Introduction to Composites

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Lecture - 54
Elastic Constants for Specially Orthotropic Materials in Plane Stress

Hello, welcome to Introduction to Composites. Today is the last day of this on-going week which is the 9th week of this course. We have been discussing the formulation stress-strain constitutive relationships for different types of materials and we started with anisotropic materials and we found that it has 21 independent elastic constants. Especially orthotropic material in 3-D has 9 material which is transversely isotropic that is its properties in a particular plane are isotropic such material has 5 elastic constants.

And today we will discuss another category of material which is known as specially orthotropic in plane stress, in plane stress.

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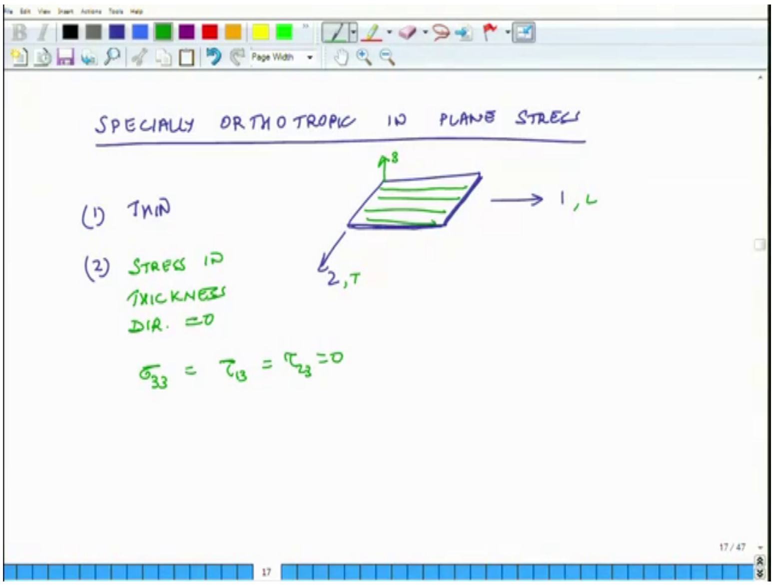

So, it is not just specially orthotropic, but it is in plane in a straight of plane stress. What is the characteristic of the material? First thing is that it is thin it is a thin thing thin material, second is it has play it is in a state of plane stress. And so suppose this is the material and because it is thin this dimension is not large suppose this is my loading direction 1 and this is 2 it is specially orthotropic. So, this is so because it is specially orthotropic let us say fibers are aligned with the 1 axis, ok.

So, 1 axis is same as the material axis L and this is same as that axis T. So, it is especially orthotropic. So, fibers are aligned with one direction it is thin and it is in plane stress. And what is the meaning of plane stress? That the stress in 3 direction in the thickness direction stresses in thickness direction are 0, which means sigma 33 is equal to sigma tau 13 is equal to tau 23 they are all 0.

Why is this kind of a material important? The reason we use we are considering this material is that a lot of times when we fabricate composites we make them using individual layers ok. We make them using individual layers and these composite layers are very thin and typically the stress in the z direction for them is very less or or 0, ok.

So, to understand the behaviour of such materials and we stack them up and then from each we makes take several such layers and ultimately we have a laminate. So, to understand the response of every single layer we have to understand how does a specially orthotropic material behave when it is in state of plane stress that is why we are trying to understand them.

So, let us see how many elastic constants such a material has. So, what do we do? We develop the stress strain relations for those.

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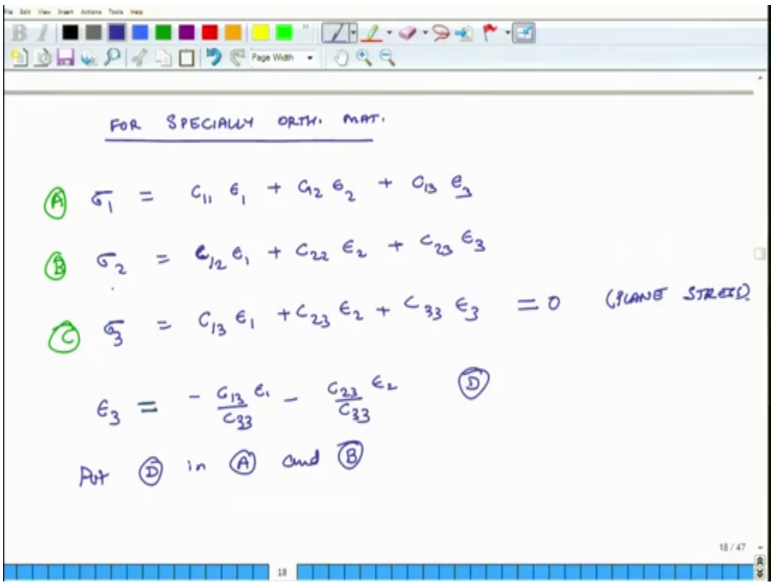

So, we know that for specially orthotropic material for specially orthotropic material which is not in which may or may not be in plane stress it has 9 constants right, it has 9 constants. So, first we do that and then we simplify it. So, sigma 11 is equal to C 11 epsilon 11 plus C 12 epsilon 22m excuse me 12 plus C 13 epsilon 13 C 14, 15, 16 they are not there because the material is a specially orthotropic it has only 9 constants, ok.

Sigma 22 equals C 12 epsilon 1 plus actually I have made a mistake in terms of notations because now we are no longer using tensor notation. So, sigma 1 is equal to this epsilon 2 epsilon 3 no shear strains sigma 2 is equal to C 12 epsilon 1 plus C 22 epsilon 2 plus C 23 epsilon 3 and sigma 3 sigma 3 is in our case it is 0 right, but if it were, but whether it was 0 or not the expression for this will be C 13 epsilon 1 plus C 23 epsilon 2 plus C 33 epsilon 3 and this is 0 for plane stress ok. This is 0 for the plane stress situation.

So, we can say, so let us say this is equation A B and C. So, from C we can see find calculate epsilon 3 we can calculate epsilon 3 epsilon 3 comes out as minus C 13 epsilon 1 by C 33 minus C 23 epsilon 2 by C 33, ok. So, this is equation D. And we put D in A and B we put equation D and A and B. So, remember how many equations are there should be 6 equations, right now we have written only equations for sigma 1 sigma 2 sigma 3 there 3 more equations for shear stresses tau 12 tau 23 tau 31 we will talk about those later but right now we are just considering the equations for extensional stresses ok.

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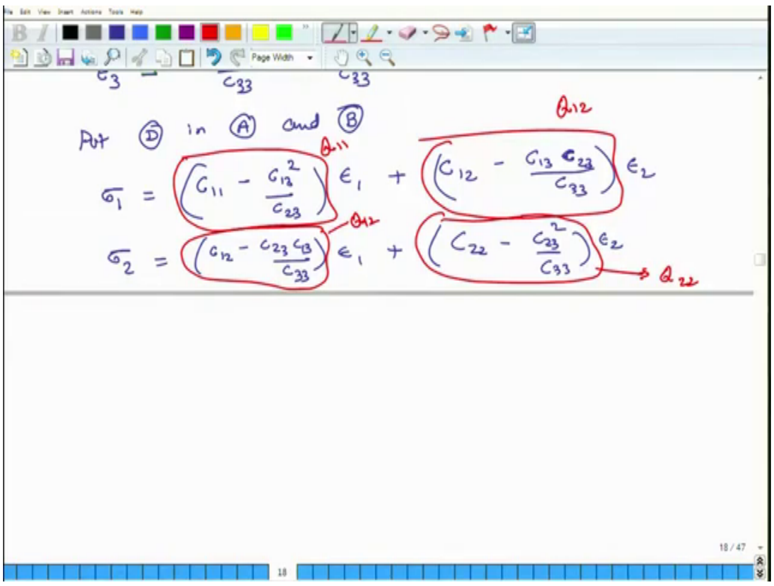

So, what we put D in A and B what we get is sigma 1 equals C 11 minus C 13 square by C 23 epsilon 1 plus C 12 minus C 13 C 23 by epsilon C 33 yeah epsilon 2 and C 2 equals

C 12 minus C 23 C 13 by C 33 epsilon 1 plus C 22 minus C 23 square by C 33 epsilon 2, ok.

We call this term as Q 1 we call a Q 11 we call this term as Q 12 this is again Q 12 and this term is called Q 22. So, we have developed expressions for sigma 1 and sigma 2 in terms of epsilon 1 and epsilon 2. For the plane stress state because we have imposed the condition of epsilon 3 you know sigma 3 being 0, ok.

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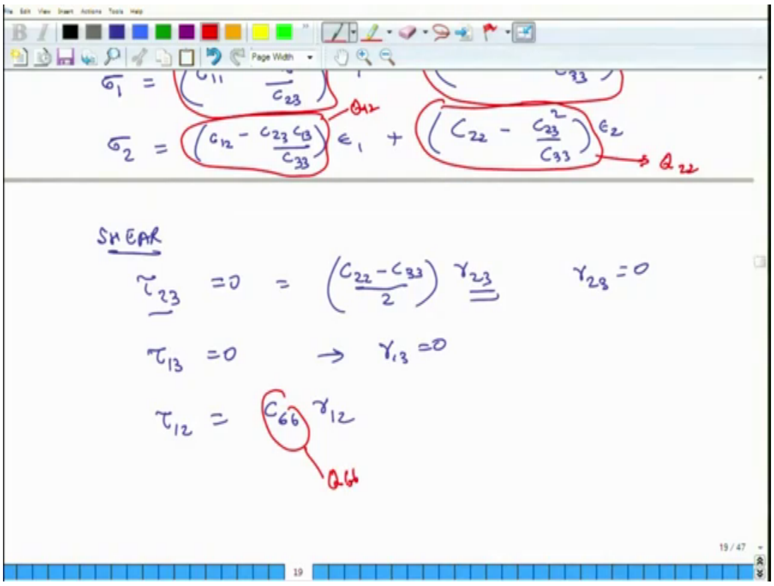

Next, look at shear stresses. So, for shear it is pretty simple sigma 23, no sigma 23 actually sorry this is the shear. So, I will use the term tau 23 equals 0 and tau 23 is what is equal to C 22 minus C 33 divided by 2 times gamma 23, ok.

Now, what this means is if this left side is 0 then there should not be any shear strain. So, which; that means, is gamma 23 is 0, not that C 22 minus C 33 by 2 is 0 that can be anything, but if I am not applying shear strain in the 23 plane then it will not an generate any shear strain. So, gamma 23 is 0. Similarly, tau 13 is equal to 0, tau 13 is 0 and that implies gamma 13 equals 0 and lastly tau 12 is equal to C 66 gamma 12 ok.

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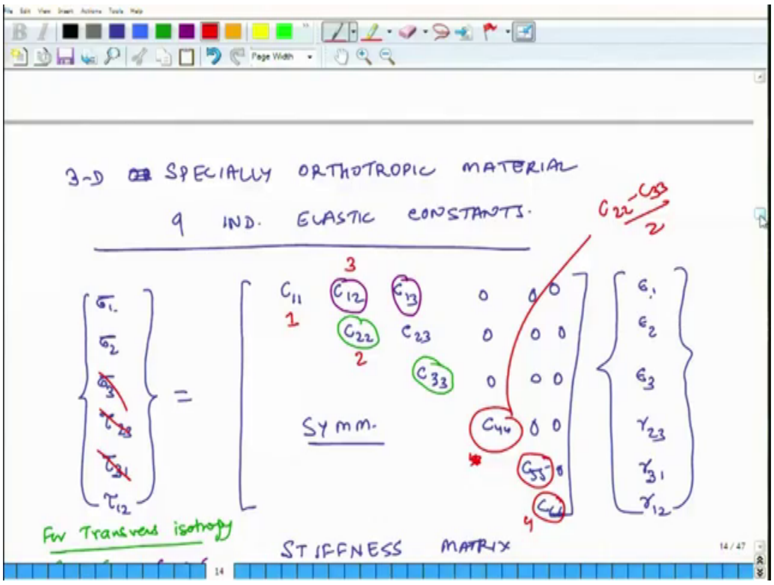

In general the full scale special orthotropic material we have shown, see this was these were the equations for full scale specially orthotropic material it had 9 constants, 6 equations, but because of plane stress condition these terms do not exist. So, 3 equations go away right and we are left with 3 equations only. So, the equations for a special orthotropic material in plane stress.

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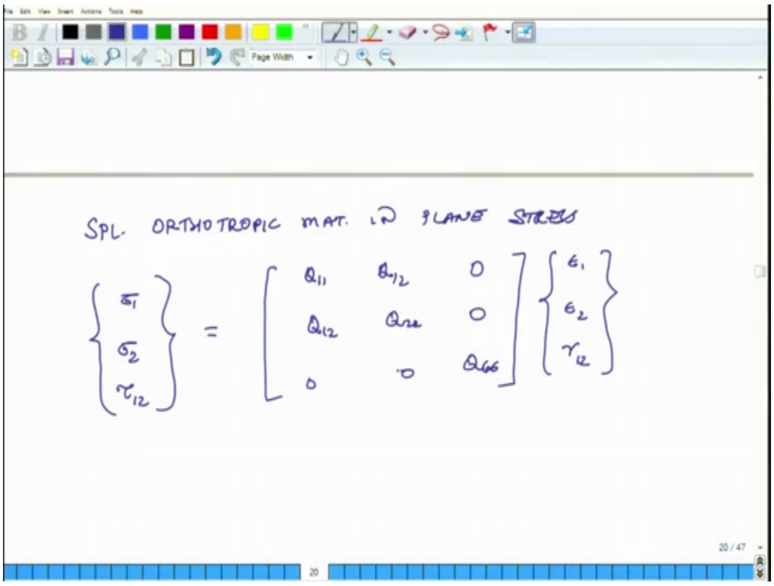

What are those equations? We have only 3 equations sigma 1, sigma 2 and tau 12 ok. And what are the 3 strains? Epsilon 1, epsilon 2 and gamma 12 ok. And the elastic constants are Q 11 Q 12 0, Q 12 Q 22 0 and Q 66 where oh, so I forgot this term C 66 we call Q 66. So, we have 3 elastic constants Q 11 Q 22 Q 12 and the 4th elastic constants is Q 66. So, these are the relations for especially orthotropic materials, ok.

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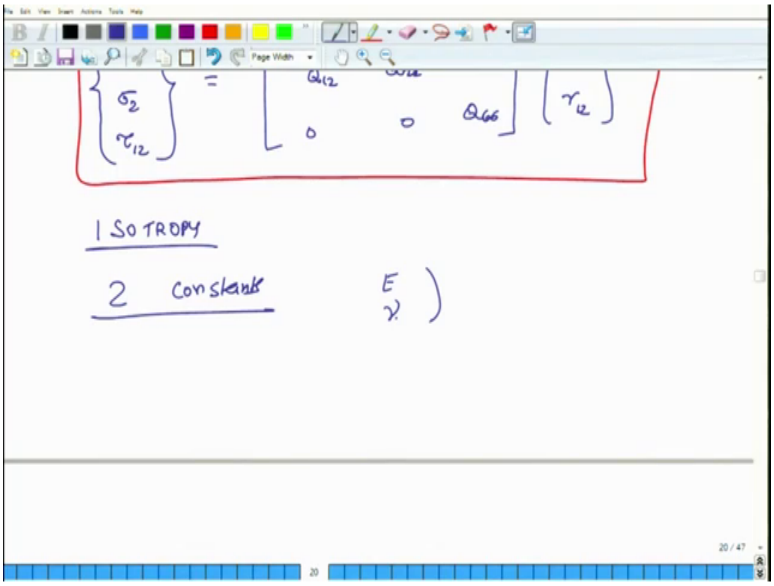

And then finally, for isotropy we have two constants and we will not discuss this because we are short on time and the focus of this course is on orthotropic materials. So, we will not talk about it. But in general E and mu are considered as 2 independent constants all other constants shear modulus bulk modulus can be expressed on terms of E and mu.

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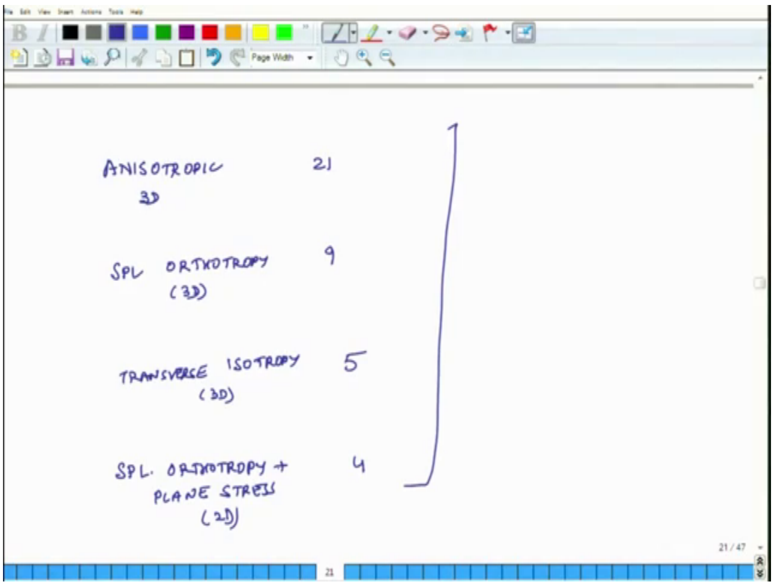

So, let us have a quick summary anisotropic materials 3-D materials we are talking about 21 constants. Then we have a special orthotropy again in 3-D, not in 2-D, we have 9 constants. Then we have in-plane isotropy. So, the material can still be thick it will can still be thick, so it is a 3-D material we have 4 constants. And then we have special orthotropy plus plane stress what that means, is that material is thin. So, this is a 2 dimensional material its very thin form a constants and no I think I this one, I made a mistake this is 5 in-plane isotropy transverse isotropy that is a more precise word transverse isotropy.

So, this is the overall picture. And what we will do tomorrow next week is we will continue this discussion maybe in the first lecture we will close this discussion on different type of materials in 1 or 2 lectures, and then we will start developing a theory for figuring out how do laminates made up of is you know either transversely isotropic materials or especially orthotropic materials which are in-plane stress, how do they behave when we attach several layers of them on top of each other. Then the how the overall plate behaves the overall composite laminate behaves.

So that concludes our discussion for today. We will meet once again next week till then have a great day and a nice weekend. Bye.