Introduction to Composites
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Lecture – 48
Transformation of Engineering Constants – Part II

Hello, welcome to Introduction to Composites. Today is the last day of the ongoing week and over last five days what we have done successfully is we have developed mathematical relationships between stress and strain for especially orthotropic lamina. We have also exhibited and shown the mathematical relationships between stress and strain for a generally orthotropic lamina and in the last class what we did accomplished was we started developing relationships between elastic constants as applicable to generally orthotropic lamina which are E x, E y, G xy, m x, m y, mu xy, mu yx and the elastic constants for especially orthotropic lamina E L, E T, G LT and nu LT and what we have shown is that specifically that E x, nu xy and m x can be exhibit expressed in terms of E L, E T, G LT and nu LT.

Today, we will develop the remaining relationships and at the end of the course at the end of this day, we will show that all the elastic constants for generally orthotropic lamina can be reduced in terms of simple functions which are applicable for especially orthotropic lamina.

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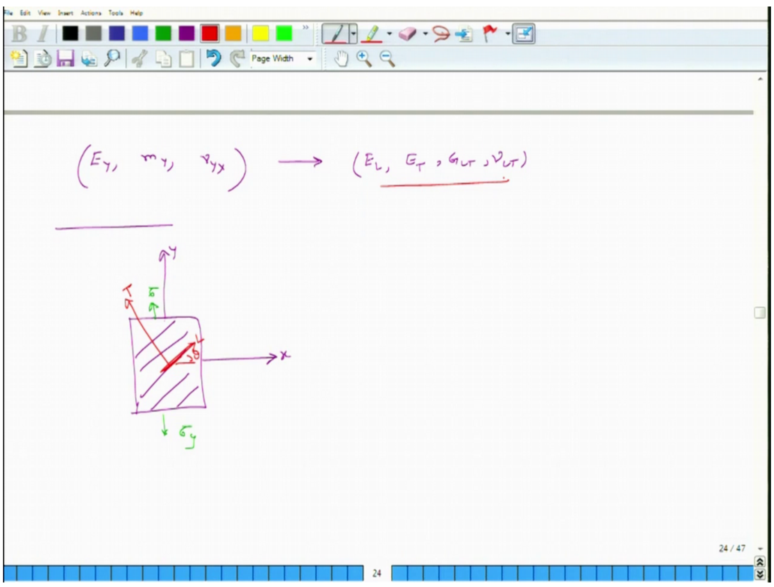

So, today, the next phase is that we will develop relationships between E y, m y and nu xy or actually nu yx. We will develop function relationships between these things to special orthotropic material properties which are E L, E T, G LT and nu L T.

So, we will develop these three relations.

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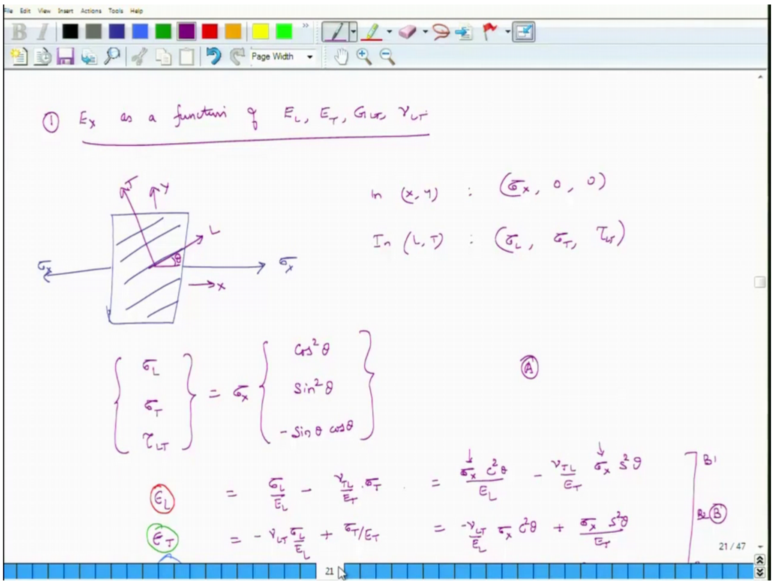

For the first set of relationships we had loaded the material in the x direction. Now, what we do is we load the material in the y direction and we will get similar relations. So, here what we have is we have this material and this is my x direction, this is the L direction, this is my transverse direction, excuse me and this is my y direction and instead of loading this sample in the x direction, I applied a pure tensile load in the y direction.

So, I apply sigma y here the angle between L and x axis still is theta. So, this angle is still theta and if we exactly follow the process which we had discussed earlier, then we end up with the relationships between doing the between E y, m y and gamma yx and these terms all the other steps are similar are very much the same. So, we do not have to repeat it. So, what I will do is, I will directly write down the relations.

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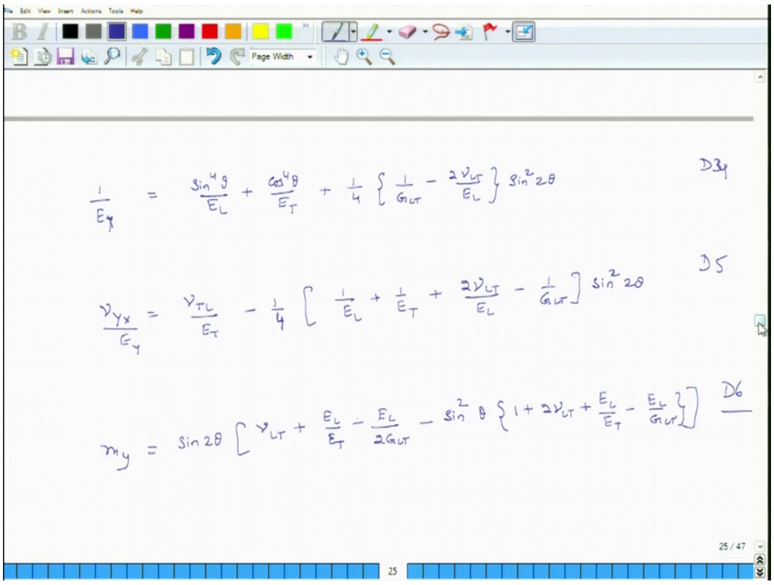

So, the first relation is for E y, Young's modulus in y direction, so, this is 1 over E y and that equals sine 4 theta over E L plus cosine 4 theta over E T plus 1 over 4 and I have 1 over G LT minus 2 nu L T divided by E L sin square 2 theta. So, this is the relation for E y and to get to this relation we have to follow exactly the same process which we did earlier.

The next relation is for m y. So, m y actually before m y we will express the relation for gamma yx. So, gamma yx is equal to nu T L over E T and this is 1 over E y on the right side minus 1 by 4. So, it is nu T L over E T minus 1 over 4 plus 1 over E L plus 1 over E T plus 2 nu L T divided by E L minus 1 over G LT sin square 2 theta. So, this is the second relation.

And, the third relation, there is little confusion on the left side, so, I will just make it explicit. So, this is nu yx divided by E y and then the cross coefficient m y equals sin 2 theta times this entire thing nu L T plus E L over E T minus E L over 2 G LT minus sin square 2 theta, oh actually I am sorry, sin square theta times 1 plus 2 nu L T plus E L over E T minus E L over G LT. So, these are the three relations and so, we will call them D3, D4, D5 and D6 and when you compare D3, D4, D5, D6 with their counterparts D1, D2, D3, what you find are some equivalences.

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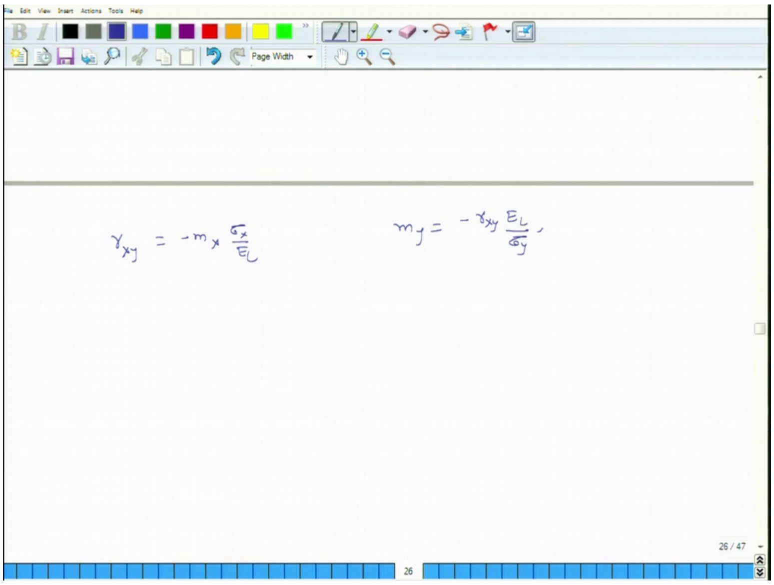

And, that is gamma xy equals minus m x sigma x over E L and you also see that m y equals minus gamma xy times E L over sigma y, I mean these are the basic definitions. So, we use these definitions to develop the relations mentioned earlier.

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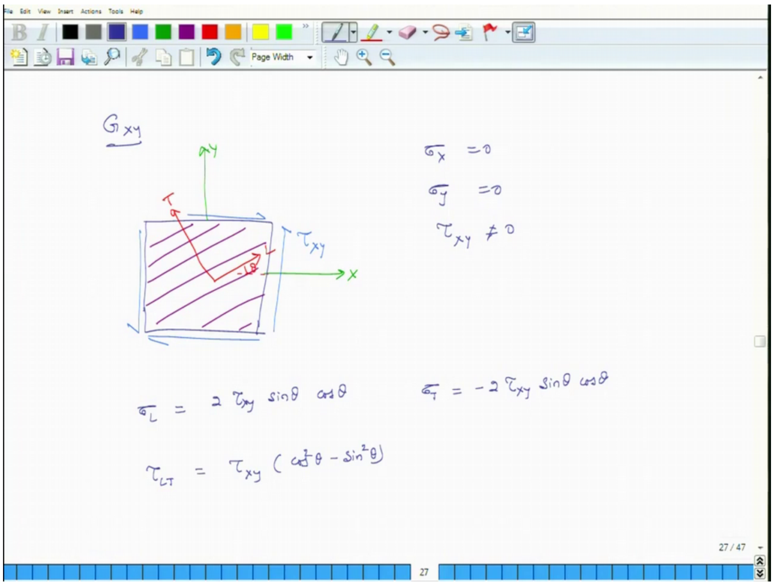

Finally, we will develop an expression for G xy, we will develop an expression for G xy. So, what do we do, we again go back to our basic concept. We have a orthotropic composite lamina. The lamina is loaded in this direction is not especially orthotropic and so, this is my x axis, this is my y axis, this is my L axis and this is my T axis.

So, the angle between x and L axis is theta and here since we are interested in finding G xy, we subject it to pure shear stress, positive shear stress. So, this is our positive shear stress is applied like this. So, this is tau xy. So, in this case sigma x is equal to 0, sigma y is equal to 0 and tau xy is not equal to 0. So, the first thing we do is we express sigma tau xy, from tau xy we compute sigma L sigma T and tau L T. So, we say from those stress transformation equations, tensor equations, sigma L is equal to 2 tau xy sine theta cosine theta, sigma T equals minus 2 tau xy sine theta cosine theta and shear stress sigma tau LT is equal to tau xy cosine square theta minus sine square theta.

So, these are the things. So, if this is the case and then we also write now sigma L is going to be a source of epsilon L and epsilon T sigma T will also generate epsilon L and epsilon T and tau LT will generate a shear strain.

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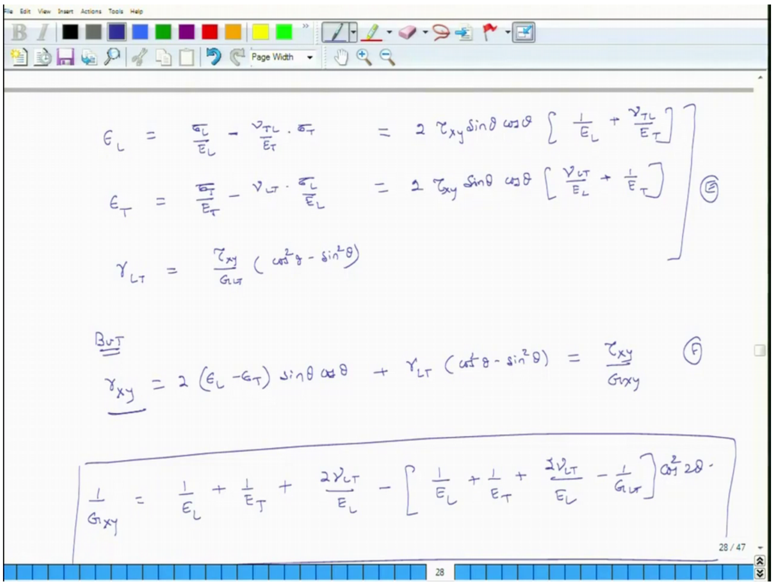

So, we can from these things we can write expressions for epsilon L. Epsilon L is what, sigma L over E L minus nu T L over E T times sigma T and from this we get 2 tau xy sin theta cosine theta, in parentheses 1 over E L plus gamma T L over E T.

The negative sign goes away because we have a negative sign here and tau sigma T is also negative. So, that is why it goes away. Similarly, epsilon T is equal to sigma T over E T minus nu L T times sigma L over E L and if we do all the mathematics we get 2 tau xy sin theta cosine theta nu LT over E L plus 1 over E T and finally, gamma LT equals tau xy by G LT cosine square theta minus sin square theta.

Now, we realize, but if we do the strain transformations calculations we realize that gamma xy can be expressed in terms of epsilon T epsilon L epsilon LT using this relation, 2 epsilon L minus epsilon T sin theta cosine theta plus gamma LT cosine square theta minus sin square theta and it is nothing, but same as tau xy over G xy. So, if I combine if we if I put, let us call this as equation E and this is F. So, if I put E in F, what I ultimately get is 1 over G xy equals 1 over E L plus 1 over E T plus 2 nu LT by L minus 1 over E L plus 1 over E T plus 2 nu L T divided by E L minus 1 over G LT cosine square 2 theta.

So, this is the last expression and that is where the shear modulus in x y coordinate system. So, this is from the third equation, this is for if we use the equation for gamma xy.

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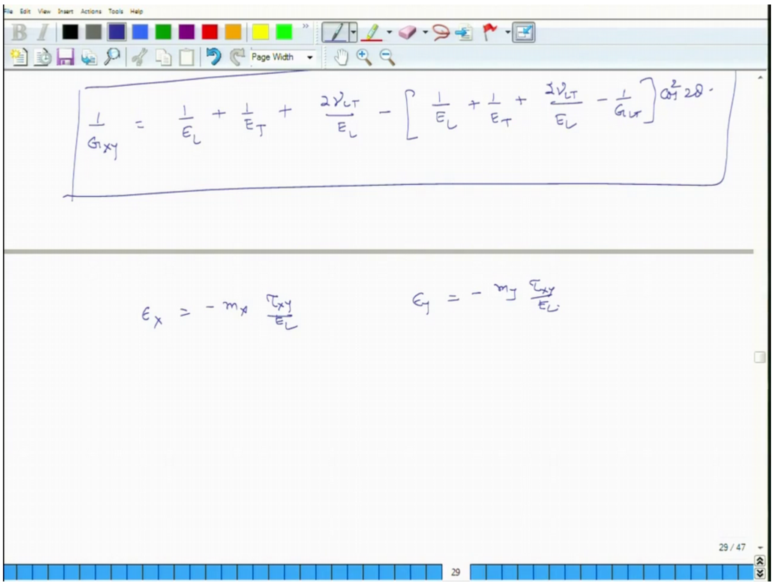

And, if we use the equation for epsilon x and epsilon y we will see from that again we will see that epsilon x is equal to it will get this relation and epsilon y equals minus m y tau xy divided by E L, we will get the same relation which we had written earlier. So, mathematically we have shown this.

So, what we have accomplished in last several lectures is we have established concrete mathematical relationships between the four elastic constants which are mutually independent for especially orthotropic plate and we have really expressed all other elastic constants E x, E y, m x, m y, nu xy, nu yx and G xy in terms of these four elastic constants.

Finally, we will do an example. So, that we become conversant with all this and here we will actually calculate strains for this system.

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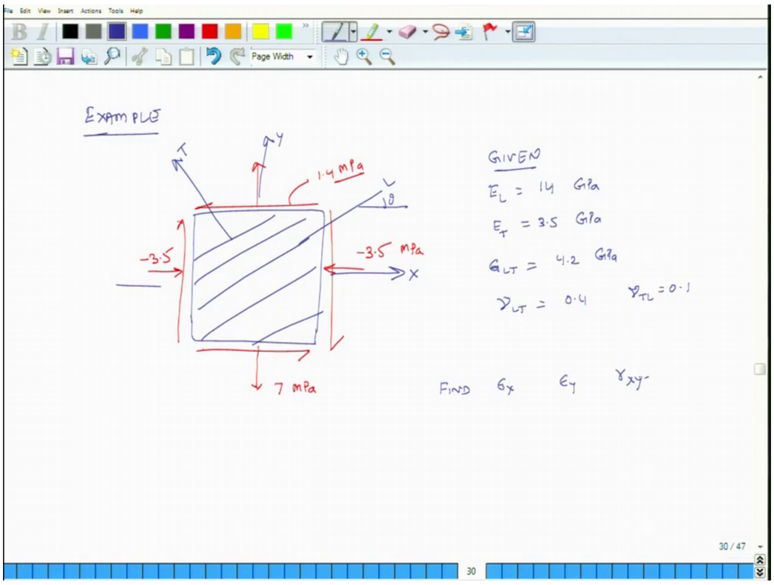

So, we have, this is my x axis, this is the y axis and this is my L axis, this is my T axis and this lamina is experiencing several stresses. So, it is experiencing negative sigma x and the value is minus 3.5 mPa, it is experiencing a positive sigma y and this values 7 mPa and finally, it is experiencing shear stress and this value is 1.4 mPa.

So, given this stress state and the material properties which are E L is equal to 14 GPa, E T is equal to 3.5 GPa, G LT equals 4.2 GPa and nu LT is equal to 0.4 and nu TL is equal to 0.1. So, these are the five material properties we already know in the L T plane. So, this is given we have to find all the stresses and strains. So, we have to find epsilon x, epsilon y, gamma xy.

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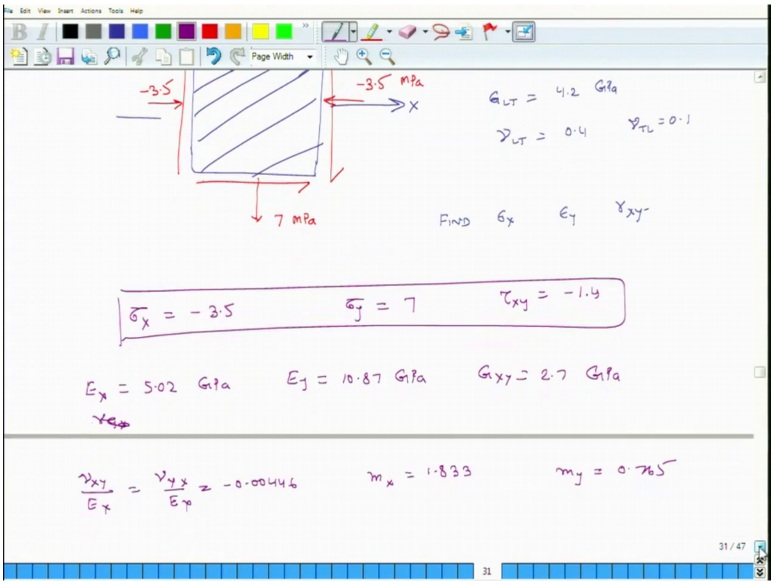

So, how do we do this? So, first we identify what are the values of sigma x, sigma y and tau xy. So, sigma x is how much minus 3.5, based on the picture sigma y equals 14 and tau xy equals.

Student: Sigma y is equal to 7.

Excuse me, it is 7 here and tau xy is minus 1.4. So, it is important, here the stresses shear stress is negative. So, this is the stress state from the material properties and theta is given to be 60 degrees. So, given this value of theta we first calculate E x, E y, E xy, G xy and so on and so forth. So, we find that E x is equal to 5.02 GPa, E y is equal to 10.87 GPa, G xy equals 2.7 GPa, nu yx, excuse me, I will write that in separate thing, nu xy divided by E x is equal to nu yx by E y and that comes out to be minus 0.00446, m x is equal to 1.8333, m y is equal to 0.765.

So, these are the values we have calculated using the relations earlier and now, from this E x, E y all these properties we can calculate epsilon x.

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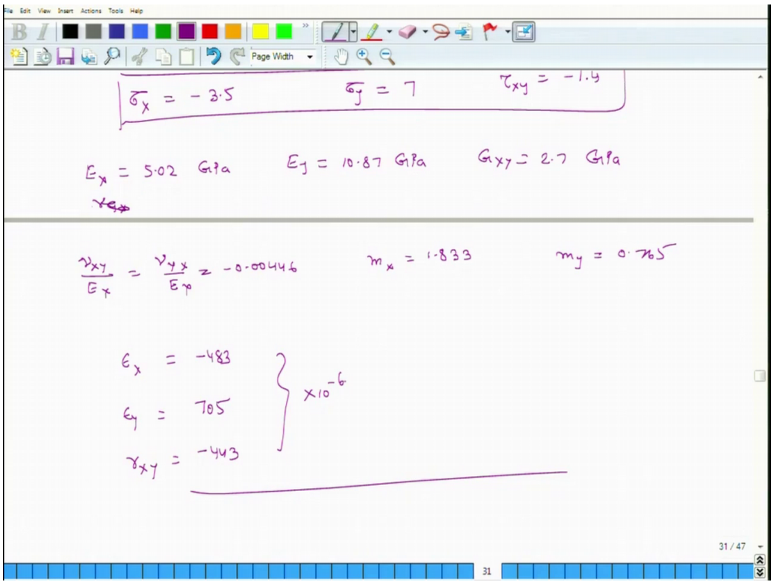

So, epsilon x comes to be minus 483, epsilon y comes to 705 and gamma xy comes to minus 443 and all this times 10 to the power of minus 6 micro strains.

So, this is how we do the computation and I hope you will find these lectures very useful in terms of computing the strains if you know the stresses in an x, y plane for a generally orthotropic plate. Next week onwards we will develop this theory further and we will start working on a formulation which will help us analyze not just single layers, but actually laminates of entire systems. So, that concludes our discussion for today and I look forward to seeing you tomorrow.

Thank you.