Introduction To Composites
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Lecture – 56
Relation Betweeen Engineering Constants and Elements of Stiffness and Compliance Matrices- Part II

Hello, welcome to Introduction to Composites MOOC course. Today is the second day of this week, which is the tenth week of this course. Yesterday, we had introduced relationships between engineering constants and members of the compliance matrix, which is the S matrix, the member, the engineering constants and the stiffness matrix elements and also the interrelationships or relationships between Q and S matrices. So, we will have a brief recap through an example and then we will move on to the next topic. So, we will do an example.

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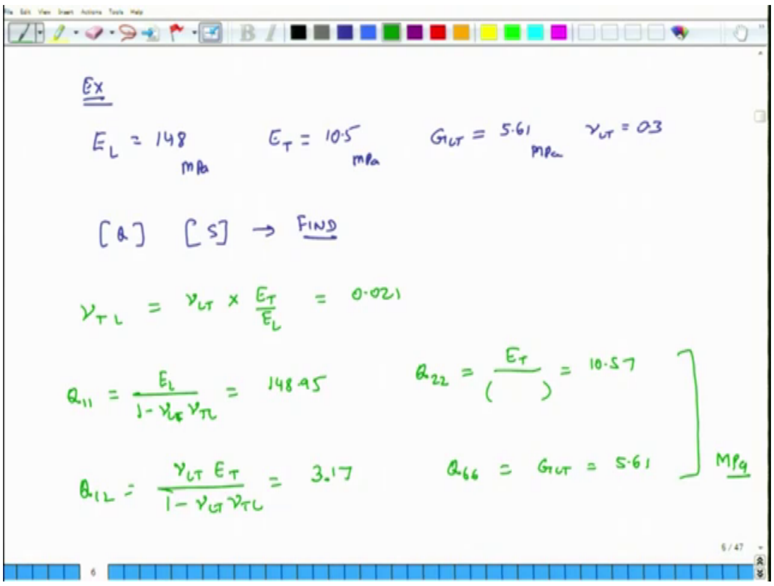

So, the example says that, we know all the engineering constants of the material. So, E L is 148 E T is 10.5 G L T is 5.61 and nu L T is 0.3 and all these are M P a of course, Poisson ratio is not in mega Pascals. It is a dimensionless number.

So, with this information, we have to find the Q and the S matrices. So, we have to find the stiffness matrix and the compliance matrix. So, that is what we want to do. So, the first thing is, we know nu L T and this nu T L gets, used a lot of times in these relations. So, nu T L equals nu L T times E T over E L, this is a relation we already know.

So, this works out to 0.021. Now, Q 1 1 equals E L divided by 1 minus nu L T, nu T L and that comes out to be 148.95 Q 2 2 equals E T divided by same thing and this is equal to 10.57 Q 1 2 equals nu L T E T divided by 1 minus nu L T nu T L and if you plug in all the numbers, you get 3.17 and finally, Q 66 is equal to G L T and G L T is already known.

So, that is 5.61 and the units of QS are M P a ok. Units of QS are M P as. So, all engineering constant do not have the same unit, because nu L T is dimensionless, but all QS, they have the same dimension and the unit, which is M P a then, next we do is S.

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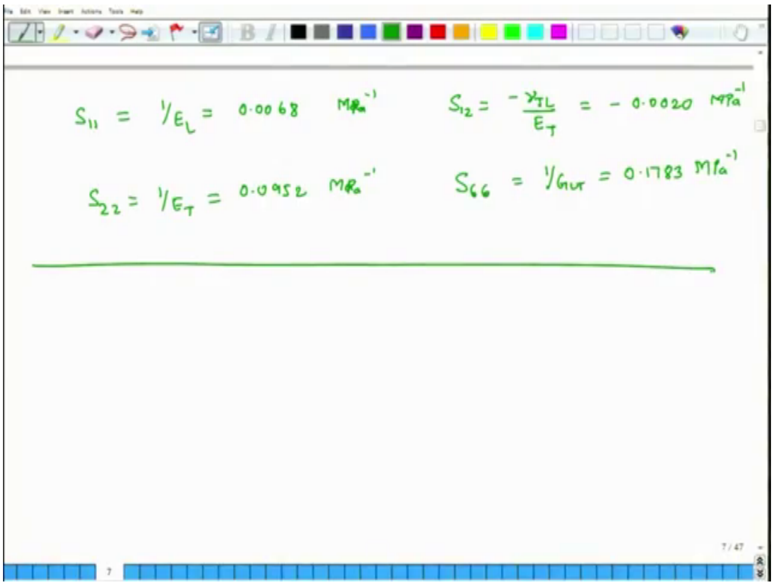

So, S 1 1 is equal to 1 over E L and that works out to be 0.0068 and this is M P a inverse, inverse of M P a S 1 2 is equal to minus nu T L divided by E T. So, remember there is a negative sign here, I do not know if in the last class I had shown that or not yeah.

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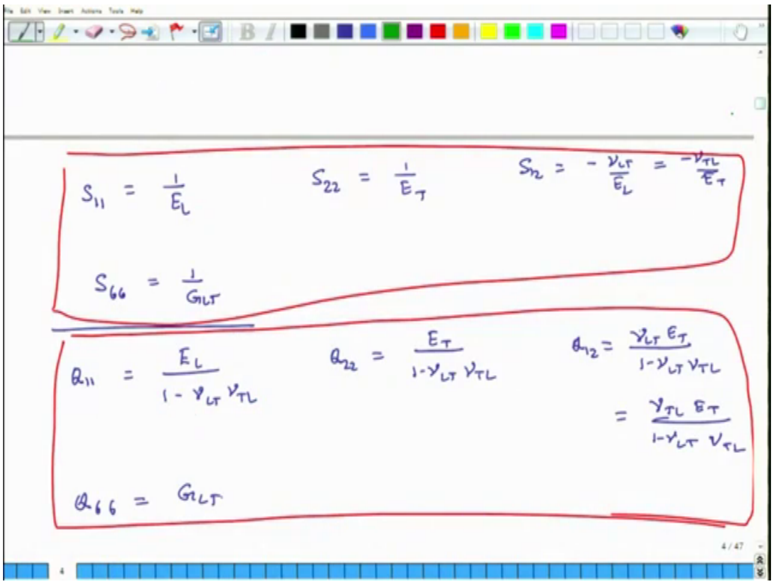

There is a negative sign I had mentioned here also; so, just wanted to reconfirm. So, it is negative of nu T L divided by E T and that works out to be minus 0.0020 and once again units are M P a inverse and then S 2 2 is 1 over E T and that is 0.0952 M P a inverse and S 6 6 is equal to 1 over G L T and that works out to be 0.1783 M P a inverse. So, once again QS and S S QS have a unit of M P a S S have a unit of M P a inverse E L E T G L T have a unit of M P a and nu L T is dimensionless. So, this is the conclusion on discussion for the how S Q and engineering constants are related to each other.

Finally, we will spend a few minutes on what kind of values, the engineering constants cannot have; so, there are some restrictions.

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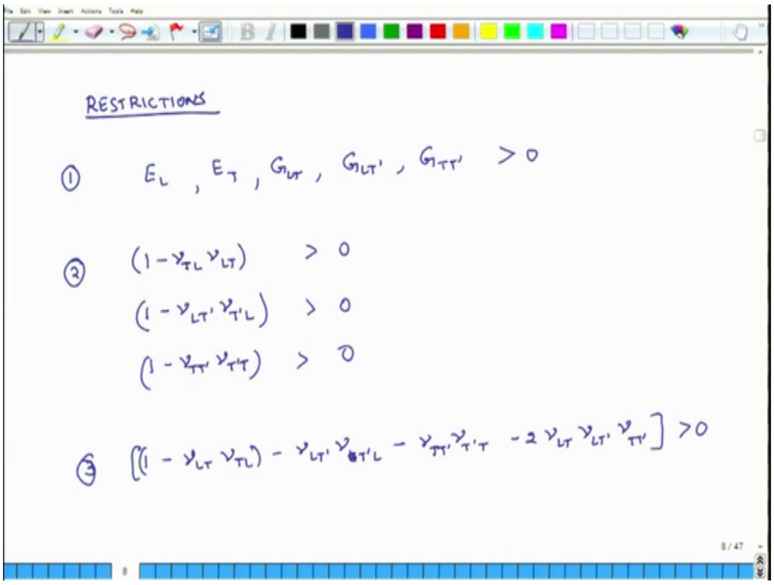

So, if you, in your calculations or in your experimental results come across certain values of E L, which violate the conditions. I am going to talk about now, then you should be sure that either they are experimental or result is wrong or your theoretical calculations are incorrect.

So, the first set of restrictions are that all the engineering constants, they have to be more than 0 elastic constants E L E T G L T G L T prime, G T T prime. So, all these guys individually, they have to be more than 0. They cannot be negative at all, just not possible; so mathematically and physically not possible.

The second set of relations are that these terms 1 minus nu T L times nu L T 1 minus nu L T prime times nu T prime L and 1 minus nu T T prime nu T prime T. So, these guys also should be individually 0, more than 0. They cannot be less than 0, it will violate some basic laws of geometry and physics. So, this cannot be negative.

And the final constraint is that 1 minus this is a little long equation nu L T nu T L minus nu L T prime nu L, excuse me, T prime L minus nu T T prime nu T prime T minus 2 nu L T nu L T prime nu T T prime. So, this long expression should be more than 0. So, I have not gone into the mathematics of all these details, but it is important that when you are doing some experiments or calculations 2 composite materials.

Make sure that none of these conditions are violated, if your system is a unidirectional lamina and it is in the state of a plane stress. So, this is very important to understand. So, now, we move to the next topic for this, course and this is about transformation of Q and S matrices transformation of Q and S matrices. So, I will explain, what does all this mean.

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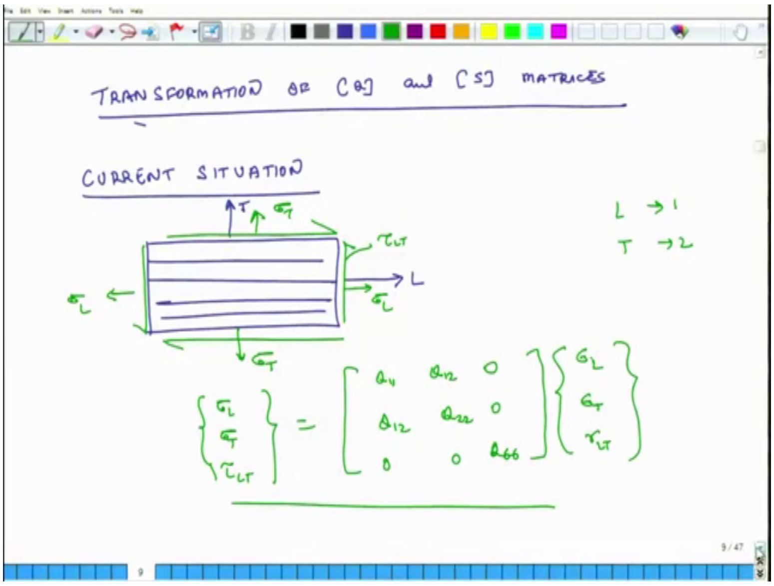

So here is the currents, current situation. So, in whatever we have learnt till so, far is that if I have a lamina, and let us say this lamina is unidirectional and I this is. So, this is my L direction, this is my T direction and let us say I have a sigma L on it, I also have, a transverse stress sigma T and I also have some shear stress.

So, this is tau L T then if I know the stresses and the engineering constants or the Q matrix of this material. I can calculate the strains in the system and vice versa and the relationship is sigma L sigma T tau L T equals this Q matrix. So, here I have used the terms L. So, L corresponds to 1 axis T corresponds to 2 axis right, that is, but here in the current situation the direction of loading and the direction of the, material axes, which in this case is L and T axes. They are aligned and if they are aligned then we can use these relations, if they are aligned then we can use these relations, but what happens, if they are not aligned?

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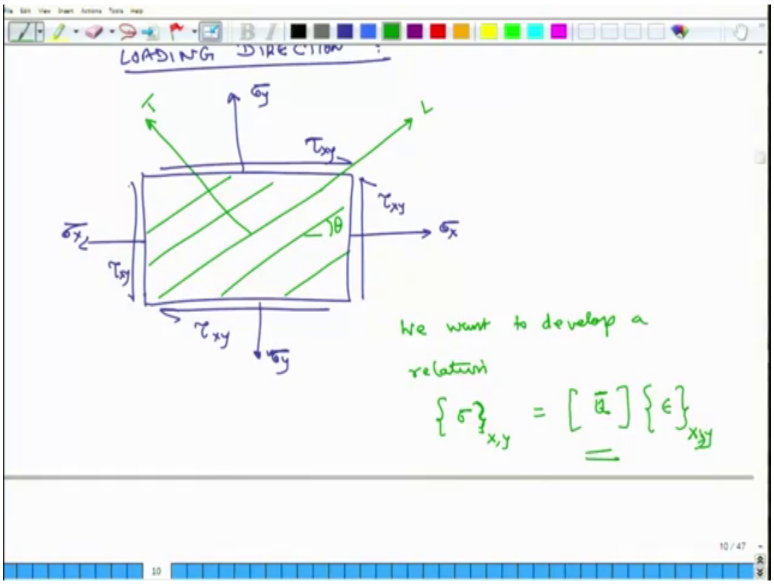

What if L T directions are not aligned and they are not aligned to loading direction, what if they are not aligned to the loading direction, that is the question. Then can we have the question is then can, we have a similar stress strain relationship and what will be the nature of QS in this case. We have Q 1 1, Q 1 2 0 Q 1 2 Q 2 2 0 and 0 0 Q 6 6.

So, what kind of stress strain relationship will be there and what will be the nature of QS, this is the question. So, I will just want to make this clearer. So, in this case the picture would be something like this. So, this is my X axis and in the X axis. I am applying sigma X, this is my Y axis. So, I am applying sigma Y sigma X sigma Y and I can also have shear stresses. So, I have all these things, but the direction of fiber is different.

So, this is the direction of fiber. So, here let us say, this is my L axis and this is my T axis. So, in this case the L and T direction is not aligned to sigma X direction, in the direction of the Y direction. Now, this equation is good only, the first set of equations, which we saw it is good only. When the alignment is there then only, we can write this equation, but in this situation, we cannot use this equation right away.

We may have to do some mathematical manipulations. So, that we can still use this, but this is what we want. So, what we want is, we want to develop a relation. What kind of a relation is that, we want to develop a relation, such that of this nature. So, here sigma is not measured with respect to L T axis, but with respect to X and Y axis and that is equal to some other matrix and we will not call it Q, but we will call it some Q bar, Q bar matrix, which is different than Q.

We do not know, what that nature is times, strain vector, which is again measured with respect to X Y coordinate system, because it is the X Y coordinate system. We are interested in, we do not care, what is the L T coordinate system ? The only thing, we have an additional thing is that the orientation of L T with respect to X Y coordinate is theta degrees. So, as theta keeps on changing, we have to figure out how would can, we relate Epsilon. The strains to the stresses in terms of this Q bar matrix, this is what we are interested in finding.

So, this is exactly, what we will start doing tomorrow and till then you please, review all the material, which we have covered till. So, far, because that will be useful also do a few questions from the book. You already have the details of the book and we will meet once again tomorrow and continue this discussion starting tomorrow.

Thank you.