Introduction to Composites
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Lecture – 36
Halpin - Tsai Relation for Transverse Modulus

Hello, welcome to introduction to composites. Today is the last day of the ongoing week which is the sixth week of this course. Yesterday, we had initiated the description on how to predict transverse modulus of unidirectional composites and using the assumption that the stress in composite matrix and fiber are the same as well as the assumption that the extension in the composite is nothing, but the sum of extensions in matrix and fibers we had developed a relation for transverse modulus and that relation for transverse modulus was one over E T where E T is the transverse modulus of composite. It equals volume fraction of fiber divided by its modulus plus volume fraction of matrix divided by its modulus.

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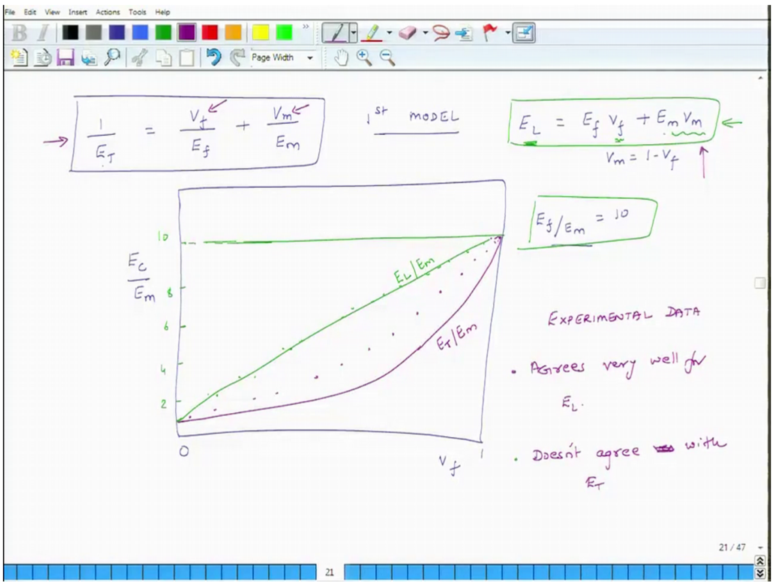

So, this is our first model first model. Now let us look at this graph. So, in this graph, what I am going to plot on the y axis is the modulus of the material of the whole composite and on the x axis. I am going to have volume fraction. So, volume fraction could vary anywhere from 0 to 1 that is 0 to 100 percent and we assume while making this graph that the fiber modulus is 10 times that of matrix modulus because you have to make some assumption.

So, E f is 10 times that of E m and once we have that we can make this plot. So, this is the relation for E T for E L; what is the relation for E L relation is E f, V m, V f plus E m V m and of course, we assume that V m equals one minus V f, we do not assume that there is any air or something like that ok.

So, first we will develop a line for E l. So, when fiber volume for.; so, this is 2, 4, 6, 8, 10, when fiber volume fraction is 0, when fiber volume fraction is 0; what is the value of E L over E m 1 when fiber volume. So, first we are going to develop a line for E L, first this is first thing, we are going to do we are going to develop a line for E L when fiber volume fraction is 0 volume fraction of matrix is 1, this number will be 1, all right. So, this is 2, 4, 6, 8, 10.

So, this curve for E L will start from here and as volume fraction keeps on increasing this is a linear relation, right as volume fraction of fiber keeps on increasing, this number will keep on increasing as a straight line and what is the maximum possible value of E L when volume fraction of fiber is 1. This term will be 0 and this is V f is 1. So, E L will be same as E f and E c over E m will become 10 because this is what we have assumed right. So, this will be a straight line. So, this is the relation for E L or rather E L over E m E L over E m.

Now, we use this relation to compute the behavior of E T over E m, we use this relation to compute E T over E m and if we do all the mathematics the graph looks something like this. So, it starts and ends at the same location, why does it start at the same location, suppose at 0 percent volume fraction this term goes to 0, this is 1. So, 1 over E m is equal to 1 over E T. So, it starts from 1 and at 10 per at 100 percent volume fraction, it ends at 10 because we have assumed that E f over E m is 10. So, this is how. So, this is the expression for E T over E m.

Now, these both lines are theoretical lines based on our models which we have developed this model when we look at experimental data for E L. So, what does experiment tell it agrees very well for E L? So, if we take sample, them in longitudinal direction and we find experimentally the values of E L you will see a lot of values, it will be along this line which means that actual value of the longitudinal modulus of a composite is pretty close to our predicted value. So, we can reliably use the equation for E l, but for does not agree very well does not agree with E T. So, if you look at the data for E T it look something like this it starts at d 0 and ends at 10.

But in general the actual value of E T transverse modulus comes out to be significantly higher than the calculated value of E T this is important to understand which means our model for E T is not a good model to predict this value.

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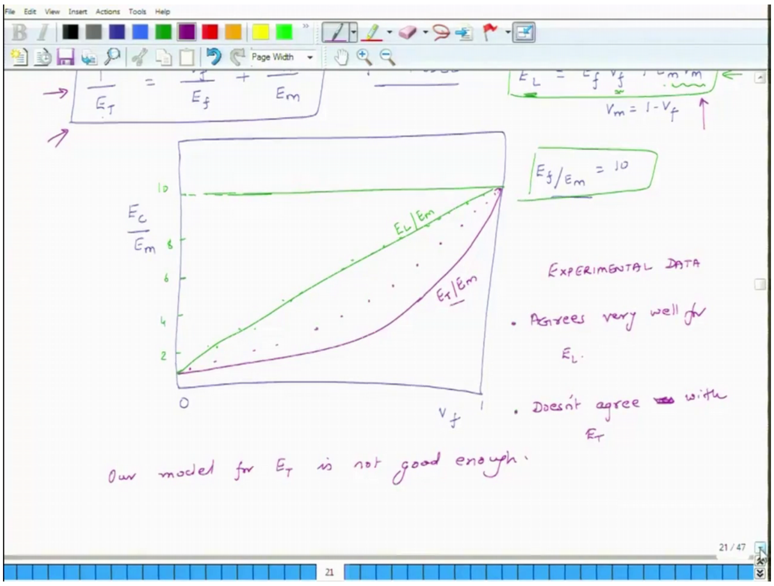

So, our model for E T is not good enough. So, we do not want to use this expression for computing E T. So, what do we do, before I explain how do we compute the value of E T we should try to understand what is the reason why this model does not predict the value of E T reliably enough there are couple of factors.

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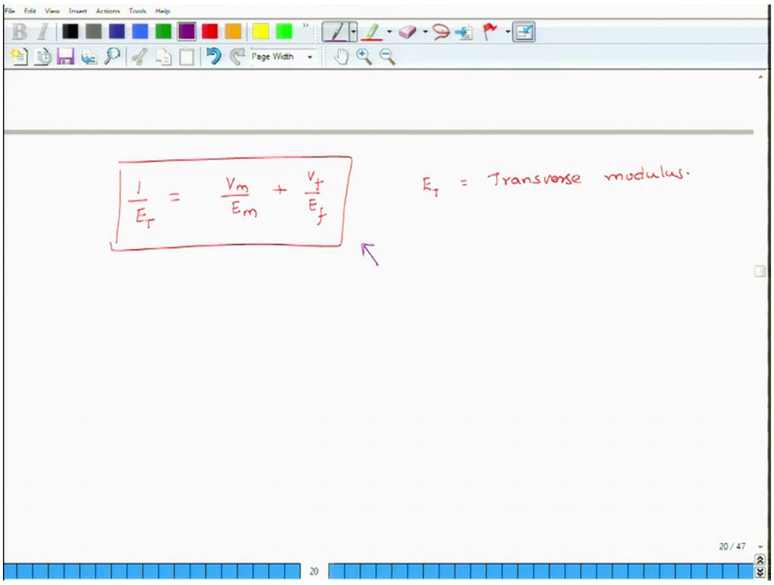

So, let us look at some of these factors reality.

So, when we were developing this expression for E T we had assumed.

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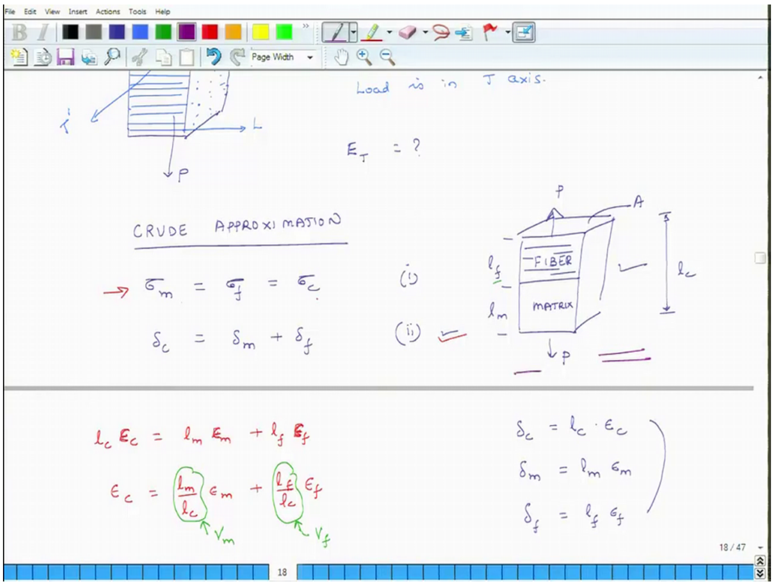

This kind of a configuration we made this picture or we made this picture.

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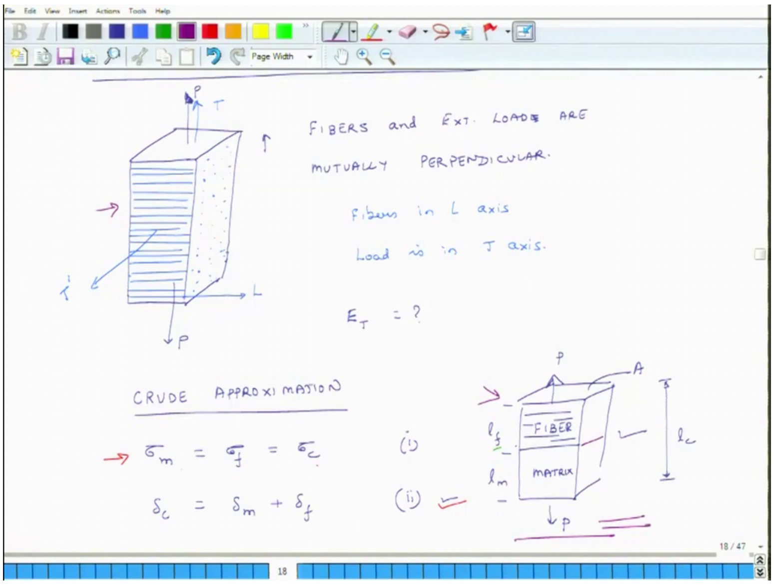

We made this picture and we made this picture and we said that there are layers of fiber and matrix layer of fiber and layer of matrix layer of fiber and layer of matrix right, but in reality, they are not layers there are fibers continuously dispersed throughout the medium and throughout the medium you also have matrix continuous the fibers are still in the transverse direction the orientation is not different, but they are continuously dispersed. So, if we do not have layers.

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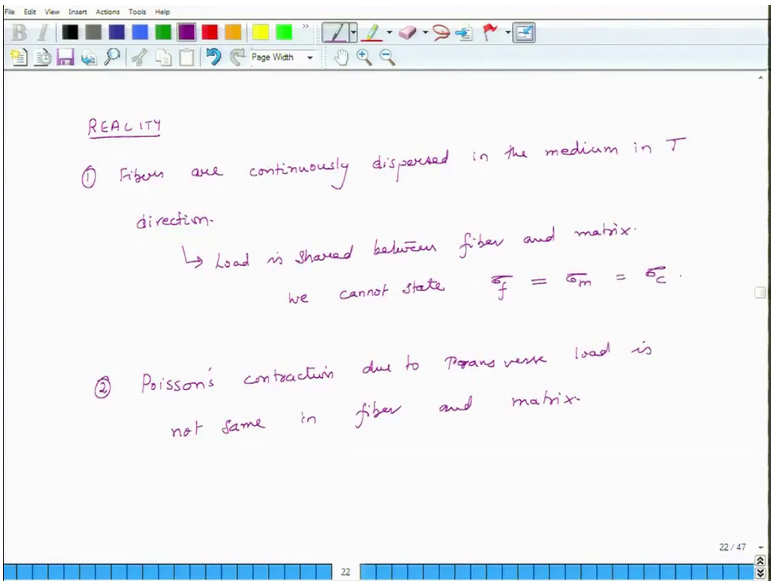

So, we cannot have this block kind of a system which we do? So, first thing is fibers are continuously dispersed in the medium in t direction now what does that mean what; that means, it that at every cross section see earlier we had assumed, what did we assume that stress in the matrix and stress in the fiber and the stress in the composite is same, but if in the same layer you have fiber and the matrix what then you cannot make this assumption, right.

Then what; that means, is some stress is being shared by the fiber and some stress is being shared by the composite right by the matrix. So, stress is actually shared it is not same because it is not layer by layer model. So, that is the one thing second thing is when we apply load on the fiber or on the system fiber expands and the matrix expands in the transverse direction right and when they are expanding in the transverse direction they will also be poisons contraction, in the other direction see when you pull something it becomes longer, but it also becomes slimmer.

So, because of this poisons effect matrix will become slimmer and also fiber will become slimmer and, but both this thing will not be same because the material properties of matrix and these things are different. So, fibers are continuous. So, what this means is load is shared between fiber and matrix. So, we cannot say state that sigma f is equal to sigma m is equal to sigma c, there is something one thing we have said.

The second thing is Poisson contraction due to transverse load is not same in fiber and matrix it is not the same. So, what does that mean typically? So, one part of the material mat you know will expand less other part other phase will expand more or contract more and so on and so forth. So, what; that means, is that fiber and matrix, they are because they are not in the layer, they interact very tightly with each other they interact tightly with each other and as a consequence because of both these effects fibers constrain expansion of the matrix they constrain expansion of the matrix.

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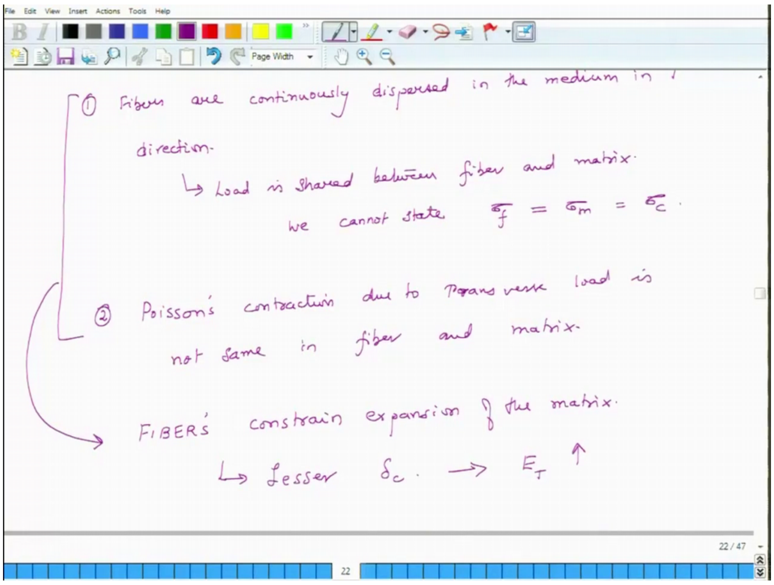

And if this constraint happens, then what will happen? The extension in the length of the composite will be less. So, this causes lesser delta c. If delta c is less, this will lead to higher E T for the same stress, there will be less extension less strain which means higher E T and that is the physical reason why a lot of this experimental data shows this is lot of this experimental data these are experimental point is higher than the predicted value of E T. So, to find the actual more reliable estimate of E T there are several ways to solve this problem, but at least in context of this course, we will not go to those methods, but rather I am going to share with you one result which gives reasonable good prediction of E T directly.

So, what we can do is. So, that method is known as it is a formula.

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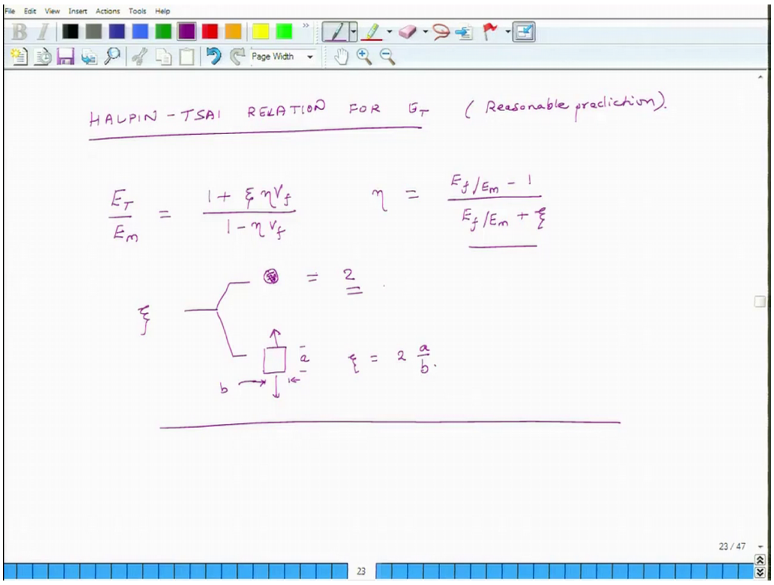

So, we will use this formula known as help in relation for E T. So, this gives reasonable prediction for E T what does this say that if you want to compute E T you can compute it using this relation E T over E m equals 1 plus eta V f divided by this term and this is the parameter. So, eta is a parameter and it is can be computed by using this relation and then you will wonder what is zeta.

So, zeta it depends if you have circular fibers which are circular cross section, then it is equal to 2, if the fiber is having a rectangular cross section, let us say the cross section is like this a and this is b and the fiber is and the fibers are oriented in such a way that the load is normal in the length of a direction, then this is equal to, then zeta equals two a over b, but most of the times we use fibers with circular cross section. So, we do not have to worry too much about this. So, we can use this.

And with these, so zeta you can say most of the fibers are circular in cross section unless you go to some fancy wires metallic wires and things like that where there may be non circular cross sections. So, you can use this two from this you can compute neta from neta you plug it in this relation and you can find et which is the transverse modulus and this relation gives you fairly good estimate of transverse modulus of composites unidirectional composites and that is pretty much what I wanted to cover for today.

Next week we will continue this discussion and we will also develop relations for other sys materials other properties of the system like transverse strength poisons ratio shear modulus and things like that. So, the discussion for this week is complete and I look forward to see all of you next week on Monday, have a great weekend.

Thank you.