Introduction to Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 38
Transverse Strength of Unidirectional Composites

Hello, welcome to Introduction to Composites. Today is the second day of this particular week which is the 7th week in this course. Today we will discuss several important things the first one being determination of transverse strength of unidirectional composites transfer strength.

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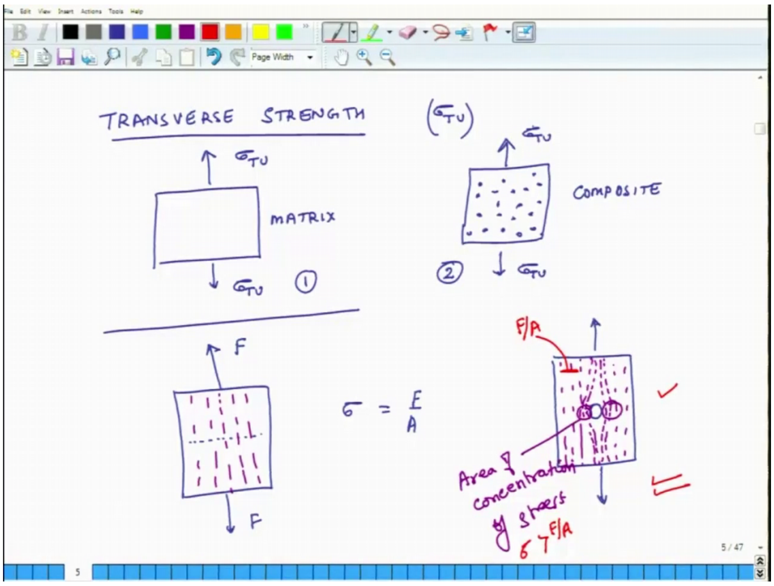

So, specifically we will be discussing parameters such as sigma T U sigma T U. Now we have seen that when we load a composite in transfer direction it is modulus is primarily determined by the matrix. And the amount and the presence of fiber does not play a very big role I mean of course, it increases the modulus, but does not play that bigger role as it is found to be in the case of longitudinal modulus.

Something similar is also true for transverse strength for composites. In fact, when we add fibers to matrix the presence of the fiber in the matrix actually makes the composite weaker not stronger in the transverse direction. So, if you have just a plain sample of material, which is just matrix. And if you have another sample and it has lot of fibers and if you are loading both of these in the tensile direction.

So, this is a composite and then then the strength in case one will be typically found to be more than the strength of the composite this observation, which has been made again and again. And now what we will do is we will try to explain the physics or the reason underlying this behavior that is that the strength of the matrix, when it is subjected to tension is better than the strength of the composite, when it is subject to tension in transverse direction.

So, there are a couple of things happening, but before that we should try to understand this thing related to stress concentration. So, suppose you have a simple matrix material and I apply some force F and it is a matrix material, which has a square or rectangular cross section then the stress let us say along this cross section is uniform and this is equal to F over A where a is the cross sectional area of this material.

But if you have unlike this suppose you have a round object cylindrical inclusion in this then, what happens even if you if the external force is uniformly applied the stress lines in the system. So, in the first case the stress lines were uniformly distributed, but here this there is a stiff material at the center and because of this the stress lines flow in a way. So, that they get concentrated around the edge of the object and as I move far away from the object they again become uniformly distributed.

So, the stress lines get concentrated in this area, area of concentration of stress, same thing is happening in this zone also. So, what; that means, is that in the region let us say far away suppose in this region stress is still f over a, but in this area of concentration stress is more than F over A in the area of stress concentration.

So, what; that means, is that typically this in this particular situation will have higher peak stresses compared to a uniform compared to a pure matrix material. And as a consequence this particular material, which has just one single solid inclusion in it itself will tend to fail at an earlier level it will fail at an earlier level, now in case of composites there are several such inclusions in the system.

So, in general the overall stress around these inclusions which are nothing, but fibers right fibers are going in the direction perpendicular to the plane of this board in the. So, stresses around these fibers are significantly higher, compared to sigma this external uniform stress which we are applying. So, what I mean to say is that when I subject unidirectional laminate in the transverse direction using a uniformly distributed force I still have a stress concentration factor SCF.

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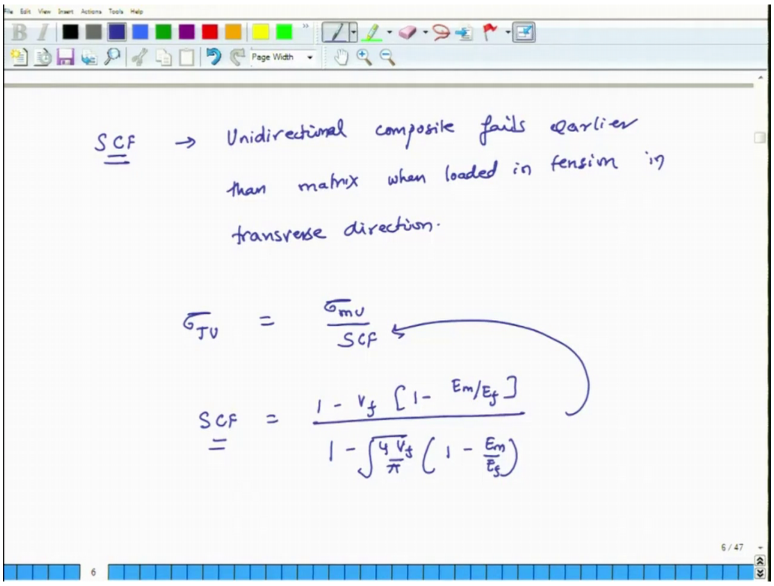

And because of this stress concentration factor unidirectional composite fails earlier, than matrix when loaded in tension with loaded in tension in transverse direction.

So, what will be the failure point of this? So, we can say that it is tensile strength in the transverse direction, sigma T U will be equal to the strength of the matrix sigma m U and it has to be less than strength of the matrix. So, divided by some factor S, now this or actually you can say it SCF Stress Concentration Factor. So, if we can know what is this stress concentration factor we can calculate sigma T U.

Now, in this course we will not learn how to compute this stress concentration factor, you can find this stress concentration factor by developing models of your system using F E A methods or there are some already some approximate solutions, which have published in existing literature. And I will share with you one estimate of such an stress concentration factor. So, that is that a stress concentration factor equals 1 minus V f divided by 1 minus E m over E f divided by 1 minus 4 V f over pi times, 1 minus E m over E f.

So, if you know the volume fraction of the material, if you know the volume fraction of the material and you know E m and E f you can find out a stress concentration factor this particular relation you plug in this relation for transverse strength and you will be able to find the transverse strength, in tensile direction for a unidirectional laminate. So, this is one way to compute the strength in tensile direction.

Sometimes it happens that we do not know at what stress the matrix may fail, but we may know at what strain it will fail. So, if the strain failure strain of the matrix is known then we use a slightly different relation.

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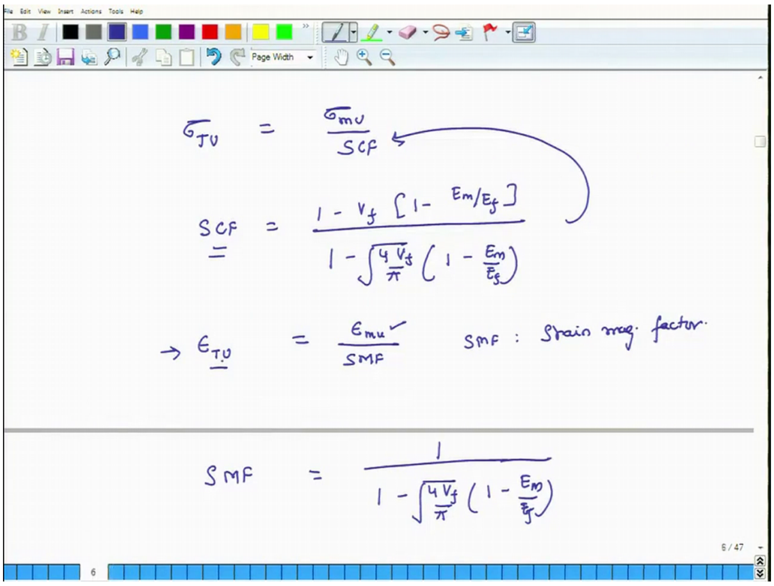

We say that sigma T U you know epsilon T U, which is the failure strain of composite is nothing, but failure strain of the matrix material divided by another factor called SMF and what is SMF?

SMF is strain magnification factor. So, this is a factor which is similar in it is nature to stress concentration factor stress gets concentrated and then it becomes high in the strain in certain areas there is more strain. So, a strain gets magnified. So, that is why we call it strain magnification factor. And so that so this particular relation we will use if we know sigma epsilon mu and how do we compute SMF.

So, SMF is equal to 1 over 1 minus 4 V f over pi times 1 minus E m over E f. It is important to understand this think about a scenario when E m and E f is the same what; that means, is the modulus of matrix and modulus of fiber is the same, what happens? Then theoretically when you look at these relations SCF becomes 1 and SMF becomes 1.

Effectively what this relation face is that if the matrix material properties become very close to that of fiber, they will be very little stress concentration or strain concentrate strain magnification. And that makes sense, because if the materials are same then when you apply external force all the force will get transmitted to the other side through uniformly spaced stress lines. So, this is with thing to understand.

So, these are 2 ways to compute the strength of the unidirectional composite, when it is loaded in transverse direction in tension.

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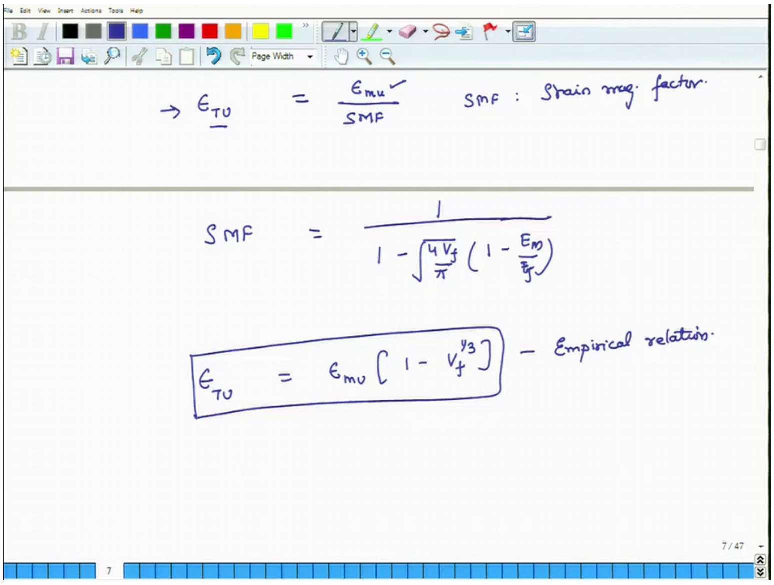

There is a third relation which is also sometimes used and this is also for strain and this is equal to epsilon mu into 1 minus V f to the power of 1 by 3. So, this is an empirical relation it is an empirical relation especially in cases when E m over E f is extremely small it is very small which is true in most of the cases?

So, if E m over E f is fairly small then this type of relation gives reasonably results and this is a simpler formula. So, we can very quickly use this formula to compute the failure strain of a unidirectional laminate, when it is loaded in tension in the transverse direction. So, this is what I wanted to cover at least in context of transverse strength of composite materials.

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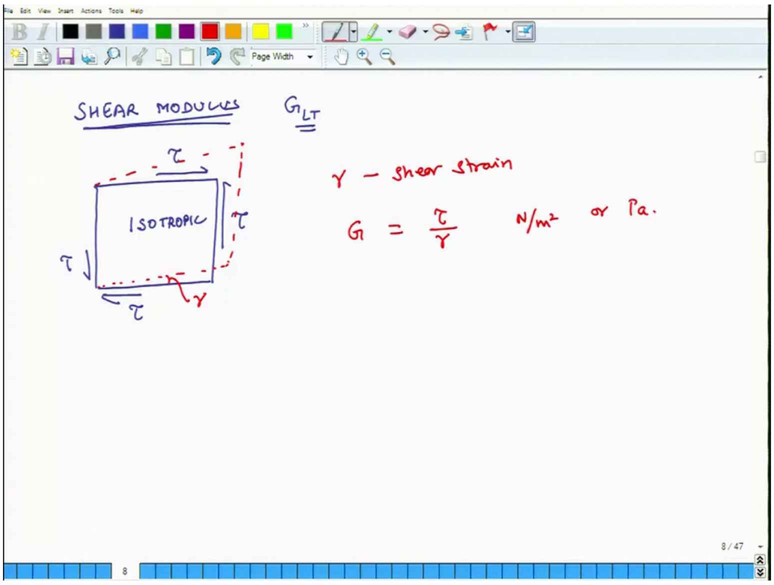

Now, we will move on to predicting shear modulus of unidirectional laminates. So, this is designated by the symbol G L T. So, how do we so before we work on G L T let us quickly recap, how do we compute G L T shear modulus of any material. Suppose this is a homogeneous isotropic material let us say for instance steel or glass or anything and if I have to compute it is shear modulus what do I do I apply on it some shear stresses.

So, I apply tau and this is remember this is isotropic material and when I apply this shear stress it this is it is deformed shape it deforms in this kind of a way and this angle which was originally 90 degrees it does not remain 90 degrees. So, there is some change in angle and let us call it gamma this gamma is the shear strain shear strain and then what is our definition for G G equals tau over gamma and the new units are still newton per meter square or Pascal's.

So, with that kind of a thinking we will now this kind of an understanding we will develop a relation for shear modulus for such composites. So, consider this picture. So, now, we are going to develop a relation for shear modulus for the unidirectional laminate.

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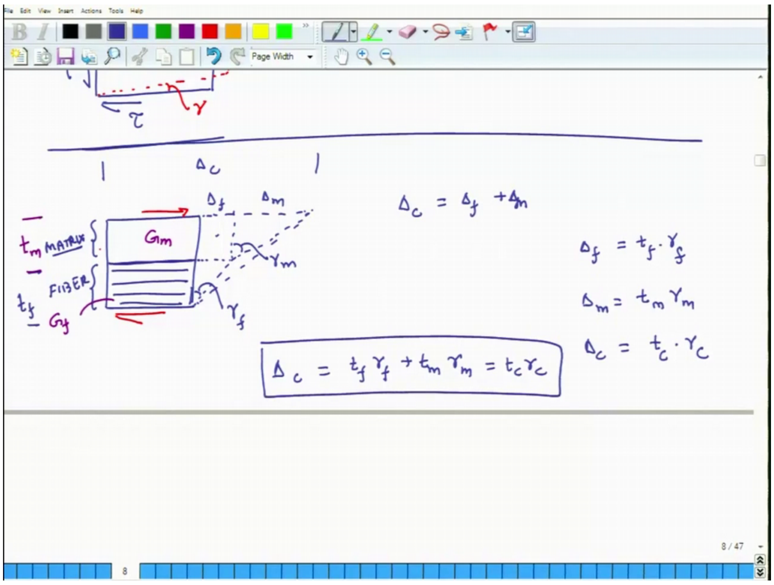

So, this is my cross section of the unidirectional laminate let us say that my fibers are running in this direction. So, this is this is fiber and this is matrix.

So, in reality the fibers and matrix are all uniformly distributed, but just for purposes of our understanding I am lumping fiber at the bottom portion and matrix in the top portion. And this kind of a lumping is valid because in reality, what will you have in reality the behavior of this system for shear will be fairly close to the actual system and why that is something we will discuss later, but for a moment just hold on to this.

So, the lower portion is fiber and the upper portion is matrix and let us say the. So, this is the lower portion is pure fiber lower portion is pure fiber and upper portion is pure matrix. And let us say that the modulus shear modulus for fiber is G f and that is it is G m for matrix.

And now I apply some shear strain to it. So, I apply some shear strain or shear stress to it. So, what happens this system deforms and as it deforms there will be some deformation, in the fiber right and there will be some more deformation, in the matrix there will be some deformation in the matrix.

So, let us say this deformation is delta f this deformation is delta m and what is the total deformation of the system it is delta c. So, delta c equals delta f plus delta m what is delta f delta f is basically this angle. So, I call it this angle is gamma f I call this angle is gamma f and this height is t f. So, delta f is equal to t f times gamma f. Assuming that these angles gamma f and shear angles are extremely small then I can directly call them, but because the shear angles are extremely small I can write this relation. So, that is the assumption.

Similarly, if this angle is gamma m and this distance is T m which is the thickness of the matrix block then I can say that delta m equals t m gamma m. So, I can state that delta c and likewise I can also say. So, delta c equals t f gamma f plus t m gamma m. And I can also say that delta c is what total thickness of the composite t c total thickness of the composite, which is t c which is what t f plus t m times the total change in angle what is this total change in angle this entire change in angle. So, this is gamma c. So, this is equal to t c gamma c.

So, this is what I have written.

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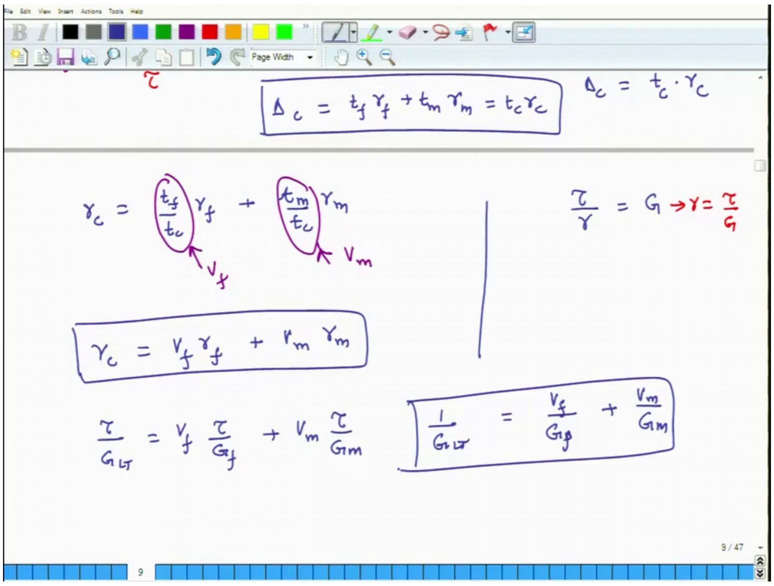

So, from here gamma c equals t f over t c gamma f plus t m oops excuse me plus t m over t c gamma m and what is t f over T c it is volume fraction of fiber. And what is tm over t c it is volume fraction of matrix. So, gamma c is equal to V f gamma f plus V m gamma m, but gamma is our change in angles or is right they are very small and their values are very small and the way we had defined earlier is that G m equals what it is equal to. So, these are fiber by itself is isotropic matrix by itself is isotropic and for isotropic materials we had said that tau over gamma is G.

Now, what is the shear stress in matrix shear stress in matrix is tau shear stress in fiber is also tau and shear stress on the overall composite is also tau. So, from so this relation gives us that gamma equals tau over G. So, we will put this relation for tau here for gamma here and what we will get is tau over G L T, this is the shear strain in the composite is what shear stress divided by it is shear modulus G L T is equal to V f tau over G f plus V m tau over G m.

Now, tau is same. So, it cancels out. So, I can say that one over G L T equals V f over G f plus V m over G m or I can also finally, write it as G L T equals G m G f by G m V f plus G f V m.

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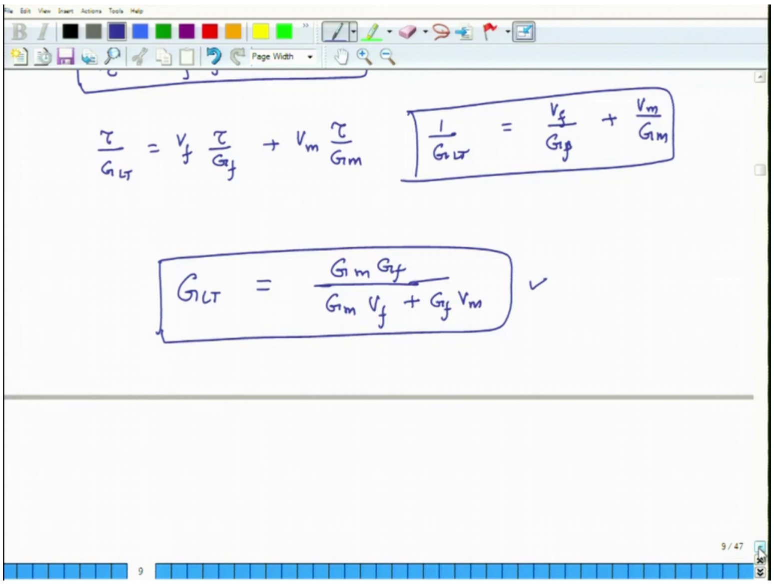

So, this is using the model which I had explained earlier that here I have separated the fiber and matrix and I have made a lot of simplifying assumptions and using this kind of an approach, I have developed the relation for shear modulus G L T for the composite.

Now, when I compare when I calculate the value of G L T from this formula and compare it with experimental values just as I found in case of transverse modulus, there was disagreement between experimental data and this data from the model the same is found to be true for G L T also. What we find is that the predicted value of G L T comes to be much less than the measured experimentally determined value of G L T.

So, based once people found this out people improved these models and then the final and a very good model was developed by Halpin-Tsai for G L T also so, not only for E T transverse models, but also for G L T. So, this is the Halpin-Tsai equation.

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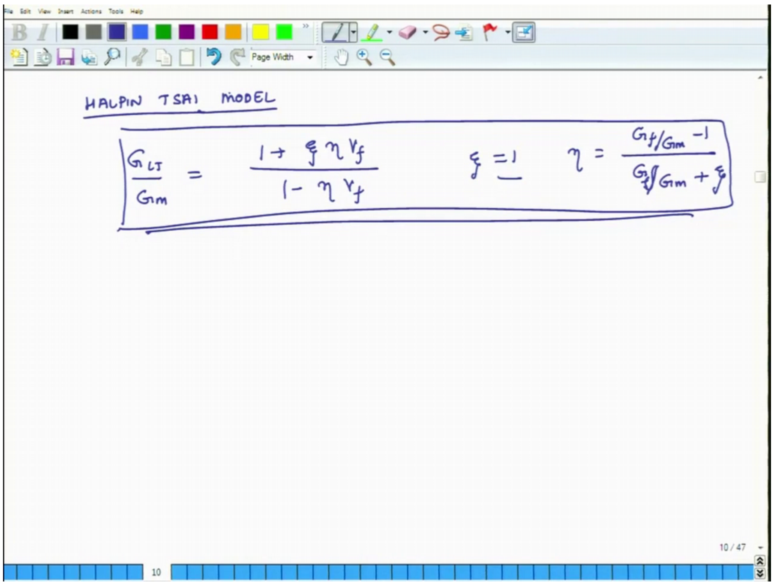

This is the Halpin-Tsai model. And this model works reasonably well. So, what does the model say it says that G L T divided by G m equals 1 plus zeta eta V f by 1 minus eta V f?

Now, here zeta equals 1 for circular fibers and eta equals G f m G f divided by G m minus one divided by G f over G m plus theta. Now remember this relation is slightly different than the relation for transverse modulus, if you look at the relation for transverse modulus there is a factor of 2 for eta, but in case of shear modulus their model says that we do not use this factor of 2, we rather just use it especially for circular fibers we just use it as 1 for non-circular fibers we use eta equals a over b not twice of a over b.

So, this is important to differentiate and if we use this relation we get a reasonably good agreement between experimental data and the predicted values. So, in this course we will use this approach to compute the value of G l T. That concludes our discussion for today, tomorrow we will continue this discussion and we will predict some more values for composites unidirectional composites and continue the same theme.

So, with that we close our discussion and we will meet tomorrow. Bye.