Introduction to Composites
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Lecture – 43
Concept of Tensor

Hello. Welcome to introduction to composites. This is the beginning of the eighth week of this course. Till so far, what we have been discussing is how some introductory information on composites, how are different types of composites fabricated typical properties of very commonly used fibers matrices and filler materials and then over the last several weeks, we have been discussing from an analytical or mathematical standpoint, the behavior of a unidirectional composite laminate or lamina when the loads which the lamina experiences are aligned to the longitudinal and transverse axis.

Starting today what we will do is go to the next level that what happens when the loads are not aligned to l or t material axis, then how does the system behave again the composite we are going to discuss is still unidirectional lamina, but the fibers the orientation of the fibers and the orientation of the external load need not be aligned to each other. So, that is what we plan to start today.

But before we do that; I will like to briefly discuss some background information about stresses and strains and what happens when we change the frame of reference and we rotate the frame of reference what happens to stresses and strains.

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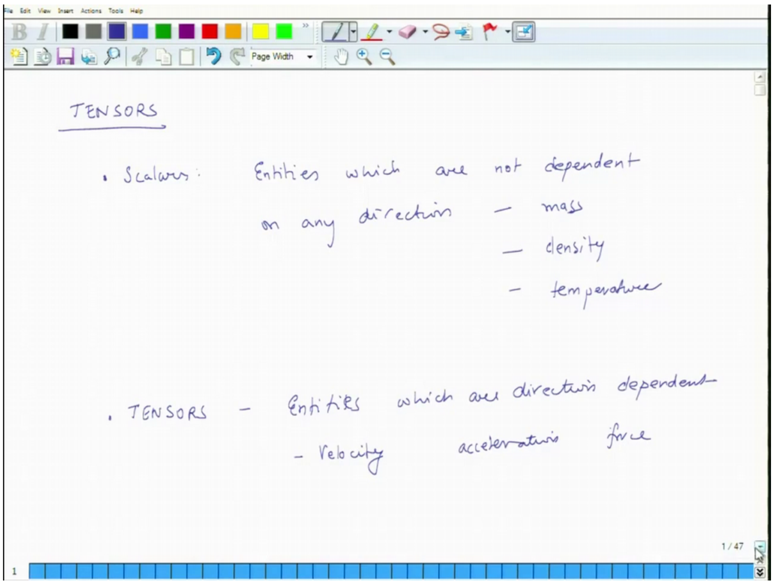

So, in that context the first concept we are going to discuss is that of tensors. Now, we all know; what are scalars. So, we have scalars these are physical entities which are not dependent on any direction.

So, an example of a scalar would be mass of an object it does not depend if you change the direction or the reference coordinate system the mass of a body does remains unchanged another example would be density or for that sake temperature. So, these are scalars then we have another set of entities which actually change which depend on direction and they are specific their values are specific to the coordinate system which we are using.

So, these entities are called tensors these entities are called tensors. So, these are entities which are direction dependent with their direction dependent. So, you may have already heard of vectors which are dependent on the direction. So, they are associated with directions, but in general vectors are a subset of the larger set known as tensors. So, example of this would be velocity ok.

Velocity of an object or acceleration or force; so, what do I mean by the dependence on direction. So, suppose there is a force.

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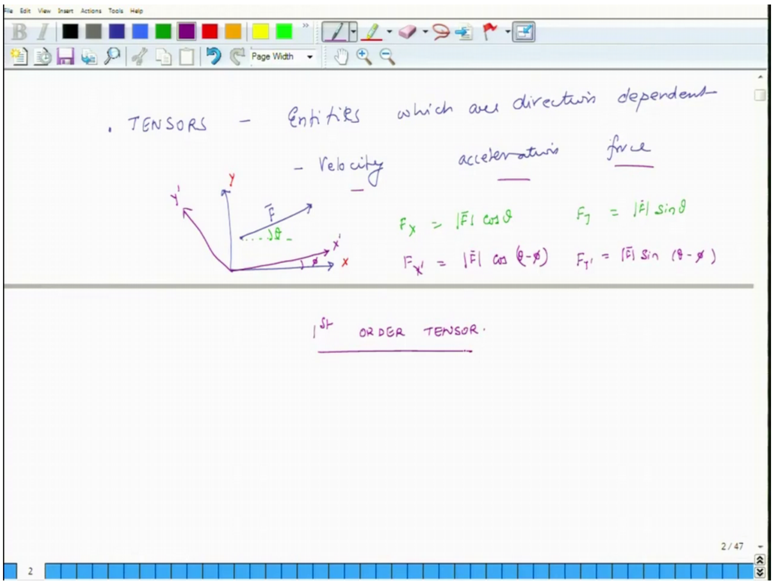

And it is magnitude is the length depicted by this, now I can have this is my x axis. This is my y axis and with respect to this coordinate system if suppose the angle with respect to x axis is theta then F x is equal to modulus of F times cosine theta and F y component in the y direction is modulus of F magnitude of F times sin theta.

Now, let us consider another situation that I have another set of another coordinate frame and this coordinate frame is basically the first coordinate frame, but it is rotated by some angle. So, let us say this is x prime and y prime and this angle between x prime and x axis is let us say phi, then if I measure the values of force along x prime and y prime direction, then I will say that F x prime is equal to modulus of F cosine theta minus phi and in the y prime direction it is modulus of F times sin of theta minus phi.

So, what we see here is that the values are the components of the force are specific to the coordinate system which we are considering if the coordinate system rotates the components of the force value they also change. So, this is an example of tensor now in this case that dependence of the entity which in this case is force it just depends on one single direction and that direction is this angle phi this direction is angle phi, but there are entities which can depend on more than one directions there are entities which depend on more than one directions.

So, in this case of velocity acceleration and force the dependence on direction is just on one direction which is associated with angle phi. So, these are known as first order tensors here the direction is only one a. So, the components are associated with only one particular direction or so, that is why it is known as first order tensor, but there could be entities which where the association of the entity could be related to more than one directions, those will be tensors of higher order not just first order and there could be second order tensors fourth order tensors and so on and so forth.

So, 2 entities which are second order tensors which we will use very frequently in this class our stress.

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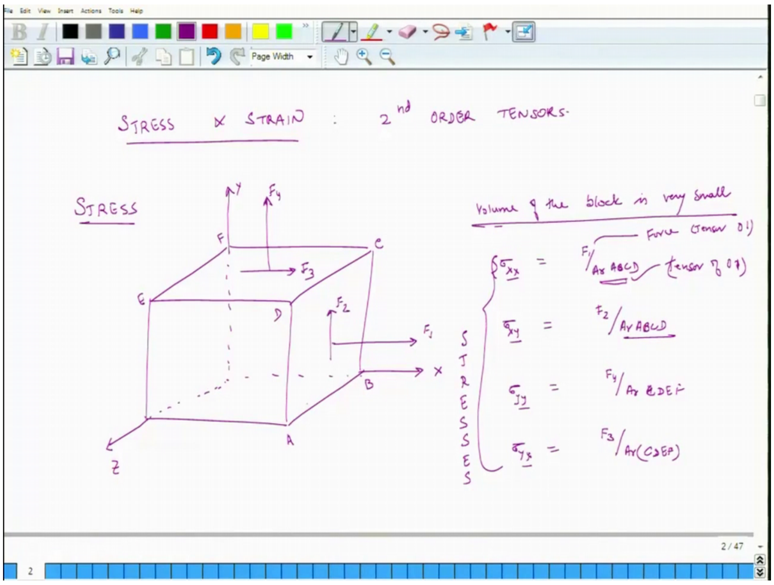

And strain and I will explain why they are second order tensors. So, these are second order tensors these are second order tensors. So, let us understand why they are second order tensors. So, first we will talk about stress.

First, we will talk about the stress. So, consider a rectangular block and let us say this is x axis, let us say this is y axis and this is actually, let us call this as z axis and this is y axis. Now on this block, I can apply all sorts of forces I can apply and let us say that this block is extremely small the block size of the block is extremely small. So, I can apply a force on this plane in this direction I can also apply a force normal to this plane.

So, let us say this we call it f. So, I can also apply a force in this plane I can apply a force on this plane I can again apply a force on this plane which is tangential to the top surface I can apply a force on the top surface which is normal to the top surface. So, let us call this F one let us call this F 2, let us call this F 3, let us call this F 4. So, we will just talk about these four forces and if the object if the size of the object if the volume of the block is very small, it is very small, then I can say that sigma x; x equals F 1 divided by.

So, let us call this area. So, A, B, C and D and this is E and F. So, this is divided by area A, B, C, D and this is a tensile stress, it is a tensile stress and here and then sigma x y is what it is F 2 divided by area A, B, C, D. So, what do we see here and then I can also say that sigma y y equals F four divided by area C, D, E, F and sigma y x equals F 3 divided by area C, D, E, F, ok.

So, these are the 4 definitions. Now these are stresses these are stresses these are stresses and we see that the value of the stress it depends on 2 elements, it depends first on the force and it also depends on the area; now here the area. So, it depends on force the numerator is force and force is a vector of first order or it is a tensor of first order and what about this area is also a vector because it has area the direction of area is typically defined as the direction of the direction of the normal which is perpendicular to that surface.

So, this denominator is also A. So, this is tensor of order one and this is also a tensor of order 1. So, both the numerator as well as the denominator is tensors of first order they are tensors of first order. So, and because the overall thing is dependent on 2 independent directions that is why we say that sigma the stress sigma xx, sigma x y, sigma y y, sigma y x is nothing, but a tensor of second order it is associated with 2 mutually independent directions; the first direction is associated with the direction of the force and the second direction is associated with the direction of the area on which the force is being applied on which the force is being applied and that is why we have 2 subscripts x x, x y and y x and so on and so forth.

So, we can do the same thing for 3 directions all the 3 directions and we will have nine different stresses. So, typically in this case we can generalize.

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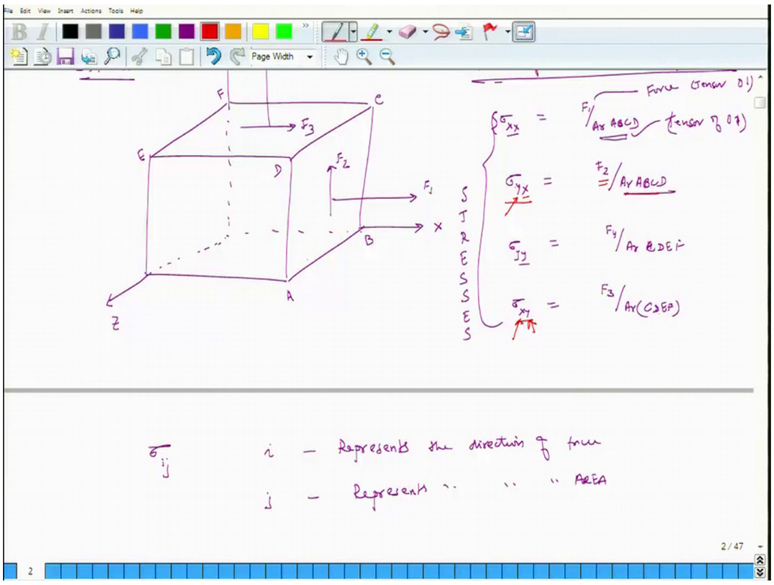

That sigma i j is a stress of second order where i; it represents the direction of actually I will just invert this. So, that we are consistent with standard literature terminology. So, this I will call it sigma y x and this I will call it sigma x y. So, i which is the first index first in the subscript, it represents the direction of the force and j represents the direction of area.

So, now this; let us look at it in this case y sigma y x is the shear stress and what is this? Y represents the direction F 2 and x represents the direction of area the direction of normal to the area which is area of A, B, C, D and in sigma x y the first index is associated with the direction of F 3 which is x direction and the second subscript is y which is associated with the direction of the normal of area C, D, E, F.

So, if there are 3 sets of orthogonal directions then totally we will have nine components of sigma i j because I can have 3 values x y and z.

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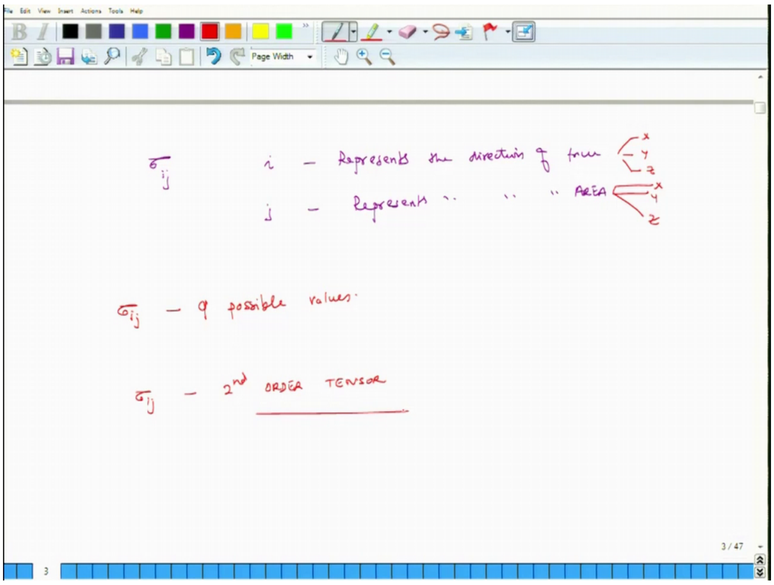

And similarly area can also have x, y, z, 3 independent values. So, total we will have sigma i j will have nine possible values sigma i j will have nine possible values. So, this is the example of a second order test. So, sigma i j is a second order tensor.

Force is a vector and vector is what it is just nothing, but a first order tensor similarly area or the direction of its normal is a first order tensor, but sigma i j or stress is nothing, but a second order tensor the other thing which is a second order tensor and it is important in context of this course is this thing called strain.

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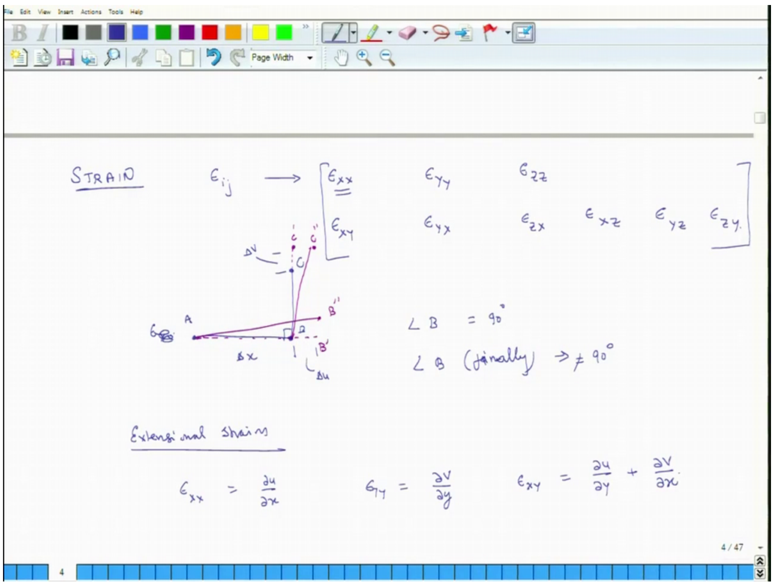

It is strain. So, strain we write epsilon i j and here you have what are these values epsilon x x, epsilon y y, epsilon z z and then epsilon x y, epsilon y x, epsilon z x, epsilon x z and epsilon y z and epsilon z y.

So, these are nine components of the strain tensor; what do that mean. So, so think about it. So, again we will explain how is the strain associated with 2 directions how is the strain associated with 2 directions. So, epsilon x x; so, suppose we have a small line element a small line element and its length is delta x suppose that line element is delta x. So, let us say this first point is A and the final point of the line element is B.

Now, I can stress this line element I can stress this line element to a place called B prime to A place called B prime and I am keeping the first point same fixed. So, that is my reference. So, I just keep it fixed, but I can not only stretch it, but I can also rotate this line element. So, I have rotated this line element and let us say. So, this is B prime prime. So, I stretch it and then I rotate it. So, this is my final position and then I have let us say another line element B C I am just going to draw this again.

So, this is the stretched position and this position was B and then starting from B let us say there is another line element and let us say this we call this v C and I can do the same thing to B c. So, I can make it longer and I can also rotate it. So, let us say this is C prime prime C prime. So, this is my this thing. So, you consider initially A B and B C, suppose they were at 90 degrees. So, you can have 2 sticks, they were at 90 degrees with respect to each other and then I stretch the first stick and also rotate it and I also stretch the second stick and also rotate it.

So, what is happening? What is happening is that the line element A B. So, so is becoming longer and it is getting rotated and the line element B C is getting longer and it is also getting rotated, but they need not rotate by the same amount, they need not rotate by the same amount if they do not rotate by the same amount initially the angle initially angle at B was 90 degrees and if they do not rotate by the same amount then the angle at location B finally, it may not be 90 degrees.

So, line elements are becoming longer and they are also the angle is also changing. So, when we talk about extensional strains extensional strains what is an extensional strain it is the original length on the change in length divided by the original length. So, our epsilon x x is what is change in length and let us say that change in length is called.

So, let us say this distance is called delta u, then it is D u over D x this is the definition and similarly, epsilon y y, if this distance is delta u, then it is it is not delta u it is delta v sorry displacement, then it is partial derivative of v with respect to partial derivative with respect to y and the change in angle away from 90 degrees epsilon x y away from 90 degrees, if you analyze this picture carefully will be nothing, but del u over del y plus del v over del x when you do the mathematics of this. So, what do you see here what you see here is that and similarly we can have other components also epsilon z z, epsilon y z, epsilon z x, epsilon y x, epsilon z y, epsilon x z ok?

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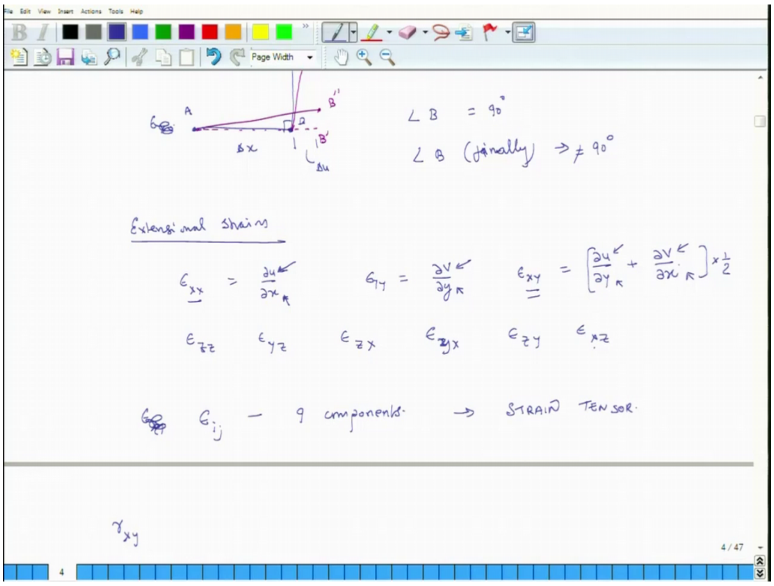

So, you can have all these nine components and all these nine components are dependent again on 2 directions one is the direction of the displacement and the second one is the direction of the original direction of the line. So, strain tensor epsilon x x is nothing, but very small change in displacement when the very small line of length delta x is becoming longer in the x direction only ok.

Instead; so, it is associated with 2 directions direction of displacement and direction of the original line element here also, it is dependent on 2 directions direction of displacement it is y the displacement in y direction is v and original length of the line element is in y direction and then x y is dependent on again 2 directions and it has 2 components displacement in x direction divided by the length of the line in the y direction plus displacement in y direction divided by length of the line in the x direction ok.

So, again these strains are dependent on 2 directions one is the direction of displacement and the other one is the direction of the line element which we are interested in which will we are interested in. So, strains we can also write a strain in general as a second order tensor and again it has nine components nine components and. So, this is a strain tensor this is the strain tensor the last thing I like to add in this context is the shear strain.

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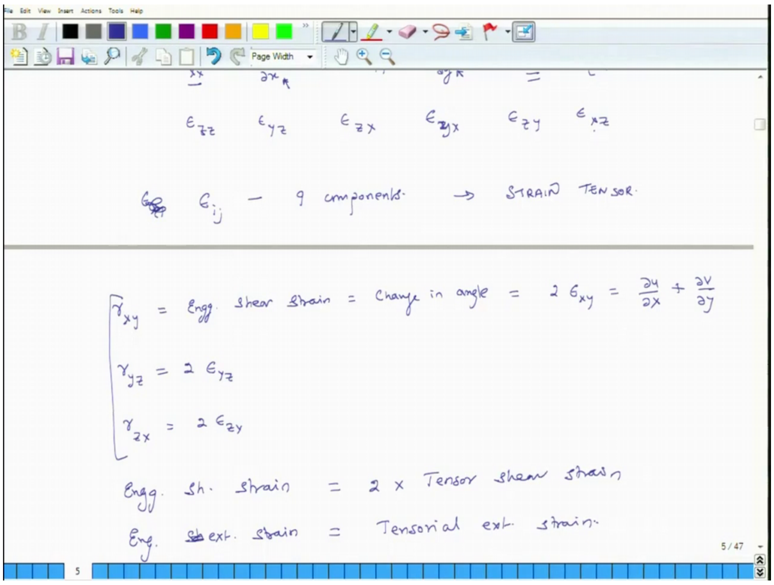

So, this; excuse me, this one, this shear strain the in terms of tensor is nothing, but defined as half of the rotation, it is defined as half of the rotation that is how the mathematics works out, but the actual shear strain this is this symbol is for engineering strain engineering shear strain.

So, engineering shear strain is what is change in angle it is change in angle and that is equal to twice of epsilon x y. So, this is what partial derivative of y with respect to x plus partial derivative of v with respect to y similarly gamma y z is equal to twice of gamma epsilon y z and gamma z x is equal to twice of epsilon z x. So, in a broad sense engineering shear strain is equal to twice of tensor shear strain and engineering extensional strain is nothing, but same as tensorial extensional strain for stresses engineering stress and tensor stress they are same, but when we talk about strains the extensional strains the definitions of definitions of strain whether we are talking about engineering strain or shear or tensor strain they are same, but the definition the value of engineering shear strain which is the total change in angle it is 2 times the tensor shear strain.

So, this is something we have to remember because this will be you will find this useful maybe next week when we discuss more about tensors. So, what we have covered today is give you a brief idea about tensors then we have said that there are entities which are direction independent, we call them scalars there are entities which are dependent on directions these are tensors if it depends on one single set of directions, then it is a first order tensor if it depends on 2 single 2 independent sets of directions, then it is a second order tensor.

As a special case you can also call vectors as first order tensor and as another special case you can also call scalars as zero order tensors. So, this concludes our discussion for today. Tomorrow, we will continue this discussion and we will revisit some of the equations which relate to stresses and strains when they are seen with respect to you know 2 different sets of directions. So, with that I conclude our discussion for today and we will meet once again tomorrow.

Thank you.