Introduction to Composites
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Lecture - 50
Generally orthotropic lamina

Hello, welcome to Introduction to Composites. Today is the second day of the 9th week of this course, and yesterday we had discussed the variation of different elastic moduli and elastic constants of generally orthotropic lamina a thin generally orthotropic lamina with respect to changes in the orientation angle of fibres which is theta ok. Now, what we had discussed is how E x, E y, nu xy, M x, M y, G xy all these elastic constants vary with respect to changes in the orientation of fibers with respect to the loading direction.

Now, what we will discuss are some more details on this and specifically we will talk about balanced lamina and what that means, in this constant context.

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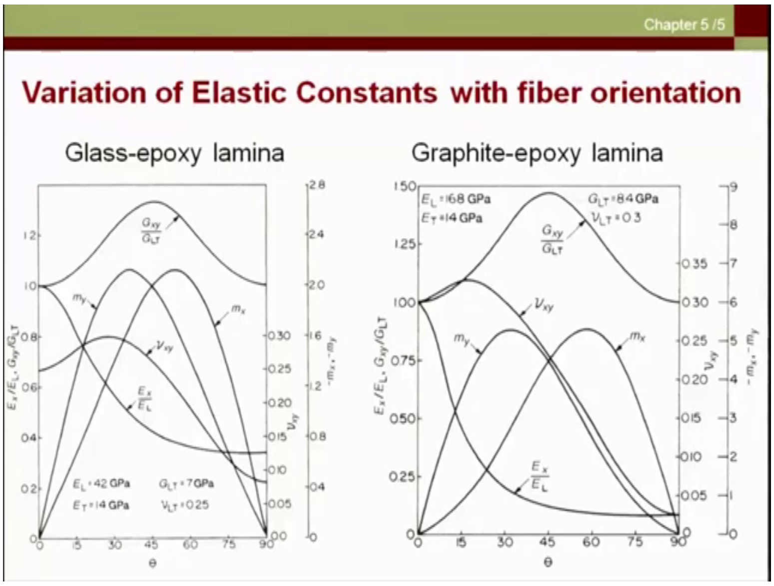

So, these are the relay plots for E x, G xy, G x, nu xy, M x, M y and so on so forth for glass epoxy lamina, graphite epoxy lamina and boron epoxy lamina ok.

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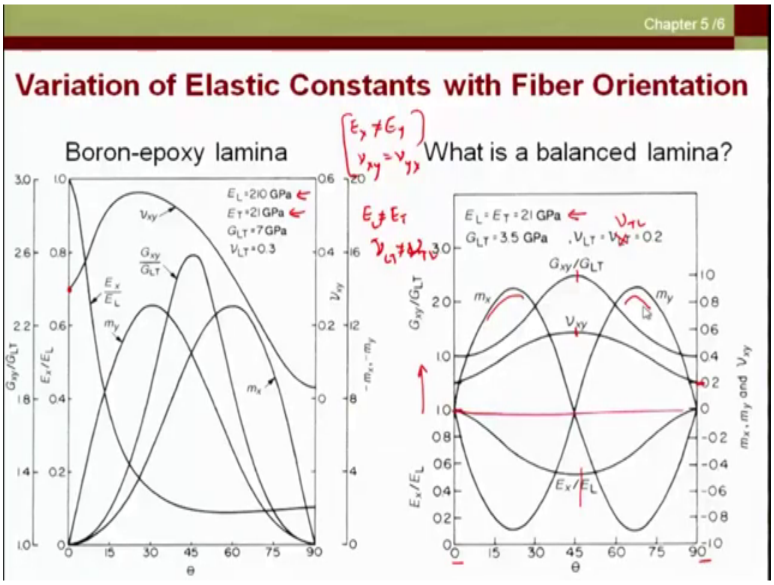

Now, what you observe something very significantly is that in these cases for all these situations E x is not equal to E y because E x at theta is equal to here you know E for instance for this thing. When theta equals 0 E x is equal to E L and when theta equals 90 degrees E x is equal to E T and E x in general is not equal to E y. And we can also say that nu xy is not equal to nu yx and all this is happening because E L is not equal to E T and nu LT is not equal to nu TL.

Now these kind, so if I have to use a single lamina for some structural application using these types of lamina maybe disadvantages in several cases because if E L is very large compared to E T, then we would also expect that the amount of load which it will be are in the L direction in the longitudinal direction will much higher and it will be a very little load in the T direction. But if it is just 1 layer we are relying on then we would like that it should have similar load carrying capabilities and stiffness properties in both the directions. So, that can be accomplished if we have a balanced lamina.

So, by balanced lamina means that it is the value of its E L and the value of E T is same and also the Poisson ratio E nu LT and nu TL they are the same. Physically this can be accomplished by having equal number of fibers in L as well as T direction. So, if you have same number of fibers in ln T direction then E L and E T will be same. And if that is the case then we would also expect that for balanced lamina E x and E y will also be same.

So, with this understanding let us look at the plots for E x, M x, M y and G xy for a balanced lamina. So, here because it is a balanced lamina E x E L and E T are same which is 20 GPa, G LT is 3.5 GPa and nu LT is equal to so this is this should be nu TL, nu LT should be equal to nu TL and that is 0.2 and if that is the case then let us look at E x. And how does E x change? It starts from 1 at 0 degrees and it keeps on going and because this is my axis for E x it also ends up at the same value 1 ok.

So, E x is symmetric the curve for E x is symmetric at around at my at the line theta equals minus 45 degrees, ditto for G x y, but G x y was anyway symmetric even for not balance unbalanced laminates, but mu xy, nu xy is also symmetric and it again it increases to a maxima, but it again becomes 0.2. So, it is 0.2 at 90 degrees and its 0.2 at 0 degrees and then M x and M y are also behaving. So, this is the relation for M x this is the equation for M y and they also behave in a very symmetric nice way.

So, these kinds of individual layers, laminas they are balanced because of the fundamental reason that the transverse and the longitudinal modulus of the lamina are same and also because the Poisson ratio LT and TL they are the same. So, this is what I wanted to discuss about balanced lamina.

Next what we will discuss is some constraints which exist on some of these materials.

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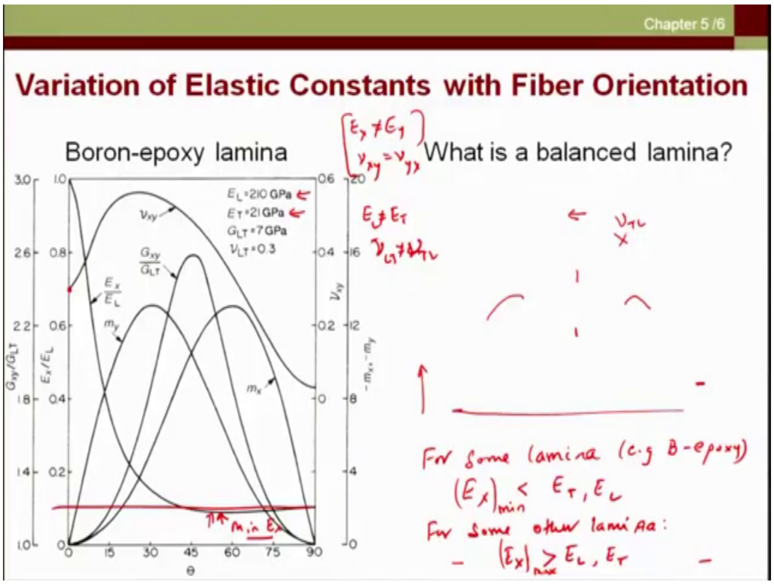

So, before we do that I like to show you once again that for boron epoxy lamina we have seen that the minima of E x is not E T right, is not E T for graphite epoxy and for glass epoxy the minimum value of E x was E t, but for boron epoxy as per this plot the minima is not E T rather it is it the minima exists when theta is somewhere between 50 and 60 degrees it exists. So, for, so what we are seeing is that for some lamina E x. So, in this case, so eg boron epoxy E x can be less than E T it can be less than C T and similarly for some other lamina E x can exceed E L and E T ok.

So, for some lamina at least in case of boron epoxy we have seen that E x. So, so E x min I am sorry I should have written min the minimum value of can be less than that of E T and E L and we can also construct some other lamina special lamina with specific properties such that their max can exceed E L and E T. So, when does this happen? This is what we are interested in.

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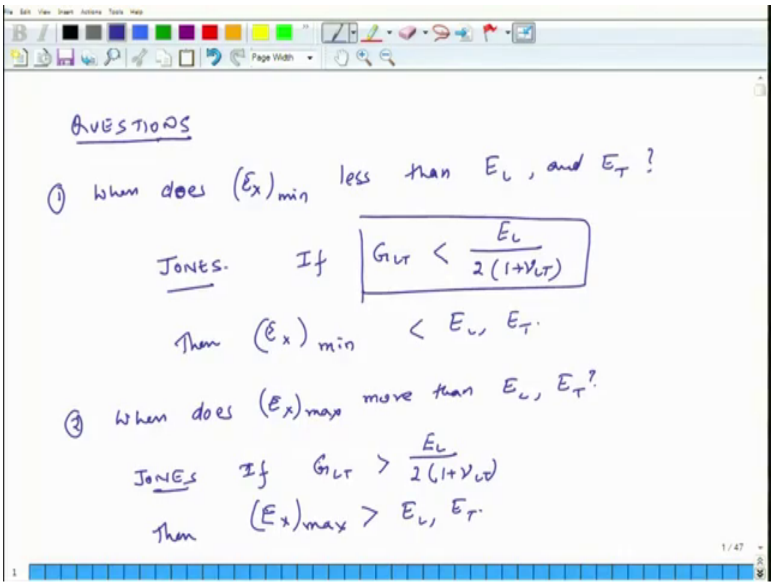

So, the question is first question when does E x min less than E L and E T. And the answer to that has been given by an expert in composite Jones, who is also the author of this RM Jones book on mechanics of composite material, and he has done mathematical collusion he says that if G LT exceeds E L divided by 2 times 1 plus nu LT then E x min is less than E L and E T ok. So, this is the first question. So, so if you find that in your composite system this condition is being satisfied then you will have a situation that for a certain value of theta the value of E x will be less than E L as well as E T ok.

The second question is when does E x max more than E L, E T. And again Mister Jones answers that it is more than E L and E T if G LT is less than oh I am sorry. So, this condition I specified incorrectly if G LT is less then this then this is less than this and if G LT is more than E L divided by 2 times 1 plus nu LT then E x max it exceeds E L and E T, ok.

So, these are 2 important conditions which help us understand what could be the extreme value of E x extreme of our E x. And how do they get it this based on mathematical analysis not very complicated analysis, but we will not discuss this at least in this class, but if you are interested you can refer to the paper by Jones and also his book and you will find the details in there. So, this concludes our discussion for this particular topic.

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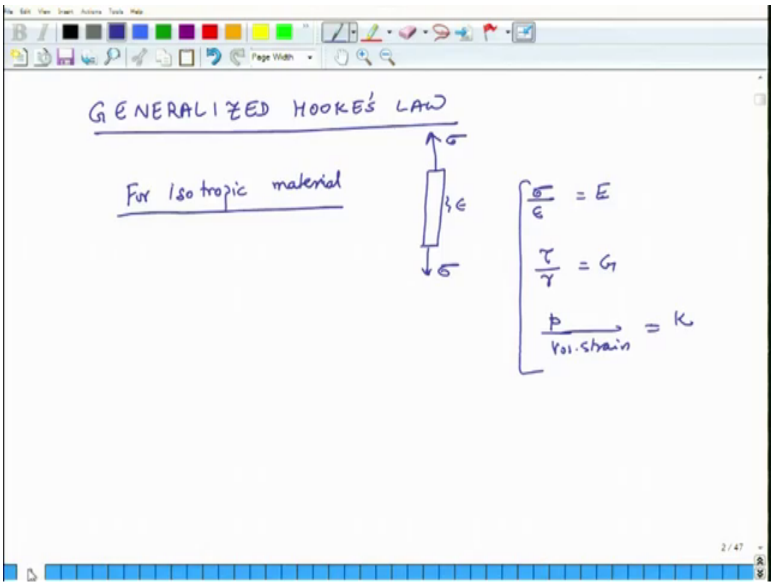

Now, what we will do is we will move on to the next topic and that is about generalized Hooke's law, generalized Hooke's law. So, why what are what do we mean imply by imply by this? So, our conventional Hooke's law which we have talked about it says that for isotropic materials; for isotropic material what is the Hooke's law? That if I have a bar and I apply a stress on it and because of that it experiences a strain, then the Hooke's law says that stress by strain equals the Young's modulus which is E, ok. So, this is the Hooke's law for isotropic material.

And an and similarly if it is in shear then tau by gamma equals G and then if it is if the material is being compressed from all the 6, all the sides then pressure divided by volumetric strain is K, bulk modulus and then. So, this is for isotropic material. So, the question is for an isotropic material we have what we have discussed till so far is only for orthotropic material and an isotropic materials. It does a similar Hooke's law exist for a fully anisotropic material that is what we are going to discuss.

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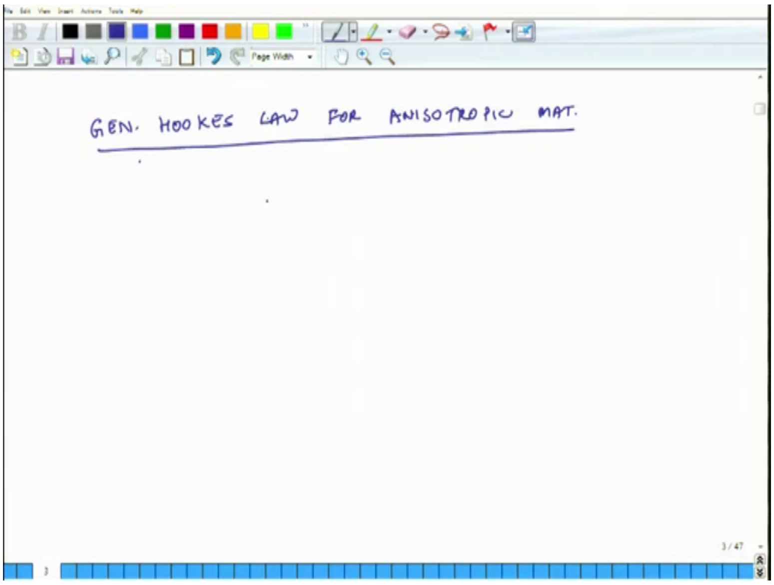

So, what we are going to discuss is generalized Hooke's law for anisotropic materials. So, before we look at it let us define our access system and what we will also do is so what we will do is what is that we will define our axis system, what are our 1 2 3 axis and with respect to axis systems we will also have a consistent way of specifying stresses and strains. And using this, such a system then we will then generate a generalized Hooke's law for an isotropic materials, ok.

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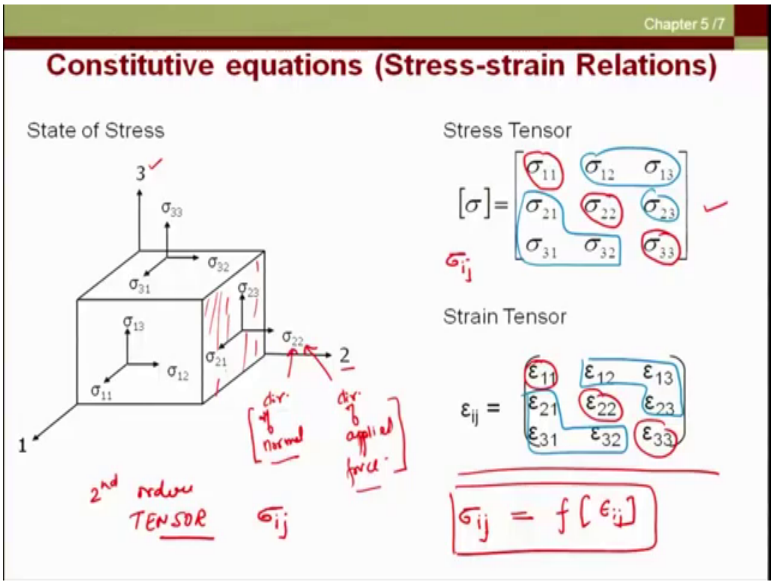

So, this is the axis system which we use for stresses. So, you have a block of material and this material can be fully anisotropic. So, it will it may have different properties in different directions, it need not be orthotropic or isotropic because those are just special cases of an anisotropic material and we will use a Cartesian system of reference. So, this is our axis number 1, this is our axis number 2, this is a axis number 3. All the stresses on this plane this plane can expressed can experience 3 types of stresses sigma 21, sigma 22 and sigma 23.

So, the first index sigma 2 corresponds to the direction which is normal to this plane ok. So, this direction of normal and the second index is the direction of force, direction of applied force ok, because stress is force divided by area. So, I am applying force in a particular direction and I am also applying it on a particular plane. So, the first index tells the direction of the plane and the second index tells the direction of the force. And as I had explained earlier the stress is associated with two different directions direction of normal of the surface and direction of applied force. So, it is a second order tensor. And in general I can specify it as sigma ij, where i represents the direction of the normal of the phase on which it is acting and j represents the direction of the force which is applied on that particular surface ok. So, this is how I define stresses.

So, we have sigma 21, sigma 22, sigma 23 on plane number 2; sigma 31, sigma 32, sigma 33 on plane number 3; and sigma 11, sigma 12, and sigma 13 on plane number 1, and likewise we have other stresses on the opposite phases also. So, these are my stresses sigma 11, sigma 12, sigma 13 and this is the stress tensor and it is represented by a 2 by 2 matrix. And in general I can also represent it as sigma ij ok. So, this is the stress tensor.

Similarly, this is our strain tensor epsilon ij. So, epsilon ij is epsilon 11. So, so what are these? So, again, so before we go to strain tensor these stresses are extensional stresses and all others these are shear stresses because the direction of the normal and the direction of the applied force are not aligned with each other. Similarly, on the strain tensor these are shear strains and these are extensional strains ok. So, this is our overall framing of the problem.

And what we have to develop is the relationship that how does sigma ij depend on, how does it depend on epsilon ij. So, this is the relation we will develop and then we will call that relation the Hooke's law for generally isotropy anisotropic material and then we will start working on that and we will finally, come down to isotropic and much simpler materials. So, that is what we plan to do in our next class. So, please remember this terminology and this is the terminology we will use starting from a next class. So, that is pretty much it for today and I look forward to seeing all of you tomorrow.

Thank you.