Introduction to Composites
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Lecture – 46
Analysis of Generally Orthotropic Lamina

Hello. Welcome to introduction to composites. This is the fourth day of the ongoing week. Yesterday, we learnt the relationships between stress and strain in context of an orthotropic lamina which was loaded in such a way that the external loads were aligned to the material axis of the system and this was the case of general orthotrophy and what we had seen was that in this kind of a situation the stress strain relationships are such that purely extensional strains stresses create only extensional strains and vice versa.

And the presence of a shear stress generates only shear strains and we had also mentioned that there are four independent elastic constants for such a material and those are E L that is the Young's modulus of the material not the Young's modulus the elastic modulus of the material in the longitudinal direction E T which is modulus of the material in transverse direction.

The shear modulus G L T and the Poisson's ratio nu L T which is major Poisson's ratio; so, now, what we will do is we will quickly do an example and then we will move on to the case of general orthotropy.

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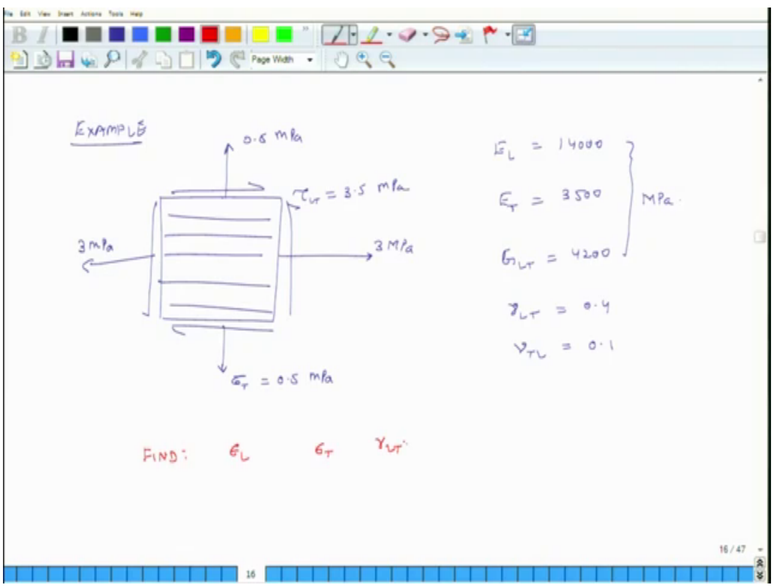

So, example; so, the problem is that we have a lamina fibers are oriented in this direction and I am loading it in such a way that sigma L is equal to 3 MPa. It also has transverse load such that the transverse stress sigma T is equal to 0.5 MPa and then there is a shear stress and the shear stress equals 3.5 MPa.

So, this is the stress state of the lamina and the material properties of the laminar such that E L is equal to 14,000 E T equals 3500 G L T equals 4200 nu L T equals 0.4 and nu T L which is not independent, we can actually calculate it, but here we are just given the value and that is equal to 0.1 and these first 3 entities E L, E T, G L T, they are in mega pascals.

So, the question is calculate. So, what do we have to find. So, we have to find epsilon L epsilon T and gamma L T. So, we start doing this.

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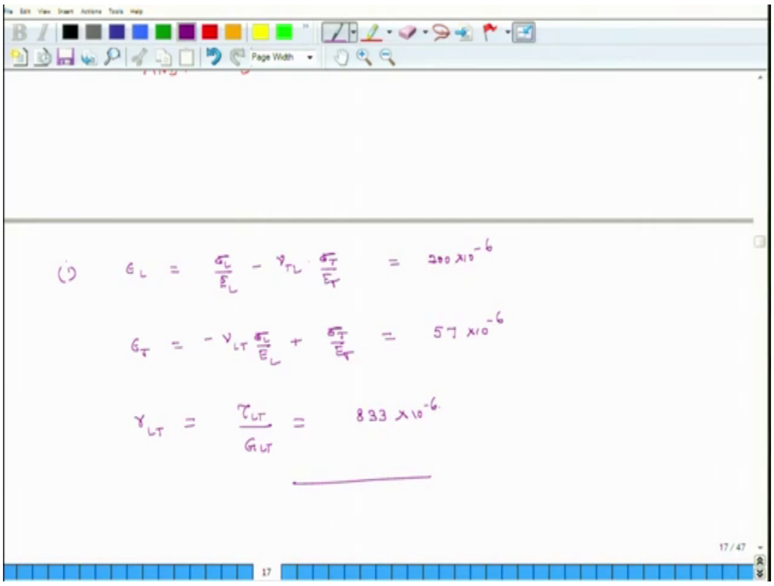

So, epsilon L equals sigma L over E L minus nu T L times sigma T over E T and if I plug in the values what I get is two hundred into 10 to the power of minus 6 and the units of strain are dimensionless. So, it is just that number, then we have epsilon T equals minus L T sigma L over E L plus sigma T over E T and once I do all the calculations I get this as 57 times 10 to the power of minus 6 micro strains and finally, the shear strain is tau L T over G L T and that works out to be 803; 833 micro strains.

So, these are the strains. So, this is how we calculate strains in an in a special in a specially orthotropic lamina, next, we will move to generally orthotropic lamina.

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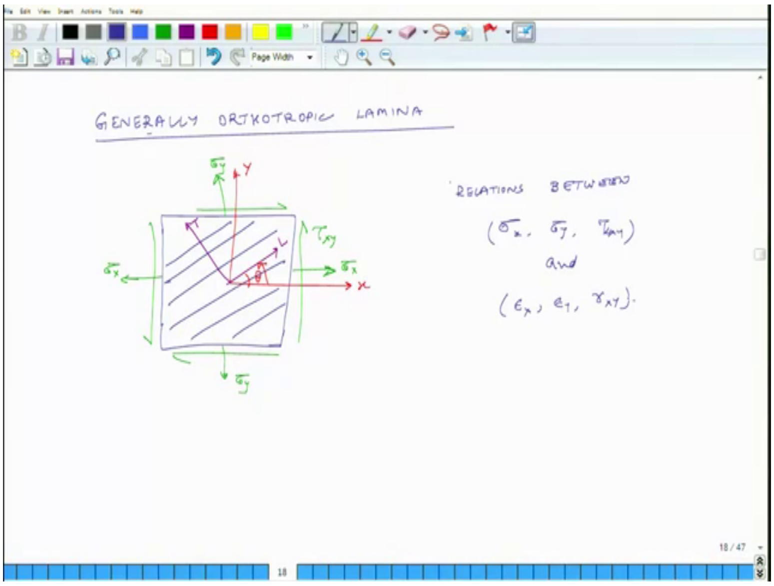

Generally orthotropic lamina and here the reference system and the convention for the angle is important. So, here we have the material sample such that its fibers are oriented like this ok.

So, my L direction this is L direction this is T direction and then my x direction is this and y direction is this and this angle is theta. So, theta is the angle between x and L going upwards in the counter clockwise direction and this plate is can be loaded by sigma x, it can also be loaded by sigma y and it can also see some shear stress. So, that is tau x y and again note the direction of tau x y; this is positive tau x y.

So, sigma x is positive sigma y is positive and tau x y is also positive this is how our sign convention is assumed to be. So, for this kind of a situation what we will do is we will write down relations between. So, we will develop relations between stresses. So, what are the stresses sigma x sigma y tau x y and strains epsilon x epsilon y gamma x y; this is what we will develop.

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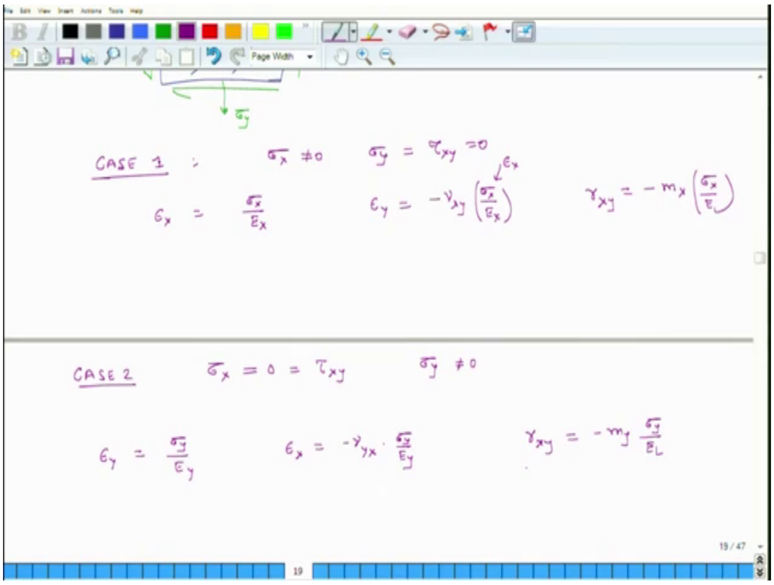

So, again we will have four different cases case one. In the first case, we have only sigma x. So, sigma x is not equal to 0 sigma y and tau x y are 0. Now, this is the case of general or orthotropy. So, when I am pulling the material it will not only exhibit extensional strains, but it will also exhibit shear strains. So, we will write down the relationship between stresses and strains.

So, epsilon x is sigma x by E x where x is the modulus of the material extensional modulus of the material in x direction epsilon y. So, when I pull it; it will also become slimmer and. So, the Poisson; so, it will exhibit Poisson strain and what is Poisson strain minus nu x y first index x which indicates the direction of the external load second index y which x indicates the direction of the extensional strain.

So, nu x y times sigma x by E x because this is nothing, but epsilon x and it will also exhibit a shear strain. So, it will also exhibit shear strain and that we define as one constant minus m x times sigma x by E L sigma x by E l. So, this is the first case two. So, in case two we have sigma x is equal to 0 and the same thing is true for tau x y the only thing which is non0 is sigma y sigma y is not equal to 0.

So, epsilon y; so, I am pulling it in the y direction. So, first thing is sigma epsilon y will be sigma y divided by E y then epsilon x will be there because of poisons effect and that will be equal to negative Poisson's ratio nu y x times the strain in the y direction and what is the strain in the y direction sigma y by E y and there will also be a shear strain gamma x y and gamma x y will be equal to minus m y sigma y divided by E l.

So, this is the second case. So, again if you double the external stress the strain will double because this is linear elasticity. So, this is the second case.

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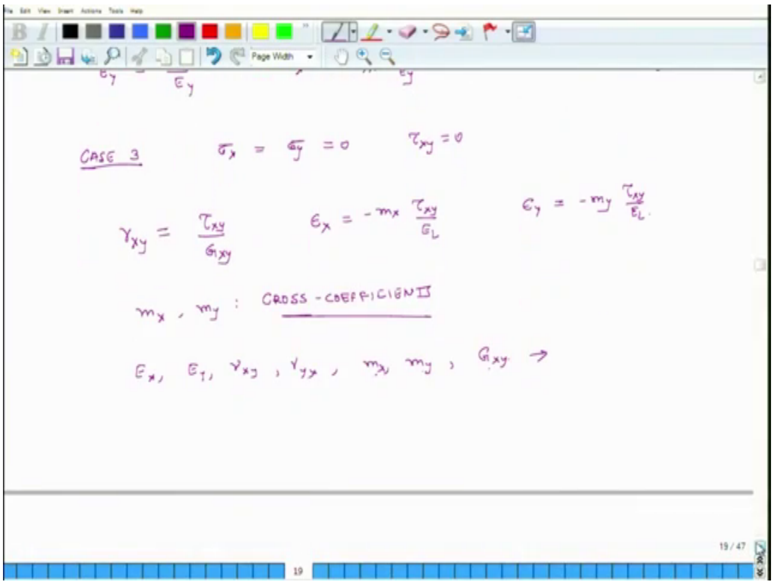

The third case is sigma x equals sigma y equals 0 and tau x y is not equal to 0 and in this case the shear stress will first it will generate a shear strain and what is the value of the shear strain gamma x y is equal to tau x y divided by G x y and as I said it also will generate extensional strains.

So, it will generate epsilon x and that is equal to when we observe it we find and this we will later prove these relations this is equal to minus m x tau x y divided by E L and a epsilon y equals minus m y tau x y divided by E L; these m x and m y are called cross coefficients because they couple the extensional and the shear responses of the system.

So, they connect the stress in longitudinal direction or transverse direction to shear strain and they also connect the shear stress to strains in x and y direction. So, that is why they are known as cross coefficients. Now, here we have how many elastic constants, we have we have elastic constant E x E y nu x y nu y x m x m y G x y. So, we have 1, 2, 3, 4, 5, 6, 7; 7 elastic constants.

But we will later see that they are not necessarily mutually independent we can reduce express all these elastic constants in terms of the four fundamental elastic constants which we had discussed and defined when we were discussing a special orthotropic case this is something we will explain if we probably start today or and then we will certainly do it tomorrow we will express these constants in terms of E L for independent elastic constants. So, these E x E y nu x y nu y x m x m y G x y is just functions of those basic four elastic constants for especially orthotropic plate.

So, these are the 3 cases when we apply only one stress and if we apply all the stresses together that would be case 4.

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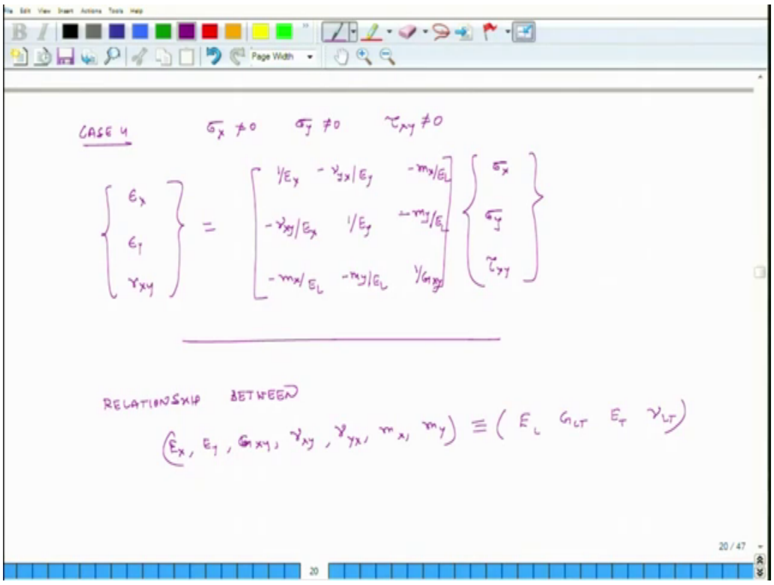

And how do we develop the strain stress relationships basically through the principle of superposition. So, here sigma x is not equal to 0 sigma y is not equal to 0 tau x y is not equal to 0 and through principle of superposition we just add up the contributions from each of the stresses.

So, epsilon; so, get this relation; so, epsilon x is epsilon x will have one component due to sigma x one component due to sigma y and one component due to tau x y. So, the component due to sigma and so though; so, here we will have a vector sigma x sigma y and tau x y and the coefficient associated with sigma x is 1 over E x and then the coefficient associated with y is minus nu y x divided by E y and then this is minus m x by E L and then the second row we have minus nu x y divided by E x 1 over E y and minus m y divided by E L and then we have m x divided by E L minus m y divided by E L and lastly we have one over G x y.

So, these are the relations and this we are getting just as we got the relation in the case of special orthothropy; you know these relations by principle of superposition similarly we have used the same approach to develop stress strain relationship for a generally orthotropic lamina when it is subjected to all the 3 stress is sigma x sigma y and tau x y. So, this is where we are the next thing is. So, now, we have the relations for a special orthotropic lamina and generally orthotropic lamina.

So, then the next question is what is the relationship between E x E y G x y nu x y nu y x m x m y this is for and how can we express these terms these things in terms of E L G L T E T nu L T which are four fundamental constants elastic constants for a especially orthotropic lamina. So, this is our next thing and this is exactly what we will start discussing tomorrow and then I hope you have a great day and we will meet once again tomorrow.

Thank you.