

Introduction to Composites
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Lecture – 42
Other Properties

Hello. Welcome to introduction to composites today is the last day of the ongoing week of this MOOC course. Today, we will discuss different relations which will help us calculate different properties of unidirectional laminates other than the ones which we have already discussed. So, we have we will be just talking about thermal coefficients moisture expansion coefficients different transfer properties including thermal conductivity electrical conductivity permeability diffusivity and so on and so forth.

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THERMAL EXPANSION COEFFICIENTS α_f α_m

$$\alpha_L = \frac{1}{E_L} [\alpha_f E_f V_f + \alpha_m E_m V_m]$$

$$\alpha_T = (1 + \nu_f) \alpha_f V_f + (1 + \nu_m) V_m \alpha_m - \alpha_L \nu_{LT}$$

If $V_f > 0.2$

$$\alpha_T = \alpha_f V_f + (1 + \nu_m) V_m \alpha_m$$

So, let us start with thermal coefficient thermal expansion coefficient and the way we are going to do this is we will not derive results for each of these properties rather we will directly share the results which you can use. So, thermal expansion; suppose you have material compare unidirectional laminate and it has an expansion thermal expansion coefficient of alpha f for the fiber and alpha m for the matrix.

Then a very general relation is alpha L which is the longitudinal thermal expansion coefficient of the material overall composite material equals one over E L alpha f E f V f plus alpha m E m V m. So, this is in the longitudinal direction and in the transverse direction, it is 1

plus Poisson's ratio of the fiber times $\alpha_f V_f$ plus $1 + \nu_m$ plus Poisson's ratio of the matrix times $V_m \alpha_m$ minus $\alpha_L \nu_L T$ and if the volume fraction of the material is very small and by small, I mean if excuse me volume fraction of the of the fiber is very large and specifically, if it is larger than 0.2, then I can rewrite the expression for α_T as something like this $\alpha_f V_f + 1 + \nu_m V_m \alpha_m$.

The impact of Poisson's ratio $\nu_L T$ it becomes negligible. So, these are the relations for thermal expansion coefficients next we will look at moisture expansion coefficients.

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MOISTURE EXPANSION COEFFICIENT (β)

$$\beta_m = \frac{1}{3} \frac{\rho}{\rho_w}$$

$\rho \rightarrow$ density of matrix
 $\rho_w \rightarrow$ water. } No voids

$$\beta_L = 0 \quad \beta_T = \frac{\rho_c}{\rho_m} (1 + \nu_m) \beta_m$$

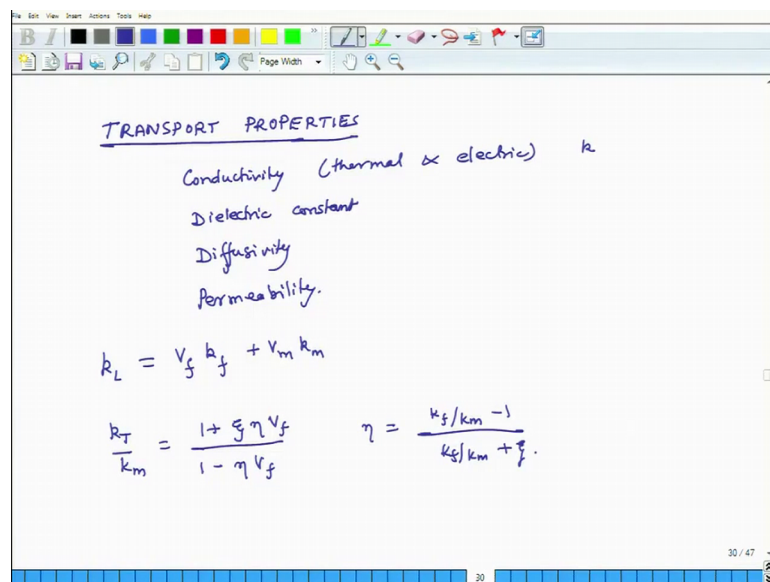
Moisture expansion coefficient; So, a lot of matrix materials when they are subjected to humidity they absorb moisture and in process their overall volume becomes larger. So, this is designated by beta. So, for the matrix beta m is what is equal to one third of rho divided by rho water.

So, what is rho rho is the density of matrix after it has absorbed moisture. So, if it absorbs more moisture beta will become larger and so on and so forth and rho w is density of water density of water. Now, here we assume that when the matrix is absorbing water there are no air spaces and voids. So, we assume that there are no voids in the system because there are voids, then first the water will go and fill up those voids and then it will try to explain the system.

So, this works when there are no voids. So, this is the relation for the matrix moisture expansion coefficient for composite we say that beta L is equal to zero in the longitudinal direction why because in the longitudinal direction you have fibers. So, when it tries to expand in the direction of fibers; fibers are very stiff and they do not let it expand significantly. So, this is approximately equal to 0, but the transverse value of beta; beta T, this is equal to density of the composite divided by density of matrix material after it has absorbed moisture times one plus nu m which is the Poisson's ratio times beta m ok.

So, using this relation we can compute how much a composite expands by when it absorbs moisture as long as the composite is unidirectional in nature.

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The next set of relations will be related to transport properties transport properties. So, in this the; I will cite a general relation and that relation will be valid for all transport properties. So, what are these transport properties conductivity and it is both thermal and electric then we have dielectric constant, then we have diffusivity and we have permeability.

So, i will write the relation longitude. So, for instance we consider conductivity. So, conductivity is k. So, k L which is the conductivity of the composite in the L direction longitudinal direction is pretty simple it is V f k f plus V m k m where V f and V m are volume fractions for the fiber and matrix respectively and k T over k m which is the transverse conductivity is given by the Halpin tsai relation 1 plus zeta theta V f divided

by $1 - \eta V_f$ where η is equal to k_f over k_m minus one divided by k_f over k_m plus ξ and here the definition of ξ is again different. So, what is it?

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Conductivity

Dielectric constant
Diffusivity
Permeability.

$$k_L = V_f k_f + V_m k_m$$

$$\frac{k_T}{k_m} = \frac{1 + \xi \eta V_f}{1 - \eta V_f} \quad \eta = \frac{k_f/k_m - 1}{k_f/k_m + \xi}$$

$$\log \xi = \sqrt{3} \log(a/b) \quad \text{except } 0 \quad \xi = 1$$

Log of ξ equals square root of three times log of a over b for all fibers except for circular cross sectional fibers ξ equals 1.

So, if it is a fiber is non circular then we use this relation.

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EXAMPLE

GRAPHITE FIBER	GLASS FIBER	MATRIX
$k_{jL} = 80 \text{ W/m}\cdot\text{c}$	$k_f = 1.05$	$k_m = 0.25$
$k_{fT} = 12.5$	$V_f = 60\%$	

GLASS BASED

$$k_L = 0.6 \times 1.05 + 0.25 \times 0.4 = 0.73 \text{ W/m}\cdot\text{c}$$

$$\xi = 1 \quad \eta = \frac{(1.05/0.25 - 1)}{1.05/0.25 + 1} = 0.615$$

$$\frac{k_T}{k_m} = \frac{1 + 0.615 \times 0.6}{1 - 0.615 \times 0.6} = 2.17 \quad k_T = 0.543 \text{ W/m}\cdot\text{c}$$

So, we will do an example; let us say I have a fiber system and let us say we are talking about graphite fibers graphite fiber. So, what is its k f? So, graphite fiber has itself is non isotropic. So, it has one conductivity in the L direction k f L is equal to 80 watts per meter per degree centigrade and k f transverse is actually pretty less, it is 12.5; 12.5 and then the other case we take is glass fiber glass fiber and it is glass is isotropic. So, k f is equal to 1.05 and then matrix. So, let us say we have a matrix material whose conductivity k m is 0.25 and we say that volume fraction is 60 percent.

So, we compute different conductivities for glass composite and graphite fiber based composite. So, first let us look at glass fiber glass based composite glass based composite. So, k L is equal to 0.6 which is volume fraction times its conductivity 1.05 plus 0.25 into 0.4 and that gives us 0.73 watts per meter per degrees centigrade to compute k T transverse conductivity zeta equals 1 eta equals 1.05 by 0.25 minus 1 divided by 1.05 by 0.25 plus 1 is equal to 0.615.

So, k T over k m equals 1 plus 0.615 into 0.6 divided by 1 minus 0.615 into 0.6. So, this gives me 2.17 or k T equals 0.453543 watts per meter per degrees centigrade. So, this is for glass which is pretty straightforward and next we look at graphite based system.

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The image shows a handwritten slide titled "GRAPHITE - Epoxy" with the following calculations:

$$k_L = 0.6 \times 80 + 0.4 \times 0.25 = 48.1$$

$$\eta = \frac{12.5 / 0.25 - 1}{12.5 / 0.25 + 1} = 0.961$$

$$\frac{k_T}{k_m} = \frac{1 + 0.961 \times 0.6}{1 - 0.961 \times 0.6} = 3.72$$

$$k_T = 0.93 \text{ W/m}^\circ\text{C}$$

Graphite epoxy system; so, k L is a straightforward k L is equal to 0.6 into 0 point; you know 80 plus 0.4 into 0.25. So, that gives me 48.1 and then for computing the transverse

thermal conductivity, we have to use the transverse direction thermal conductivity of the glass fiber.

So, first we compute η . So, that is equal to 12.5 divided by 0.25 minus 1 divided by 12.5 divided by 0.25 plus 1 which works out to be 0.961 . So, k_L over k_m equals 1 plus 0.961 into 0.6 which is the volume fraction divided by 1 minus 0.961 into 0.6 .

So, this is equal to 3.72 . So, k_T is equal to 0.93 watts per meter degrees centigrade. So, what do we see that for fibers which have different conductivities in different directions we have to use appropriate value of the conductivity in the correct direction. So, so this should be k_T over k_m finally, we summarize whatever we have learnt over this week and also part of the or over this week and last week and we are going to do it through a table.

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		S - STRONG	W - WEAK	N
		FIBER	MATRIX	INTERFACE
TENSION	PROP.	S	W	N
	E_L	S	W	N
	σ_L	W	S	S
	E_T	W	S	S
COMPRESSION	σ_T	S	W	N
	E_L'	S	S	N
	σ_L'	S	S	N
	E_T'	S	S	N

So, what we are going to do is we are going to construct a table first thing is that we will look at different properties; tension properties in compression and properties in shear and then we will see the influence of fiber influence of matrix and influence of interface. So, by that I mean how good the bond is between the fiber and the matrix.

So, first set of properties are related to tension. So, one is E_L young's modulus in the longitudinal direction second property is σ_L strength in the longitudinal direction in tension third is tensile modulus or transverse modulus and transverse strength in

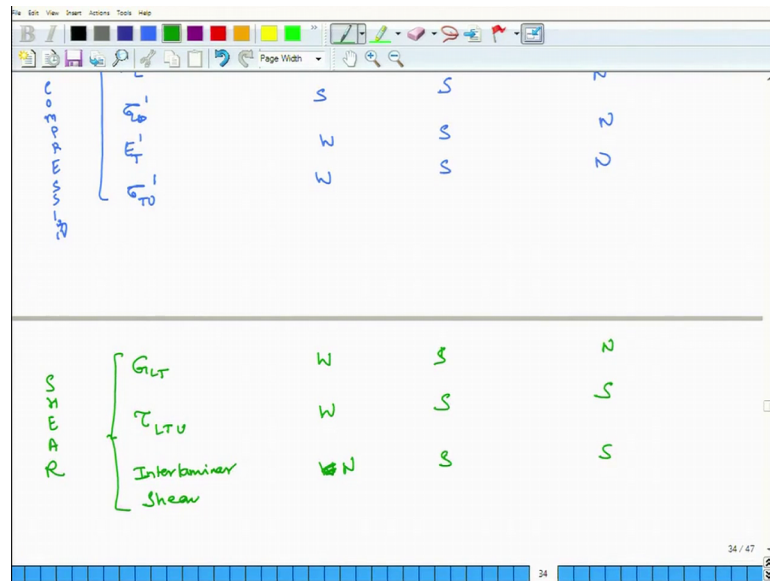
tension. So, these are five four properties in tension. So, we will write some letters. So, how strong if the influence of fiber is very high, then we write S as far as strong if it is not high, then we write W; W for weak and if it is negligible then we write N, ok.

So, fibers role is strong matrix role weak you have seen it right the longitudinal modulus is strongly determined by properties of the fiber and interface role negligible $\sigma_{L u}$ same thing strong weak negligible, but when you look at the transverse modulus then fibers role is weak matrix role is strong and interface is negligible and lastly if you look at the transverse strength when you apply x tensile load then once again the role of fiber is weak what is really important is how good your matrix is. So, strong and how good you stick, I mean join the two things together. So, this has a strong role.

So, these are tensile tension related properties next we look at compression related properties. So, $E_{L u}$ prime $\sigma_{L u}$ prime $E_{T u}$ prime and $\sigma_{T u}$ prime. So, all these are compression related properties. So, once again in compression strong role of fiber weak role of matrix negligible role of interface longitudinal strength in compression strong role of fiber and also a strong role of matrix why is it a strong role of matrix because if the matrix is very weak the fibers will buckle very easily?

So, it depends how strong and stiff your matrix is that has a big role and again negligible role of interface and that the third one is transverse direction compressive modulus weak role a strong role negligible role weak role a strong role negligible role and then the last one is shear properties. So, what are shear properties $g_{L T}$ and $\tau_{L T u}$ shear strength?

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Actually the other third one is inter-laminar shear; inter-laminar shear between two layers. So, these are shear properties. So, in this case the role of fiber is not that high G_{LT} because most of the shear is resisted by the matrix. So, this has a strong rule and negligible rule of the interface weak rule of fiber a strong role of matrix and a strong role of interface because if the interface is not strong, then everything will slide over each other. So, it will be and finally, inter-laminar shear which is the shear strength between two layers weak or actually here it is actually negligible strong and strong ok.

So, this gives you a comprehensive overview in a subjective sense and with this we conclude our discussion for this week we will meet once again on Monday and we will start discussing how to model composites which have several layers of unidirectional laminate stacked on top of each other and that is the type of discussion which will be extended over several weeks and I look forward to seeing you on Monday, till then, have a great weekend, bye.