Introduction to Composites
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Lecture - 53
Elastic Constants for Specially Orthotropic Materials

Hello, welcome to Introduction to Composites. Today is the 5th day of the on-going week, which is the 9th week of this course. And we have been discussing over this week about generalized Hooke's law and what we have shown is that for a fully anisotropic material we require 21 independent elastic constants, because the total number of constants initially we have calculated was 81, because of stress symmetry it came down to 54 because of strain symmetry it came down to 36. And then for the strain energy consideration reasons the number finally, came down and settled at 21.

Now, we will look at some special types of materials and see that for those types of materials, what is the total number of elastic constants? So, we will start with anisotropic material 21 constants and for special materials how does this number come down to.

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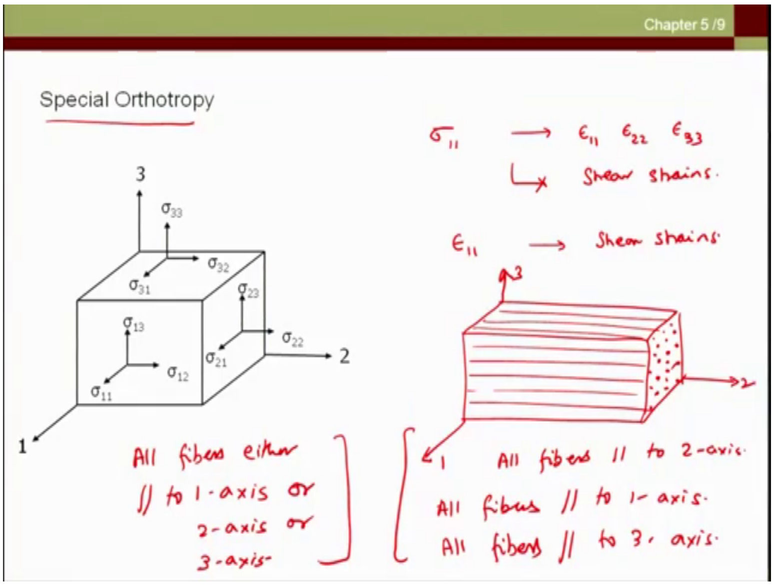

So, the first case we will look at is the case of special orthography ok. Now, what did we discuss in case of a special orthography? That if I apply an extensional stress sigma let us say I apply only sigma 11 then it will only generate epsilon 11, epsilon 22, epsilon 33, but it will not generate any shear strains ok.

So, only, so extensional stresses cause extensional strains and shear stresses cause shear strains similarly if I pull it in epsilon 11 directions only then it will generate only stresses in other directions, but it will not necessarily generate shear strains it will not generate shear strains ok. This is what it means. Physically why would that be? Possible suppose the material is a piece of wood ok, and let us say this is my 2 axis to be consistent with this picture this is my 3 axis and this is my 1 axis and suppose all fibers are parallel to 2 axis.

So, when I look at this picture and I then the fibers add this and they look like this we just see the tip of these fibers and the fibers run parallel to the 2 axis this. So, this is the physical structure and if I pull such a structure in 2 direction if I pull such a structure in 2 direction and I only I am pulling it in the length direction there is no need that we should expect that it should deform in a shear right, physically intuitively that is what we would expect and that is exactly what we see in actual experiments. So, if fibers are aligned the material axis is aligned to the 2 axis then I will not see any coupling between extensional strains stresses and shear strains and shear stresses and extensional strains related to the two direction.

Another scenario could be, another scenario could be that all fibers are parallel to 1 axis then also the coupling between shear and extension will not exist. And another case could be all fibers are parallel to 3 axis in that case also the same thing would be expected and then the 4th condition could be combination of these.

So, here all fibers either parallel to 1 axis or 2 axis or 3 axis. So, it can be, so you have some fibers going vertically up, another bunch of fibers could be going like this along parallel to the 2 axis, and the third set of fibers could be going perpendicular that is parallel to the 1 axis. But they are not at any angle they are only running either parallel to 1 axis or 2 axis or 3 axis they are not at any other angles. And even in this case if you pull a thing it will only become longer, it will not you know shear stresses will generate only shear strains extensional stresses will generate only extensional strains and that is all dependent on the structure of the material and the direction and if the direction of the forces is aligned to the material axis. Even in such complicated material system even then shear stresses will produce only shear strains extensional stresses will produce only extensional strains right. So, this is what it means.

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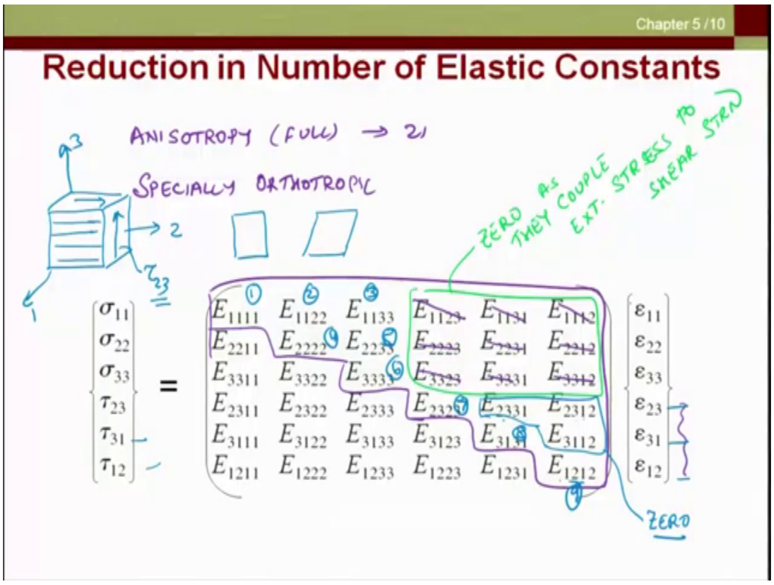

If that is the case then how do these equations work out? Now, earlier we had shown that the number of because of strain energy considerations these are the only independent elastic constants right because this matrix is symmetric. So, for anisotropy full anisotropy we have 21. Now, if it is especially orthotropic what will happen? If it is especially orthotropic let us see how these number of what is especially orthotropic, for a material where fibers are running parallel to 1 axis, 2 axis, 3 axis or all of them simultaneously, but not at some other angle ok. So, it is not that some fibers are at 45 degrees to 2 axis they are either at 0 or they are at ninety those are the only two options available.

So, if that is the case then shear strains and extensional stresses will not be coupled and shear stresses and extensional strains will not be coupled that is what it means, which means that this term will be 0, this term will be 0, this term will be 0 because this term couples sigma 11 with the shear strains ok.

Similarly, this term will be 0, this term will be 0 this term will be 0 ok. Similarly this these 3 terms will be 0 because all these terms all these terms they couple the shear stresses with shear strains. So, I will put that in a different color. So, all these terms they will be 0. Why? As they couple extensional stresses to shear strain ok, and if we want that their connection should be not existent then these terms have to be 0 ok.

And then there are some other terms which are also going to be 0. So, what would those be? So, these terms will also be 0, these terms will also be 0. Why because if you, so consider a situation this is a material and suppose this is a simple material all the fibers are running in this direction and I apply a shear strain like this, this is my 2 axis, this is 1 axis, this is 3 axis. So, I am applying tau 23, when I apply tau 23 in such a material the only first thing is there is no going there is no extensional strain which will be generated because I am applying only tau 23 and it is especially orthotropic. The second thing is it will this tau 23 will only generate. So, initially the block is like this and later the block will become like this. It will not shear in the 3 1 plane or in the 12 plane and these terms 23 31, 23 12, 31 12 they couple shear in other planes also. So, they will also be 0 ok.

So, tau 23 will only generate epsilon 23, tau 31 will only generate epsilon 31 and tau 12 will only generate epsilon 12 ok. So, if that is the case then the total number of elastic constants in such a case is 1 2 3 4 5 6 7 8 and 9.

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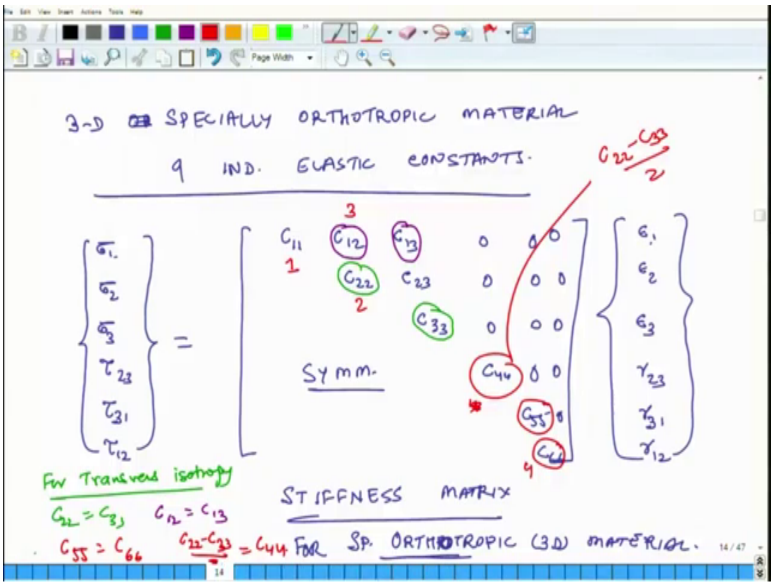

So, a 3 dimensionally especially orthotropic solid will have 9 independent elastic constants. So, a 3-D specially orthotropic material, 3-D specially orthotropic material it will have 9 independent elastic constants, ok.

Now, either we can express the equations in the tensor form which we just saw or now I am going to make my notation a little simpler because otherwise I have to write sigma 11 and then I have to write E 11, E 11 12 and all that you know 4 notations. So, in non-tensor notation, in matrix non-tensor form I will just write sigma 1, sigma 1 means sigma 11, sigma 2, sigma 3, tau 23 and here again remember I am not writing sigma 12 I am writing explicitly tau 23, I am not writing sigma 23, tau 31 and tau 12. So, these are 6 engineering stresses and they are related to a big matrix which is of size 6 by 6 and these are and the strains are epsilon 1, epsilon 2. So, this is not tensor notation its engineering notation epsilon 3 and here I write gamma 23 engineering strain gamma 31 gamma 12 and the elastic constants are. So, we are now going to get rid of E notation and we will use C notation C 11.

First index relates to stress second index relates to strain, and there are total 9 right. So, C 12, C 13 this matrix is symmetric C 22, C 23, C 33, C 44, C 55 and C 66 and all other terms in this matrix all other term this is all symmetry ok. So, these are the 6, 9 constants, ok.

So, this is the stiffness matrix for a specially orthotropic and its 3-D orthotropic material. So, these are not tensor equations. So, we have developed used the concept of tensors we reduce the number of constants to 9. And now we are expressing it in non-tensor notation you are not using. And this is special orthotropic which means that the material axis are aligned to the direction of loading, this is important to understand.

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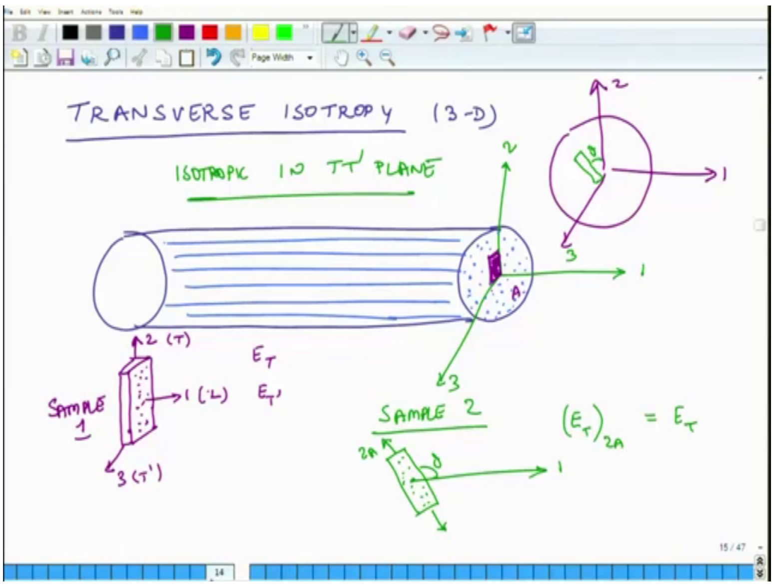

The second case is we will talk about is transverse isotropy, transverse isotropy. So, first case which we had discussed was fully anisotropic then we simplified it and especially orthotropic in 3-D, now we will reduce it make it even simpler and this is the case for a transverse isotropy. So, this is again for 3-D materials. So, what does that mean?

So, so I will explain this by example. Suppose there is a material. So, let us take a rod let us say it is a wooden rod and this is my axis 1, this is my, so I have just changed the orientation of the axis this is my axis 2, this is my axis 3 and this is a wooden rod. So, all the fibers are there and they are running along the length of the rod along the length of the rod. So, when I see from this end I just see dots each dot represents a single fiber and it is the end of the fiber, and all these fibers are held together by an adhesive a raisin or a matrix material, ok.

Now, think about it suppose I take a material like this sample I take a sample like this. So, I am looking at the rod and I am cutting I am cutting a material from the end and I take a sample like this ok. So, in this case, so this is sample A. So, in sample A, in sample A this is my 2 axis. So, it has a thickness, so actually sorry, I will just erase this. So, I will. So, let us just assume let I take I cut a sample which is of like this orientation ok, this is sample A and it is a thickness in the 1 direction ok.

So, how does sample A look ok? This is how it looks, and all the dots are here this is my 2 axis, this is my 3 axis and this is my axis 1. And now in this if I test it. So, 2 axis, so 1 axis is what it is the longitudinal axis the direction of fiber right. What is 2 axis? It is let us say transverse axis and 3 axis is transverse prime right. So, if I want to find E T and E T prime just by looking at the structure we will see that E T and E T prime will be same, ok.

Now, what I do is instead of this sample I take another sample, but, this is sample 1 and then I take another sample and here. So, just to make picture clear this is my this thing this is 1, this is 2, this is 3 and my sample is somewhere like this like this. So, sample 2 is like this ok, so but this normal to the surface is still direction 1, normal to the surface is still direction 1 and if normal to the surface is still direction 1 then I still see all these fiber ends and if I pull it in this thing in this direction. So, let us say this is direction 2 A and right this is direction 2 A, then E T in direction 2 A will be same as E T ok. It will be same, because it is essentially the same material and I have just the orientation and all these things does not matter.

So, regardless of this angle theta, regardless of this angle theta as long as I am rotating the sample in the 23 plane, right. So, if I am just rotating the sample in 23 plane that material properties in along the length of the sample will not change and this will. So, the material properties E T and E T prime will remain exactly the same as long as I keep on changing the material in 23 plane ok, which means that the material is isotropic in TT plane which means that the material is isotropic in TT plane. That is why it is called transverse isotropy as transversely isotropic and if the material is transversely isotropic then we have to go and see what further simplifications we can make.

So, if the material is transversely isotropic then what does that mean? So, suppose the material is transversely isotropic then C 22 and C 33 they will be same because 2 corresponds to the T axis and 3 corresponds to the T prime axis. So, C 22 and C 33 will be the same. So, far for transverse isotropy C 22 is equal to C 33. The other condition is C 12 and C 13 will be same C 1 is 11 corresponds to 1 direction, 2 corresponds to T direction, 3 corresponds to T prime direction. So, C 12 equals C 13 that is C LT equals C LT prime ok, so that is there. And C 55 and C 66 are same and finally, we can show that C 22 minus C 33 divided by 2, divided by 2 equals C 44. So, this is equal to C 22 minus C 33 divided by 2.

So, in this case how many constants we have? 1 2 3 4, 4 constants, everything else is expressed in terms of others.

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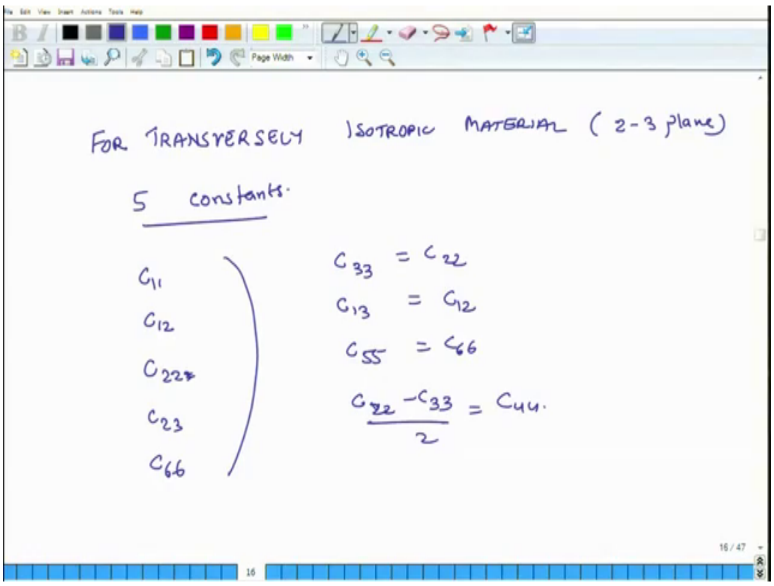

So, for a transversely isotropic material, so for transverse actually it is 5 I counted wrong 5 met constants for an for especially orthotropic in 3-D, 9 transversely isotropic 5 constants. So, and this is in 23 plane. And what are, so what are these constant? C 11, C 12, C 22, C 23 and C 66, these are the 5 constants. What is C 22, C 33? C 33 equals C 22, C 13 equals C 12, C 55 equals C 66 and C 22 minus C 33 divided by 2 is equal to C 44, ok. So, for transversely isotropic material, let me think.

So, we will continue this discussion tomorrow. And tomorrow we will cover two more different types of materials, one is isotropic material and the other one is especially orthotropic material in a plane stress straight which is the most important material from the standpoint of this course. And that is what we will discuss today tomorrow, and till then please have a great day, have a great time and we will meet once again tomorrow at the same time.

Thank you.