Introduction to Composites
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Lecture – 45
Analysis of Specially Orthotropic Lamina

Hello, welcome to introduction to composites. Today is the third day of this ongoing week and today, what we will discuss is stress; stress strain relationship for an orthotropic lamina. So, that is the focus of our discussion.

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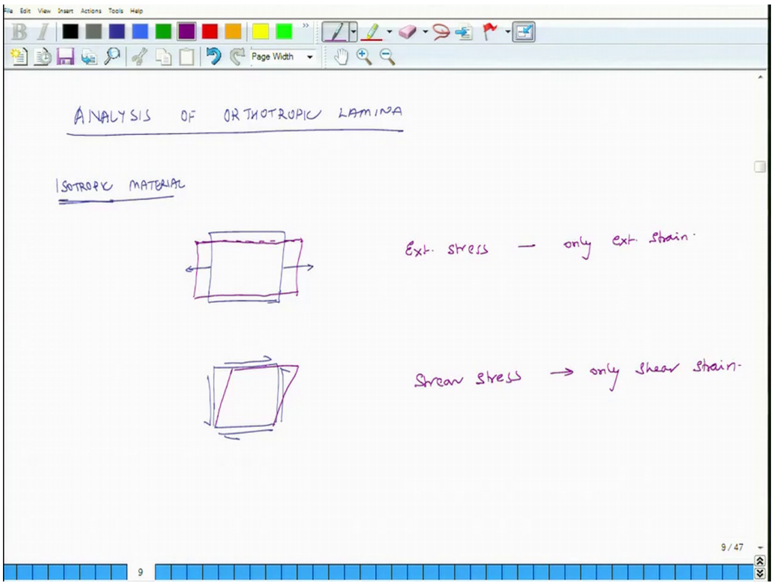

So, essentially what we will like to understand is that if I apply some stresses on an orthotropic lamina how can we predict the strains in it and vice versa.

So, analysis of orthotropic lamina and we have discussed what is an orthotropic lamina, but for purposes of completeness we will just quickly recap what orthotrophy implies. So, first we will consider an isotropic material isotropic material and in for such a material and this is an isotropic material and if I pull it. So, I just pull it I only apply extensional force on it, then it becomes longer and also it becomes thinner or slimmer.

So, this is a response. So, extensional stress it causes only extensional strain. So, the thing becomes longer that is an extensional strain and it becomes slimmer that is also an extensional strain, but in the negative direction. So, this is how isotropic materials behave and if I have the same material and I subject it to a pure shear stress if I apply and I apply I subjected to pure shear stress then I get pure shear strain.

So, the thickness and the length of the object they do not change. So, shear stress, it generates only shear strain this is the behavior observed behavior of isotropic materials in contrast if I have anisotropic material.

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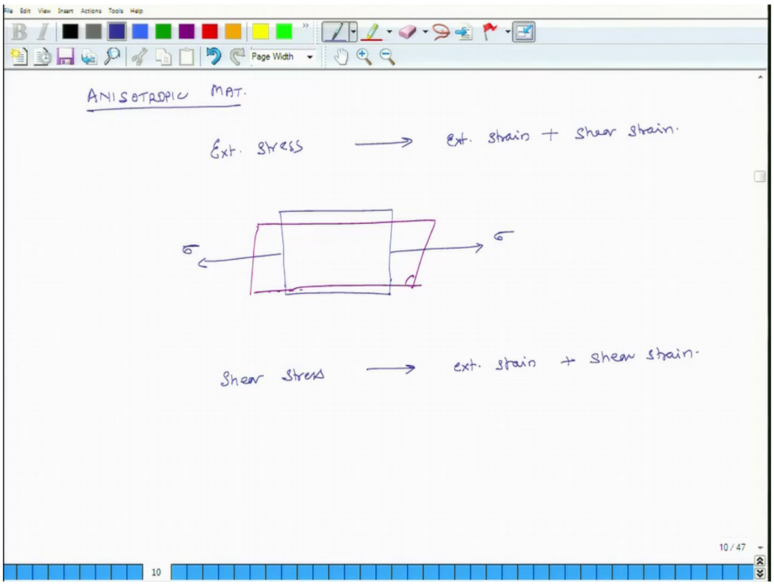

If I have anisotropic material, then extensional strains stress it generates extensional strain and it also generates shear strains, it also generates shear strain. So, to visualize it if I have a rectangular block and I subject it to sigma x, it will not only become thinner and longer, but it will also distort in shape it will also distort in shape.

So, this angle will also change. So, external stress causes not only extensional strain, but also shear strain and if I apply a pure shear stress, it what does it do it also generates extensional strain plus shear strain it generates extensional strain plus shear strain. So, these are anisotropic materials then we have orthotropic materials this is the third case.

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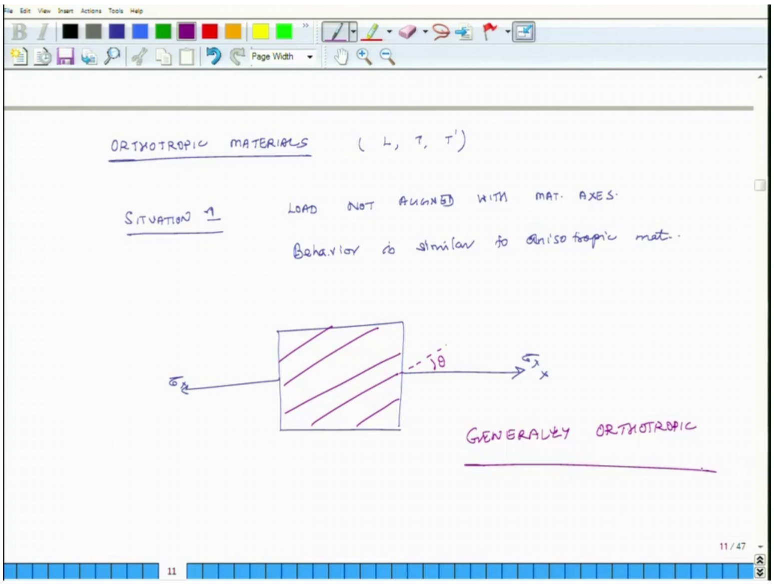

Orthotropic materials and when we say orthotropic materials what does it mean we had discussed this earlier that they have three different material axes ha. So, what are those material axes L T and T prime and in general. So, so we will consider two special case two situations here situation one in a situation one load is not aligned with it is not aligned with material axis and in this case the behavior is similar to anisotropic materials similar to anisotropic materials.

So, what does he do I mean when load is not aligned with material axis. So, let us say this is the material and I am pulling it in x direction. So, so this is my x direction sigma x sigma x and the material axis is not aligned. So, the material axis could be at some angle. So, maybe this is the L direction. So, fibers are running at an angle theta to the thing. So, in that case the behavior is similar to anisotropic materials ok.

So, this is situation one. So, in this case the response of the material is that of a orthotropic material, but this is general orthotrophy. So, it is generally orthotropic orthotropic and then we have situation 2.

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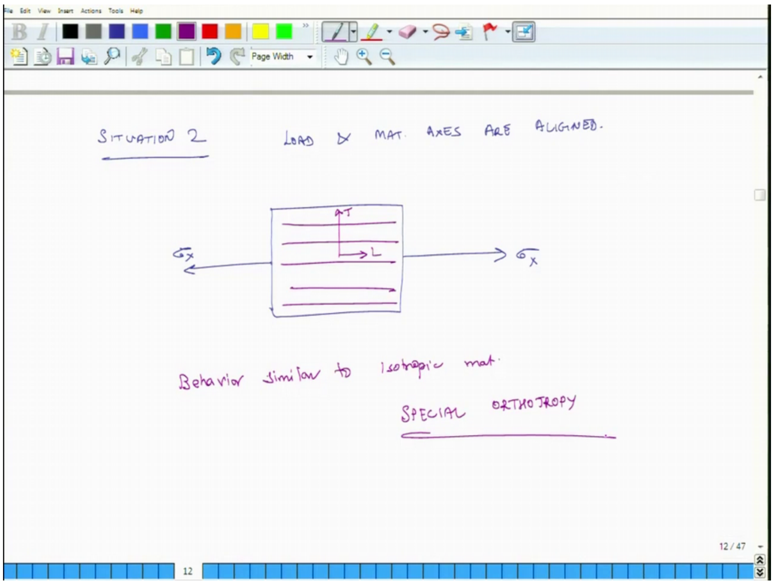

We are load and material axis are aligned. So, how does it look like you have the material let us say this is sigma x sigma x and the fibers either they are in the L direction or they are in the T direction.

So, in this case they are in the L direction. So, this is my L direction and this is my T direction. So, when load and material axes are aligned then behavior is similar it is similar to isotropic material. So, this is the case of special orthotropy. So, in this case if you produce apply an extensional stress it will only generate extensional strains it will not generate shear strains and if you apply a pure shear stress such that the shear stress directions are aligned with material axis, then it will generate only shear restrains it will not generate extensional strains.

So, these are the different situations these are different situations. So, we have the case of isotropy, we have the case of anisotropy and then we have the case of orthotropy and in orthotropy, we can have two kind of situations general orthotropy where the behavior of material is similar to that of anisotropic materials and special orthotropy where the material behaves as an isotropic material.

So, here now what we will do is we will discuss first special orthotropy and then we will go to general orthotropy.

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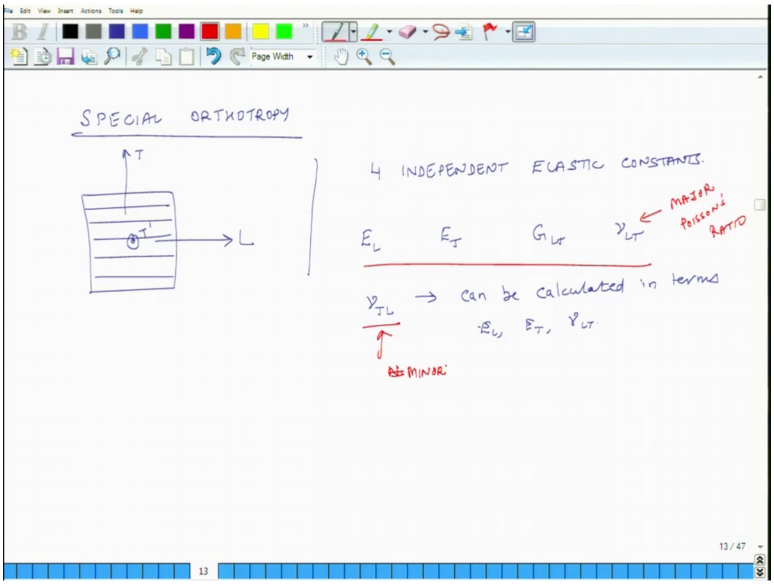

So, we will start with special orthography. So, this is my material and let us say; this is the direction of the fibers. So, this is my L direction, this is the T direction and the direction normal to the plane of this board is T prime direction. So, this is T prime direction normal to the plane of this screen which you are watching now we had mentioned earlier in one of the classes and this later, we will actually prove it is that especially orthotropic material. So, it has 4 independent elastic constants.

So, what are those these are E L E T and we have learned how to calculate these if we know the material properties G L T and nu L T and then nu T L can be calculated in terms of E L; E T and nu L T. So, these are the 4 independent elastic constants, this one nu L T is called major poisons ratio we have again discussed this earlier and this one is known as minor, it is a minor Poisson's ratio.

Now, let us look at the stress strain behavior of this material.

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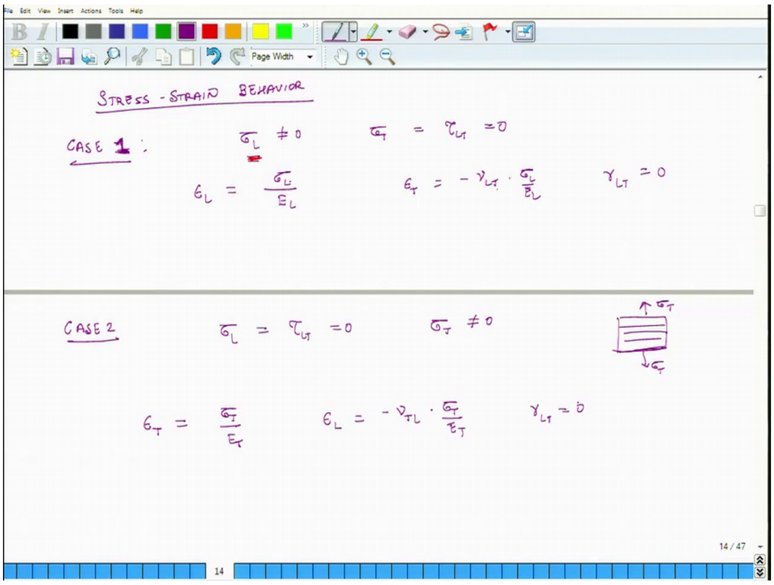

And we will look at three actually 4 different cases case one. So, if sigma L is not 0 and. So, this kind of a material we can apply three different loads on it three different types of stresses sigma L sigma T and tau L T those are the three things. So, if sigma L is not 0, but sigma T equals tau L T equals 0 then what will happen this system will generate strains in that case epsilon L is what sigma L divided by E L; this is how we have defined E L and because the thing becomes a little longer, it will also become slimmer because of the poisons effect.

So, the transverse strain epsilon T will be what it will be minus nu L T times epsilon L why is it nu L T the first subscript tells us the direction of the loading direction of loading is L direction this is the direction of the loading L direction and the second subscript tells us the strain direction. So, strain is happening in the transverse direction because it is shrinking. So, epsilon T equals my negative of nu L T times epsilon L and what is epsilon L epsilon L is nothing, but sigma L over E L.

So, this is what I will write; now this is a especially orthotropic material the load direction and the direction of the fiber is aligned. So, in this case I am applying only extensional stress in it. So, there will be no shear strain. So, gamma L T equals 0 gamma L T is equal to 0. So, this is case one next look at case 2; in case 2, we apply a pure transverse stress only a transverse stress.

So, in this case sigma L is equal to tau L T is equal to 0, but sigma T is not equal to 0. So, when I apply a pure transverse stress; what will happen it will have a transverse strain? So, epsilon T equals sigma T by E T as this thing becomes longer in the transverse direction it will also become narrower in the longitudinal direction because of Poisson's effect.

So, epsilon L is equal to minus nu T L times epsilon T, but epsilon T is what sigma T over E T. So, I will just erase this and replace it by this. So, it is sigma T over E T and once again. So, here what is how are we loading it my fibers are like this and I am loading it in transverse direction. So, here also the loading is aligned with the axis here the load is aligned with the transverse axis and. So, I am just applying a pure extensional strain and because it is the case of special orthotropy, there will not be any shear strain gamma L T.

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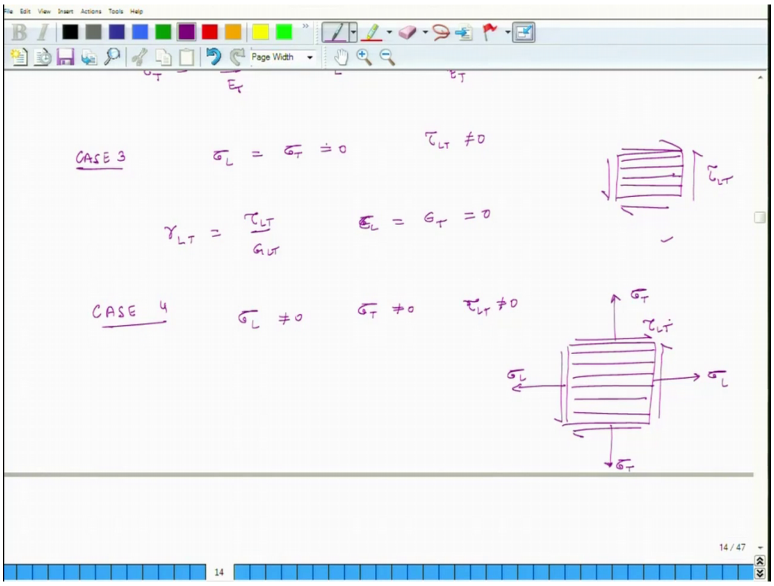

So, gamma L T will be 0 the third case is that of a pure shear stress. So, here sigma L equals sigma T is equal to 0 only shear stress is present only shear stress is present. So, and; so, this is my L axis and I am applying a shear stress. So, please note this convention of shear stress is positive if I applied shear stress in the other direction then it would be negative shear stress. So, this is a positive shear stress this is our convention and in this case gamma L T equals tau L T by shear modulus of the material and the extensional strains in transverse and longitudinal directions they are both 0 because of a special orthotropy.

So, in the first case I applied only an extensional stress in the length direction longitudinal direction here pure extensional stress in the transverse direction and here pure shear stress and now we will discuss a fourth case; case 4 and what is case 4 all the stresses sigma L sigma T and tau L T are present. So, sigma L is not equal to 0 sigma T is not equal to 0 and tau L T is not equal to 0.

So, how does the loading looks look like. So, this is my orientation of fibers I am applying a stress in the length in the longitudinal direction, I am also applying a transverse load or stress and I am also applying a shear stress and I am also applying a shear stress. So, this is what I am applying. So, in this case I am interested in finding out the strains.

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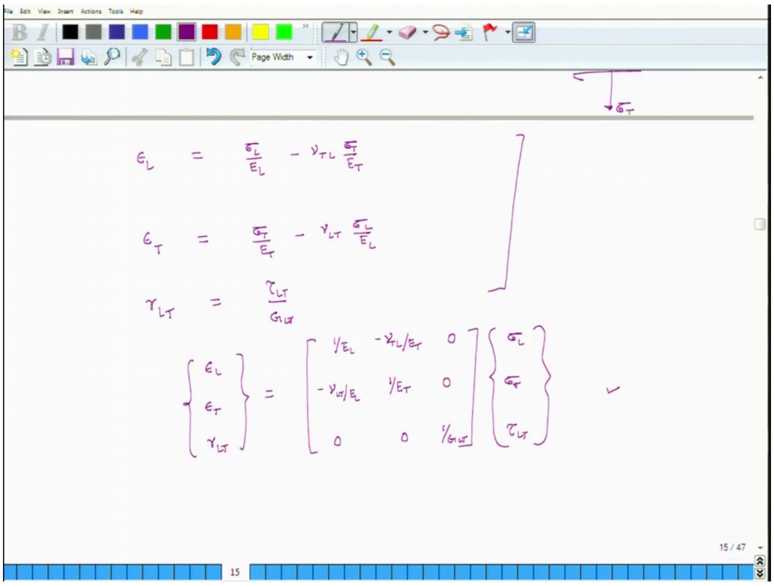

So, I want to find out epsilon L I want to find out epsilon T and I want to find out gamma L T. Now how do we do this we already know that if there is pure stress in L direction, then these are the strains if there is pure only stress in the transverse direction then these are the strains and if there is a pure shear stress if there is a pure shear stress then these are the strains. So, and in the fourth case all the three stresses are present. So, we just use the principle of superposition and just add up all the values and we get the overall strains.

So, overall strain in the length in the longitudinal direction is strain in L direction due to sigma l. So, what is that sigma L divided by E L plus strain in L direction due to sigma T and that will be the Poisson's component. So, it is nu T L sigma T over E T and the shear stress will not create any extensional strain similarly epsilon T is equal to sigma T over E T this is the component just due to sigma T plus the effect of Poisson plus the; Poisson effect when you are applying sigma L and that is minus nu L T sigma L over E T our sigma L over E L and this is equal to tau L T over G L T, ok.

So, these are the overall strains in the whole system. So, we can write this also in a matrix form epsilon L epsilon T gamma L T equals. So, on my this side I will have vectors sigma L sigma T tau L T and here I have 1 over E L minus nu T L by E T 0 minus nu L T by E L 1 over E T 0 0 0 1 over G L T. So, this is the situation. So, what we have shown till so far is now with these relations.

So, this is the matrix form or this is the general relation a conventional form if we have especially orthotropic lamina which is subjected to stresses in longitudinal transverse and shear directions then we can predict its strains using these equations. So, this concludes our discussion for today tomorrow we will continue this discussion and we will also start exploring the relationships for general generally orthotropic lamina. So, with that I closed my lecture for today and I look forward to seeing you tomorrow.

Thank you.