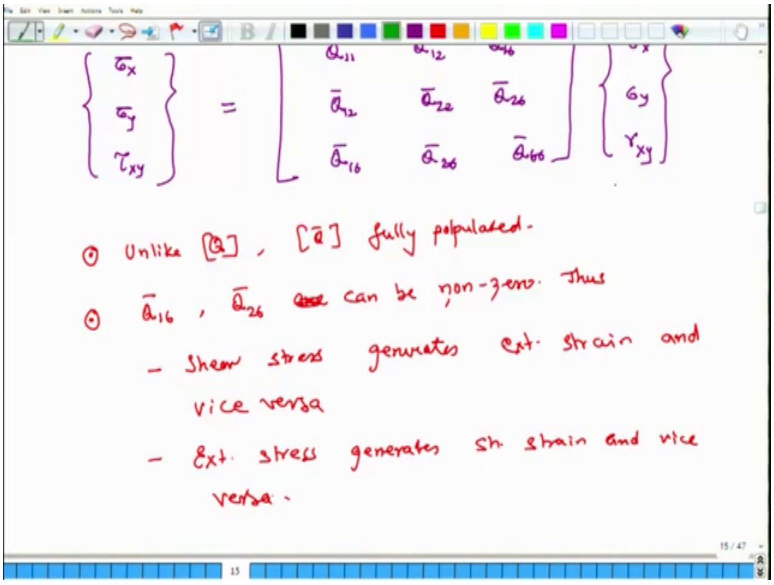

Then we use this relation and take the inverse of this relation and we can calculate sigma x, sigma y and tau x y, but we will use a slightly different method today.

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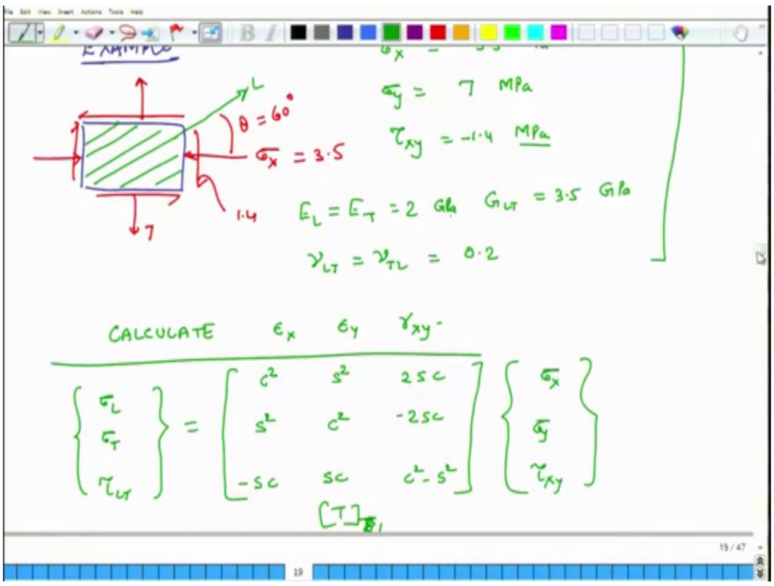

So, what we will do is. first we will calculate stresses with respect to L and T coordinate system ok. So, sigma L sigma T tau L T is equal to. So, this is the relation we had talked about earlier, this is equal to sigma x, sigma y tau x y and this matrix is what the transformation transformation matrix T 1 and what is the what are the coefficients for this transform I mean the members of this transformation matrix, cosine square theta. Sin square theta 2 sin theta cosine theta minus 2 sin theta cosine theta cosine square theta minus sin square theta, this is cosine square theta, this is sin square theta and these guys are minus sin theta cosine theta and this is sin theta cosine theta.

So, first step is we are computing sigma L, sigma T tau L t, so if we.

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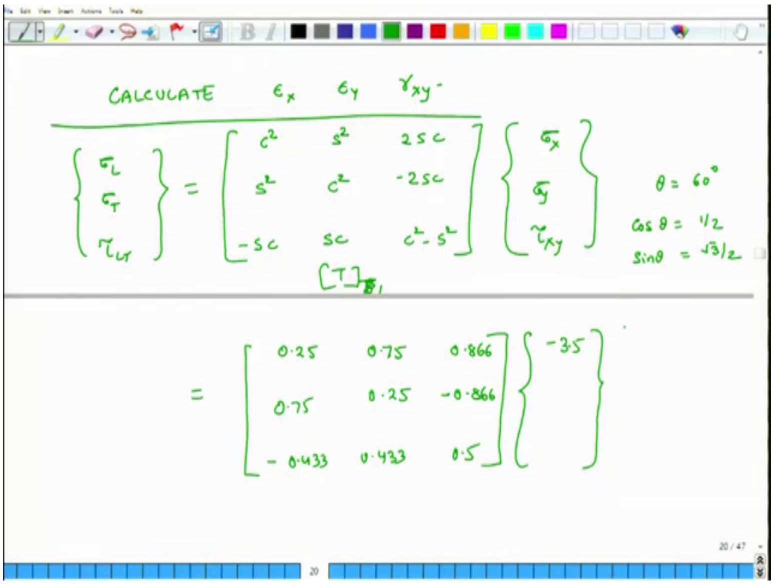

So, what is theta? Theta is equal to 60 degrees. So, cosine theta is how much? Cosine theta is.

Student: (Refer Time: 11:29).

Half sin theta is.

Student: Root 3.

Root 3 by 2 ok. So, from this we find out the members of this matrix. So, what are those values? So, this is 0.25. So, I have 0.25, this is 0.75 and this is 0.866, 0.25 minus 0.866 and this is. So, cosine square theta minus sin square theta is cosine of 2 theta. So, it is will be cosine of 120 degrees right. So, this is 0.5 and this is 0.433, minus 0.433 and this is 0.75 times, times sigma x sigma x is how much negative 3.5, sigma y is 7 and this is minus 1.4 ok.

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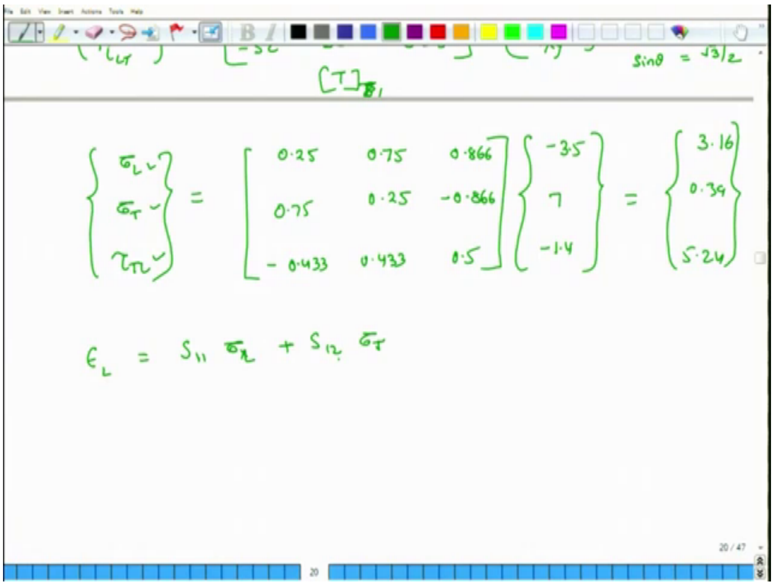

So, if you do all this then what you get is 37 minus 12.05 and 57.4 this is 5.24. So, these numbers are incorrect. So, this is 3, 16 this is 0.39 and this is 5.24 ok. So, now, that we have. So, these are what these are sigma L, sigma T and tau L T or tau T l.

Next what we do is we compute, now that we know this epsilon L epsilon T and tau L t. So, what do we do, we know that epsilon x no epsilon L equals S 1 1 sigma sigma L plus S 1 2 sigma T. S 1 6 is 0 right and S 1 1 and S 1 2 we have already shown how to calculate u from engineering constants. So, from these engineering constants we can calculate S 1 1 and S 1 2.

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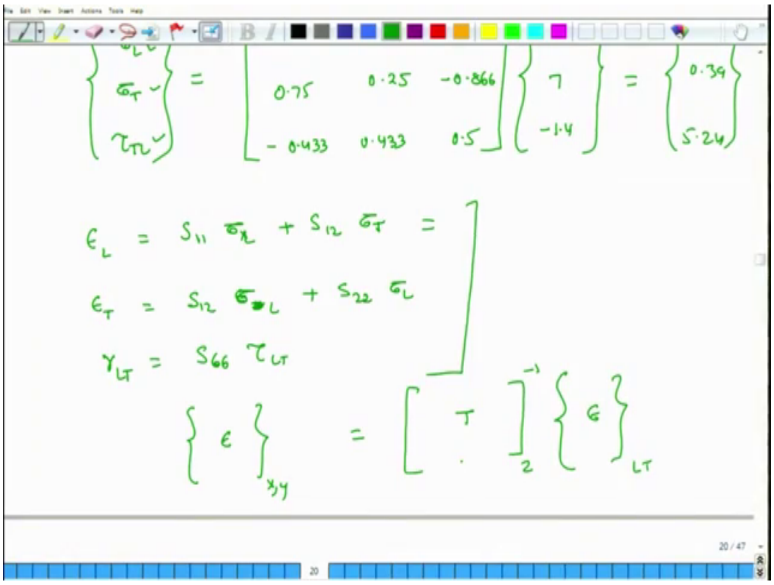

So, this works out to be something similarly epsilon T is equal to S 1 2 times sigma T no excuse me sigma L plus S 2 2 sigma L and gamma L T is equal to S 6 6 tau L t. So, we can calculate these things from here.

And once we have computed epsilon L, epsilon T and gamma L T then from these we calculate epsilon x y using the in the x y coordinate system. What is that? it is the transformation matrix to its inverse times strains measured with respect to L T coordinate system ok.

Now, I will give you a trick to how to very quickly compute the inverse of these transformation matrices which are dependent on theta. So, suppose you want to calculate the inverse transform of inverse of this first transformation matrix T 1, effectively what; that means, is that you replace theta by minus theta and that will be will give you the inverse of it. So, the inverse of T 2 when theta is equal to theta is nothing, but you calculate theta by replacing it by minus theta. So, you put minus 60 degrees and you will be able to calculate the inverse of that and that will very quickly give you the solution.

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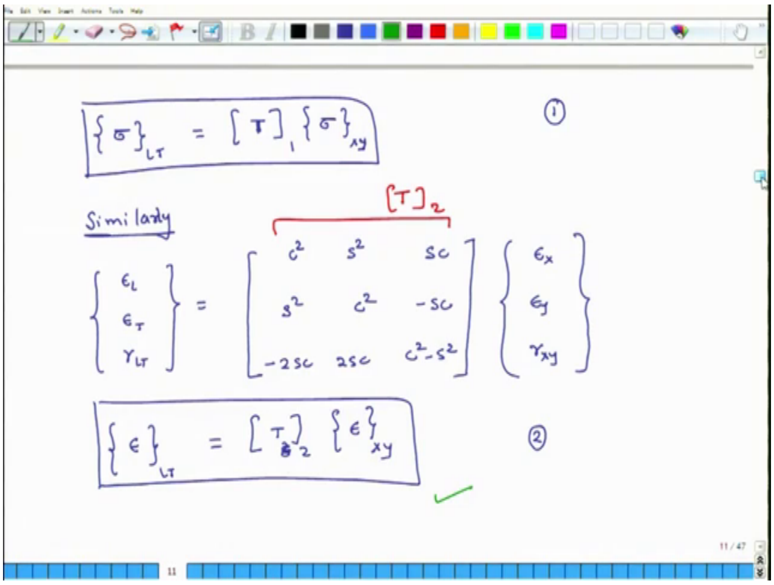

So, if you do all that mathematics and what is the, what are the elements in the T 2 matrix we had shown earlier. So, here if you replace theta by minus theta what are the terms which are going to change, this will go from positive to negative, this will go from negative to positive, this will go from positive to negative this will go from negative to positive all other things are going to remain the same ok. So, you do those replacements and. So, that will give you inverse of T 2 and then you multiply that by epsilon L T which we have know how to calculate and then you get epsilon x epsilon y epsilon and gamma x y.

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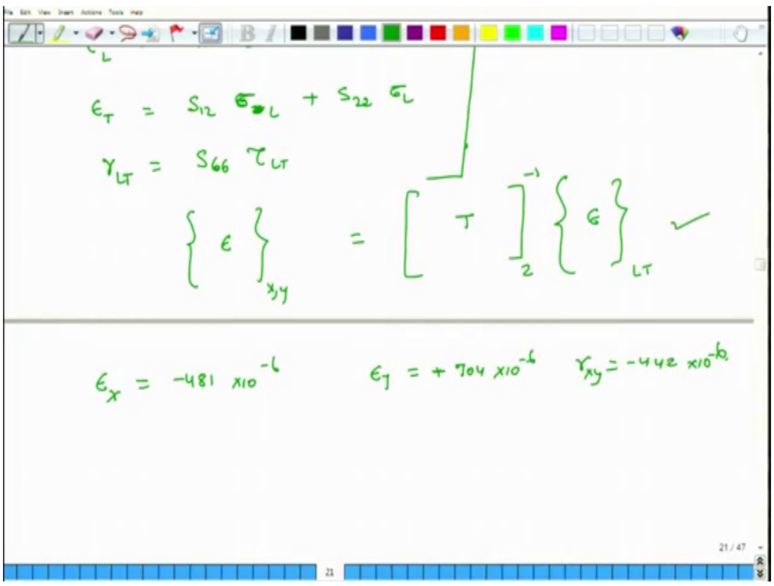

So, epsilon x it works out to be minus 481 into 10 to the power of minus 6 epsilon y equals minus no its actually plus 704 times 10 to the power of minus 6 and gamma x y is equal to minus 442 times 10 to the power of minus 6 ok.

So, this concludes our discussion for today and this essentially had tells us how we can compute strains and stresses in a system even if it is not, even if the direction of stresses are not necessary aligned with the material axis of the system. So, first thing is we from in terms of engineering constants we calculate the Q matrix and the S matrix and from that Q matrix and S matrix we can calculate Q bar and S bar matrices and once we have those matrices in place then we can relate strains and stresses in the way we want them to connect.

So, this is the conclusion of our today's lecture we will start a new topic tomorrow and that is about failure of unidirectional lamina and different theories associated with failure of unidirectional lamina. So, with that I conclude our discussion and I look forward to seeing all of you tomorrow.

Thank you.