Introduction to Composites
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Lecture - 49
Variation of Elastic Constants with Respect to Fiber Orientation for Generally Orthotropic Lamina

Hello, welcome to introduction to composite materials. Today is the start of the ninth week of this 12-week course, and in the last week we had covered stress strain relations, stress strain relations for generally orthotropic lamina. And what we had shown was that for a generally orthotropic lamina, there are 7 elastic constants which help us determine the stresses in the lamina, if strains are known and vice versa.

So, these elastic constants are Ex, Ey, G xy, mx, my, nu xy, nu yx. So, these are the 7 constants and, what we had also shown is that these elastic 7 elastic constants can be expressed in terms of 4 independent elastic constants, which they are measurable in the LT plane, and these are EL, et, G LT, nu l t. So, these are the 4 constants.

So, let us quickly review all this material once again.

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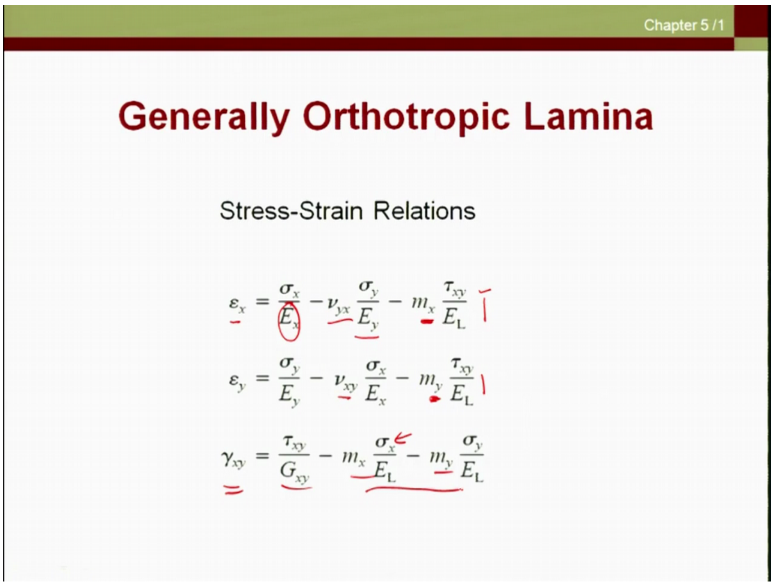

So, what we had shown was that the stress strain relations for a generally orthotropic lamina. Look, something like this, where epsilon xx is equal to sigma x divided by Ex minus nu yx times Ey over Ey minus mx times shear stress divided by EL. And similarly, we have stress strain relationships for the y direction and for the shear strain.

So, using these relations and also the concepts involving stress transformation and strain transformation we had developed expressions for sigma x sigma y tau xy all these things. So, specifically we had developed relations for Ex Ey nu x y, nu y x, mx, my and G xy, and these 7 relations are shown here.

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Chapter 5 /2

Engineering Constants for Orthotropic Lamina

$$\frac{1}{E_x} = \frac{\cos^4\theta}{E_L} + \frac{\sin^4\theta}{E_T} + \frac{1}{4}\left(\frac{1}{G_{LT}} - \frac{2\nu_{LT}}{E_L}\right)\sin^2 2\theta$$

$$\frac{1}{E_y} = \frac{\sin^4\theta}{E_L} + \frac{\cos^4\theta}{E_T} + \frac{1}{4}\left(\frac{1}{G_{LT}} - \frac{2\nu_{LT}}{E_L}\right)\sin^2 2\theta$$

$$\frac{1}{G_{xy}} = \frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \left(\frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}}\right)\cos^2 2\theta$$

So, these are the two relations for Ex and Ey. So, here if we know all the parameters on the right side of the equation specifically l EL ET G LT ULT and the value of theta, we can compute Ex and Ey. These are relations we had developed. Similarly, what you see is this is the relation for G xy, ok. The shear modulus for the material when we are working in an xy plane rather than an LT plane. So, these are the relations, and then we had also developed relations for Poisson ratios nu x y and nu yx.

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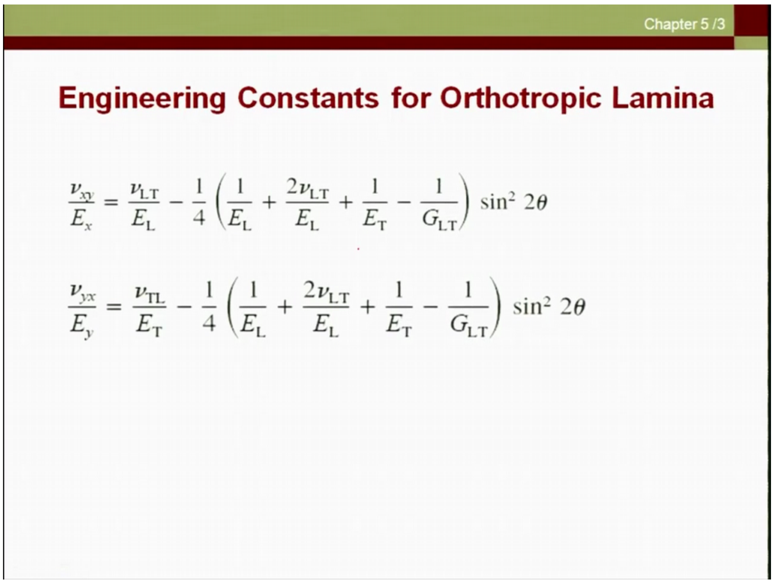

And which are these, and what I will do is that we will email you these equations also once this class is over. So, that you may refer to these, and you can practice these equations for purposes of derivation, and also for purposes of calculation calculating these elastic constants, then your axis system is not specifically aligned with the material axis of the orthotropic material. So, these are 2 additional material properties elastic constants.

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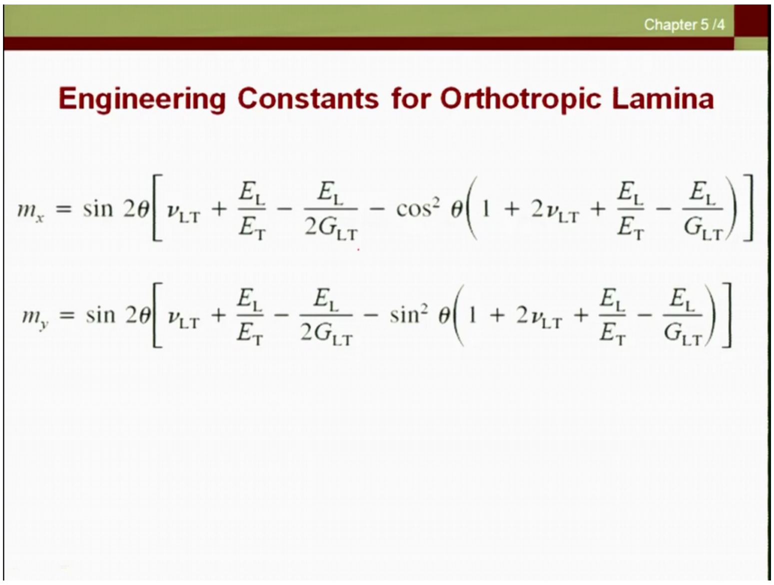

And finally, the last 2 elastic constants known as cross coefficients are mx and my. Now I would like to make some specific comments on these all these equations.

So, let us start with Ex and Ey.

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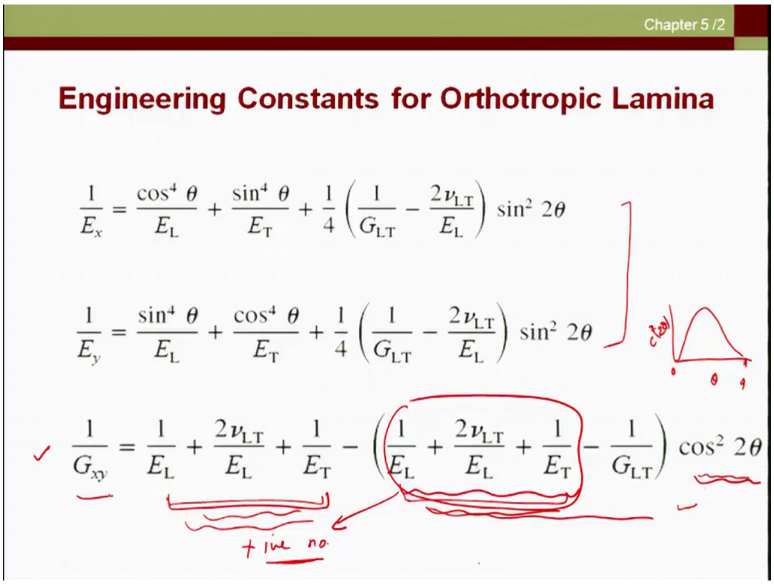

So, what you see here is that if theta is equal to 0. So, then 1 over Ex all these sine theta terms go away. So, 1 over Ex becomes 1 over EL or Ex equals EL, which makes sense because when theta equals 0 then the x axis, and the l axis are mutually aligned. So, Ex will be equal to EL. Similarly, when theta equals 90 degrees, then the x axis will be aligned to the transverse axis which is the t axis, and in that case Ex will equal ET these are 2 observations.

Similarly, when theta equals 0, let us look at the third equation. So, when theta equals 0 then cosine square theta becomes 1 and what happens is that the terms outside the bracket and the terms in the first 3 terms inside the bracket they cancel out each other. And essentially what that implies is that when theta equals 0 then G xy is same as G LT, ok.

Similarly, when theta equals 180 is equal to 90 degrees. That is when the x axis is aligned to the transverse axis of the system then cosine square of 2 theta, 2 theta becomes 180 degrees. So, cosine square of 2 theta equals 1. And once again the terms involving

EL, ET and nu LT they cancel out, and once again we have Gxy equals G LT so, this is one important thing.

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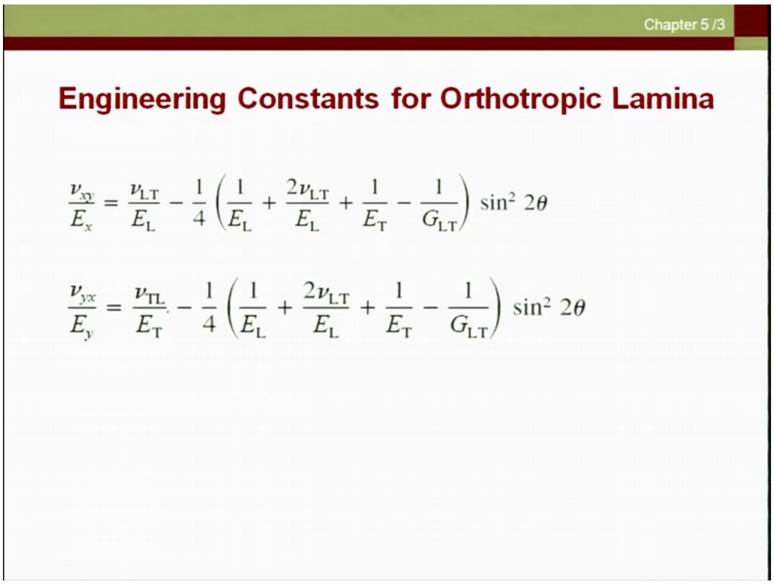

Next look at Poisson ratios and once again when theta equals 0 the nu xy equals nu LT, and when theta equals 90 degrees, then once again sine squared 2 theta becomes 0 the nu yx becomes nu tl and so on and so forth.

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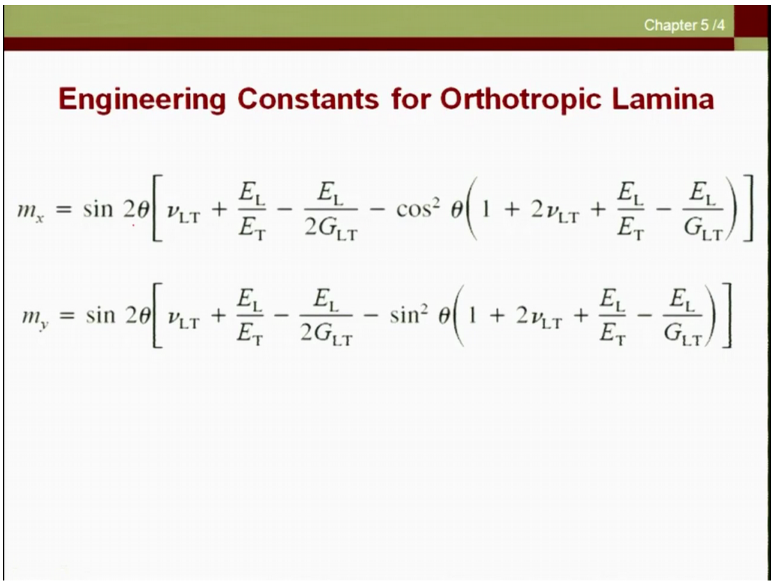

And for mx and my what we see is that when theta equals 0 or theta equals 90, then for either of those cases mx and my they are 0. So, they assumed non0 values only when theta is not equal to 0 or 90 degrees.

So, this is consistent with our original stress strain relations, because here what we see is, that when theta equals 0 or theta equals 90 degrees, our excess system will be aligned with the material axis of the system; that is, we will not have the generally orthotropic lamina situation rather that will simplify into the especially orthotropic lamina situation. And in that case when we have stress the coupling between shear stress and the shear strain ah and the not the coupling between the shear stress which is tau xy.

And the extensional strain is through mx, and the role of mx, and my becomes 0 for theta equals 0 and 90 degrees. And similarly, as we would expect that in a special orthotropic situation, a pure shear stress will produce only shear strain. And that is exactly what we see with the third equation that when mx and my are 0, then tau xy will only produce gamma xy.

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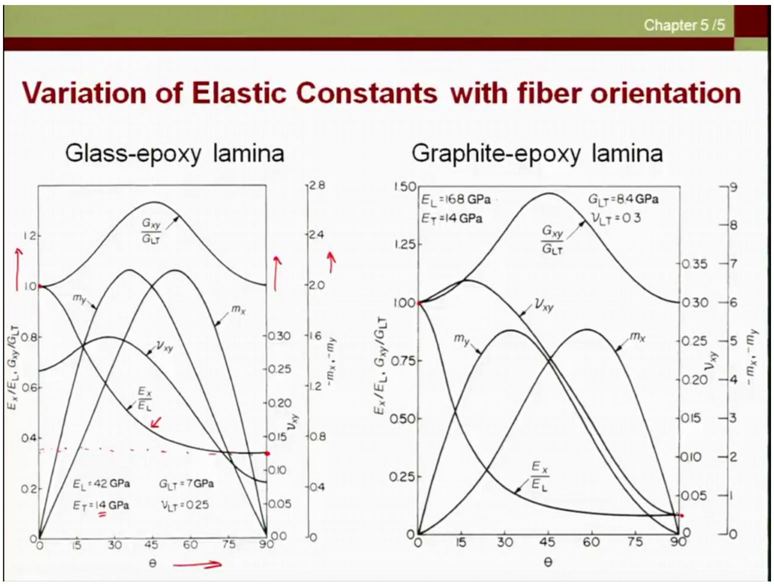

So, these observations which we made just now on mx and my are consistent with our original equations, which connect stresses and strains for a generally orthotropic lamina, ok.

Next let us look at how these elastic constants vary with fiber orientation. And what we will do is, we will look at 3 material systems. One is a glass epoxy material system, the other one is graphite fiber epoxy material system composite material system. And the third one we will use boron fibers and epoxy is the matrix materials.

So, we will consider 3 categories of materials glass epoxy graphite epoxy and boron epoxy. And we will see plots of how Ex poisson ratio nu xy the 2 crossed coefficients mx and my and the shear modulus g xy they change with respect to changes in theta. So, let us look at this variation through these figures.

So, first we will look at just this figure, and there are several lines on this. So, it is important to understand what has being plotted on the x axis what we are seeing is the variation of theta. So, theta is increasing from 0 to 90 degrees, ok. And then on the y axis I have 2 parallel x actually 3 parallel axis. So, this axis it provides us the scale for Ex normalized with respect to EL, and G xy normalized with respect to G LT ok.

Then there is a second axis, because on this on this graph I am plotting all Ex nu xy mx my and Gxy. So, here the second vertical axis is for nu xy. So, nu xy is changing from 0 to let us say 0.6 or something like this. And the third parallel axis is for mx and my. And here the value of negative of mx and negative of my is changing from 0 to 2.8, ok.

So, let us look at first the variation of Ex which is this curve ok, and what do we see. So, Ex divided by l is equal to 1 at this point, which means that when theta is equal to 0 degrees Ex will be equal to EL, ok. And as my theta increases, the value of Ex continuously keeps on dropping. And then it slowly starts becoming flat, and in ends up at the value of something like 0.3, ok. So, maybe I didn’t draw this line straight enough, but it is about 0.3 because ET is 14 GPa. So, ET over EL s 1 by 3 so, 0.33 something in that range 0.33, ok

So, at 1 x so, at one extreme the value of Ex is 14 g. I am plotting Ex divided by EL and at the other extreme it is 0.33, the ratio is which means that it is about 14 GPa because the material system which has been chosen such that els 40 2 GPa els 14 GPa G LT 7 GPa and nu LT is 0.25.

So, this is one important observation that for glass epoxy, and for glass epoxy Ex continuously decreases, it continuously decreases and it becomes achieves a minima for and the minimum value is same as that of ET. Let us look at the same graph, but this is for graphite epoxy lamina, now what is the difference here the EL is 168 ET is still 14 because we are using the same matrix material. G LT is 8.4 so, it little higher and nu LTS 0.3.

So, once again it starts from one and it goes down to pretty small value because EL by ET by EL is very less it is less than 0.1, 14 divided by 168 is less than 0.1. So, it has it shows the same trend so, this is one important parameter. So, for glass epoxy and graphite epoxy because these values are of EL ET and g LT nu LT are like this. Ex divided by EL monotonically continuously it keeps on decreasing and achieves a minima, then theta equals 90 degrees.

But the same cannot be said for the third material system which is boron epoxy.

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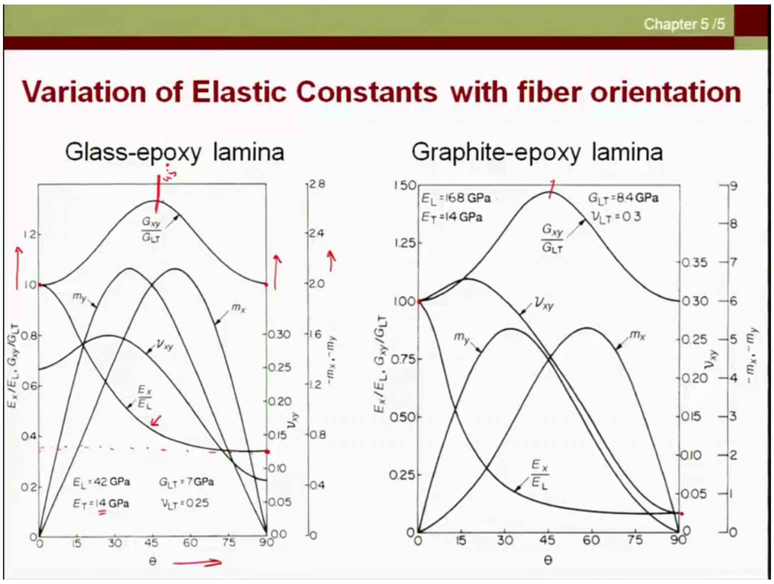

So, what do we see in boron epoxy? So, this is the graph for Ex divided by EL, and what we see here is that it once again as expected it would start at 1.0, which means at 0 degrees Ex will be same as EL which is 210 GPa, and then it is minimum value is about 0.1 because ET is 21 GPa, but it, but sorry, I am sorry. So, it is not a minima at 90 degrees it becomes 21 GPa, but it goes below that minima, somewhere between 40 and 60 degrees, ok.

So, we should so far boron epoxy, because these material properties EL ET G LT nu LT are like this. The value of Ex actually goes and becomes less than that of ET, add some value of theta which lies between 45 and 60 degrees, ok. So, this is an important thing so, lot of times we think that Ex will be always at a minimum when theta equals 90, but that may not necessarily be always the case, as we have seen in this particular example, where Ex can be less than ET in certain special cases such as in boron epoxy situation so, this is one conclusion

The next thing we will look at is variation of G xy. So, G xy it starts so, when I divide the value of G xy by G LT at theta equals 0 this ratio is one, which means that at when the direction of loading and the direction of fibers are mutually aligned. Then G xy and g LT is the same. And the same thing is true when theta equals 90 degrees. So, theta equals 0 and theta equals 90 degrees G xy and G LT are identical, which is consistent with our relation which is shown here. That when theta equals 0, these terms and these terms cancel out terms involving EL and ET and only G LT is left. And the same thing is true for theta equals 90 degrees.

The other thing to note is the curve for G xy is symmetric at 45 around 40 theta equals 45 degrees, ok. It is symmetric around theta equals 45 degrees. And why is that? The reason for that is again attributable to the mathematical relation which we see here that here cosine. So, so we have this term which does not involve the theta term and then this term involves the theta term, and this theta term involves cosine square of twice of theta.

So, if we plot cosine square of twice of theta. So, if I plot it like this from between 0 and 90. So, it will be something like this. So, because so this is cosine square twice of theta ok so, because this curve is like this it is for this reason 1 over G xy is the plot for 1 over G xy or for that sake even from the plot for G xy is symmetric around this vertical line, which is theta equals 45 degrees, ok, this is the second thing.

The third thing is, that the maximum value of G xy is always at around 45 degrees, it is at 45 degrees. It is not at either theta is equal to 0 or at theta equals 90 degrees.

Maximum value is not at theta equals 0 or theta at 90 degrees. So, this is another important feature there so, lot of times we may think that shear modulus will be maximum and theta equals 45 0 degrees or 90 degrees, but that is not true. It is maximum when theta equals 45 degrees. And the reason for that is once again from a mathematical standpoint, it is once again embedded in the equation for G xy.

See what happens is that this is the relation for 1 over G xy. Now this is a positive number ok, and this is this term multiplied by cosine square 2 theta is getting multiplied by it and it gets subtracted from this positive number ok. So, what typically happens is that as this theta starts increasing from 0 degrees. This term tends to reduce the magnitude of this positive number. And as this number becomes smaller in it is magnitude, it is inverse starts increasing, because this is 1 over G xy, ok.

So, this is the mathematical reason and the physical reason for that is something we will discuss later, but because of this G xy maximizes at 45 degrees. And this is not particularly true for a specific material, but it is true for all generally orthotropic materials. So, we will see that the same trend is true for graphite epoxy lamina, and for glass epoxy lamina so far graphite epoxy lamina also it is maximized that 45 degrees. And we see the same thing for boron epoxy lamina. The same thing is true for boron epoxy lamina so, this is another important observation.

The next trend we will like to see is for mx, or for that sake we can also look at the graph for my they are similar, but they are shifted by certain degree is, you know. So, what we see is, that mx in case of glass epoxy lamina increases from 0 degrees to as much as 2 it is value becomes as high as 2 for glass epoxy lamina.

For graphite epoxy lamina, the value of mx it goes up to as high as 5. And for boron epoxy, it is even more extreme it exceeds 12. So, the point what I am trying to make is and what is this mx what is the role of mx and my? The role of mx and my is apparent from these stress strain relations, and what these relations tell us that if I apply a shear strain just a shear strain on the specimen, it can create an extensional strain if I just apply a shear stress. And if I just apply an extensional stress it can create an shear strain in the system and because this mx. And my can be very large what; that means, is that shear stresses can create significant extensional strains, and extensional stresses can create significant shear strains in a generally orthotropic lamina. And so, we cannot assume that the effects these these coupling effects between shear and extension, they can be ignored, they can these coupling effects can be very significant, and if they are very significant then we have to be cognizant of that and taking into account for those factors for those effects.

So, this is there then, the last thing is that we will make some comments on poisson ratio. So, poisson ratio it again it starts in glass epoxy lamina from some value. So, it starts from so, the axis for poisson ratio is here so, it starts from about 0.25. In case of glass epoxy, it actually increases as theta increases, see this is so, this is the peak value it increases. So, it becomes maximum at around 30 degrees, right? And then it starts falling and it keeps on falling till it is value becomes something like, somewhere between 0.05 and 0.1, ok

So, the maxima of nu xy is not necessarily either at 0 degrees or at 90 degrees. It can be at some value in between 0 and 90, in this case it is maximum is maximizing at around 30 degrees. For graphite epoxy let us again look at it, and we see that it again starts from 0.3, this is the starting point, and the poisson ratio it keeps on decreasing and it is pretty small bit at around 0.025 at 90 degrees, but it maximizes at around 15 degrees for graphite epoxy lamina, ok.

So, in glass epoxy now this is again dependent on actual material properties, for different glass epoxy lamina these values may vary. But the point is that the maxima of nu xy may need not be at 0 or 90. And for boron epoxy, it starts from 0.3 at 0 degrees, and it actually rises very significantly, and it approaches something like 0.55, and then at around 25 degrees. And then from there it starts decreasing, and it becomes extremely small ok, it becomes extremely small to about 0.03 somewhere in that range when theta equals 90 degrees. So, these are the overall trends related to variation of Ex, Ey, mx, my, Gxy, nu xy and so on and so forth, with respect to changes in theta.

So, this is the first thing I wanted to mention. What we will do is, we will continue this discussion on variation of elastic constants tomorrow as well. And once we have completed that then we will move into the next segment of our lectures which will be a generalized hooks' law for orthotropic systems and anisotropic systems. So, that concludes our discussion for today. I will look forward to seeing you all tomorrow. Until then have a great night, and bye, bye.