Introduction to Composites
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Lecture – 35
Transverse Modulus of Unidirectional Composite

Hello. Welcome to introduction to composites. Today is the fifth day of the ongoing week which is the sixth week of this course. Till so far, we have been discussing the behavior of unidirectional laminates, particularly when they are loaded in the longitudinal direction that is the direction of the fibers and specifically, we have computed 2 important properties of such composites in the longitudinal direction. One is the Young's modulus; not the modulus of the material in the longitudinal direction not the Young's modulus, but the modulus and also the strength of the composite in the longitudinal direction.

Then when we are discussing the strength of the system we should be aware that we assumed and we never explained it, we assumed internally a lot of things while developing the expressions for the composite loaded in the longitudinal direction.

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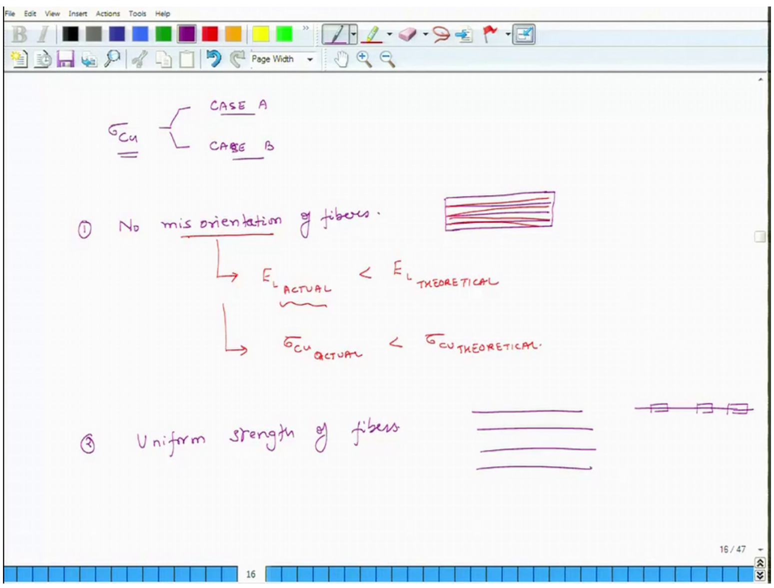

And our estimates of sigma cu as calculated in 2 cases which we have developed case A and case B, they are the highest possible estimates based on some assumptions, but if those assumptions which we will very quickly discuss now are not necessarily true, then the calculation of sigma cu based on case A and case B may be an upper limit, but real values will be somewhat lower.

So, what are the assumptions which we assumed and we never explicitly stated we had assumed one that all the fibers were in exactly 0 degree direction. So, there was no miss orientation of fibers, we assume no miss orientation of fibers so, but tip. So, we assume that all the fibers were exactly straight, but in reality the fibers may be at some direction you know they may be more or less in the 0 degree direction, but not necessarily perfectly aligned with the 0 degree direction and if there is miss orientation the first thing and we will see it later in the course if the fibers are not exactly aligned in the 0 degree direction our computation of E L, actual it will be less than E L theoretically which we computed and we will actually see how this thing gets influenced.

And once this E L is going to be less it will affect all other calculations also in k or in context of the strength of the composite also if the fibers are misoriented, if the fibers are misoriented, we will not only have a pure tensile stress in the system, but there will be also some shear stress in the system. So, because of that the strength of sigma cu actual will be somewhat le less than sigma cu theoretical because the loading also. So, first thing is modulus will be less second a strength will be less because the loading will be more complicated there will be a shear element and things like that. So, this is one thing.

The second thing we assumed was all fibers have uniform strength what does uniform strength mean. So, we assume that uniform strength of fibers what does it mean it means that if you take ten different fibers individually and you pull them, they will break at the same stress level, this is one the second thing is that if you from the same fiber if you take different pieces and you pull individual piece they will also have the same strength.

So, for a particular fiber different sections have same strength and also if you take different fibers the strength is same this may not be or entirely true in reality. So, if you take some especially ductile fibers for made of metals they may have this property their strength may be more or less uniform.

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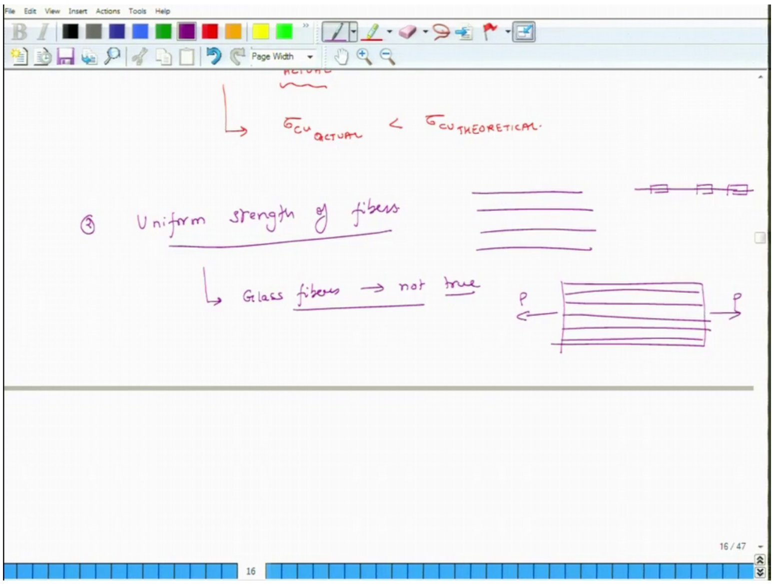

But for glass fibers, this may not be true there is a significant statistical variation you take twenty different fibers there are lot of imperfections in glass fibers especially because they are they have high abrasion and that that damages the fibers.

So, for glass fibers and some other types of fibers the strength of different fibers may have a significant variation when this kind of variation happens consider, what will happen. So, you have different fibers of different strengths and you pull the composite in length direction some fibers will break at an earlier load level because they have lesser strength because of this variation and when they fail they transfer the load to other fibers. So, which means that at earlier loads compared to what we calculate theoretically at earlier loads some fibers break they transfer load to other fibers and these fibers develop higher stresses than theoretically predicted.

Now, if you keep on pulling it further, they may also fail earlier because now in reality they are taking more load then they were supposed to. So, because of this thing also again we may have a case that sigma cu actual may be less than sigma theoretical this is the second assumption which we had made third assumption.

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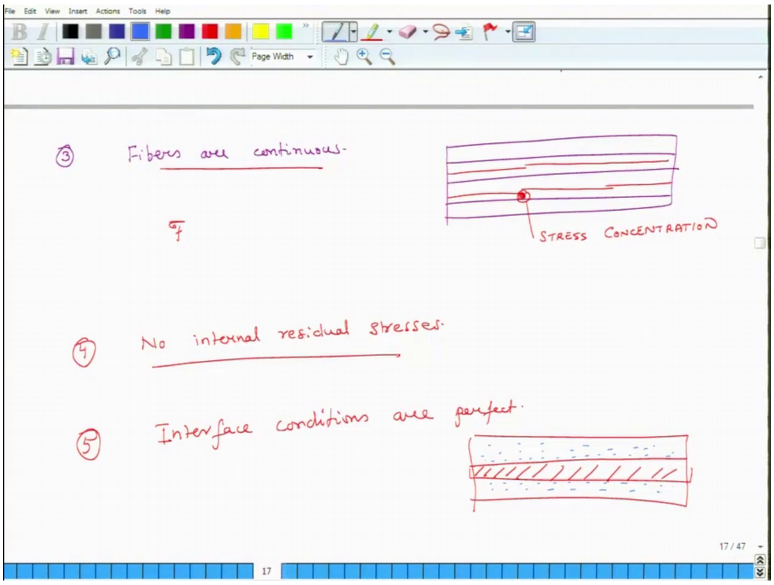

Which we made were that fibers were continuous this is what we had assumed which means that if you take a sample of composite fibers run throughout the length of the composite.

Again in reality that will not be true they will always be fibers which may run something like this something like this. So, there will be broken fibers they may be still long fibers with sufficient length. So, that we cannot call it a short fiber composite, but there will be there will be some breakages because of processing also because of production things like that when this happens at their end points you have sources of stress concentration and the end points, there are high stresses then what you will be predicting what you predict theoretically is that sigma f, the stress in the fiber you can compute based on all the discussion which we had, but that is a theoretical prediction assuming that fibers are continuous.

But the actual stress especially at their endpoints will be much higher than the theoretically computed stress what; that means, is that as I keep on increasing the stress these fibers will tend to fail prematurely earlier at their end locations and once some things start failing, they cause other failures to happen also once again in this case also you will have the same situation the actual strength of the composite will be lesser than what we are computing theoretically.

The fourth assumption was that there are no internal residual stresses there are no internal residual stresses. So, when we were computing we were saying that I all the stress in the fiber and matrix was caused only because of external load that is what we have assumed we did not say that when there is 0 load on the fiber, there are still stresses in the matrix and fiber, but in reality there are always internal stresses in fibers and in matrix why because typically these composites get cured at elevated temperatures and they solidify at elevated temperatures and then they shrink from high temperature to room temperature when the shrink matrix tries to contract more fiber tries to contract less because the coefficient of expansion of fiber is typically less than that of matrix as a consequence matrix and fibers develop internal stresses.

Even though the overall external stress on the system is 0 some parts are in compression some parts are in tension and such a system when we apply some external load it again has internal stresses plus external stresses due to external load. So, this also causes change in the failure stress of the composite.

And lastly an important condition is interface conditions are perfect what does that mean what; that means, is that you have a fiber and it is surrounded by matrix material. So, this is my fiber and it is surrounded by some matrix and once again, we assume that the matrix is perfectly bonded to the fiber and what this means is using this perfect bonding assumption we were able to compute we were able to assume that the strain in the matrix is same as a strain in the fiber and from that we developed all expressions related to Young's to modulus longitudinal modulus as well as the strength.

But in reality there may be areas which are actually debonded even at 0 loads even at 0 loads and once this happens at the points where it is debonded, they are those points again act as sources of stress concentration and once that happens again that causes reduction in the overall strength of the composites. So, these 5-6 factors miss orientation non uniform strength of the fibers imperfect interface conditions discontinuous fibers as well as residual stresses all these factors influence the overall strength of composites in longitudinal direction.

So, this is some important insight I wanted to provide because a lot of times you will find that whatever computations you do for calculating the strength of these fibers, you will find invariably that will be an upper estimate of the composites when you compare to experimental data and you should be cognizant that this difference is attributable to several important factors and some of the more important factors we have already discussed.

So, with this we now move on to the next topic and that is about transverse modulus of unidirectional composite.

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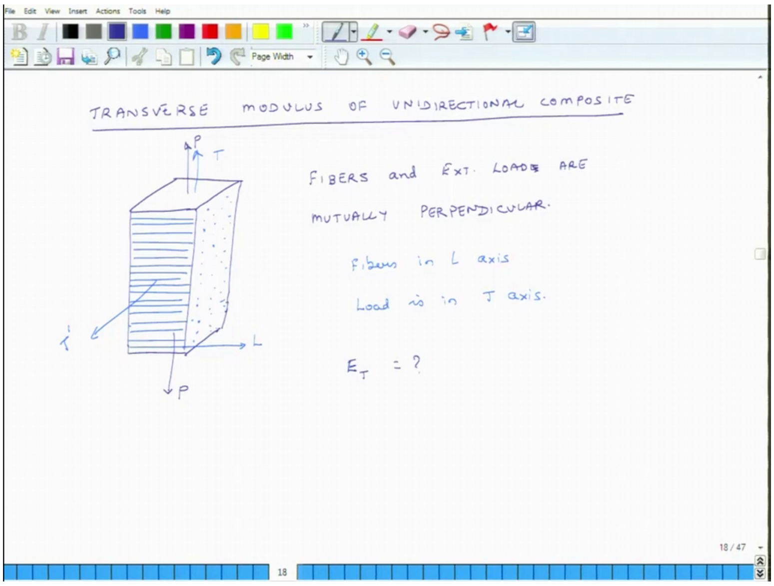

So, what do we mean by transverse modulus what we imply is. So, let us say this is a sample of composite material and here the load is in the transverse direction what does that mean. So, let us say I am still loading it in this direction, but the fibers are not aligned with the direction of the load rather fibers are at ninety degrees to the load.

So, my fibers are aligned like this and so on and so forth. So, here fibers and external load are mutually perpendicular. Now the direction of the fiber, oh, excuse me, sorry, I made a mistake in this picture. So, when I see it from the cross section, I will only see ends of the fibers I will not see the whole fiber yeah.

So, this is if I have an access system which is aligned to the direction of the fiber then this is L axis and. So, fibers are in L axis and one of the directions normal to is T axis. So, load is in T axis and of course, there could be a third direction also this is T prime; T prime axis. So, what we are interested in finding out is ET transverse modulus of the fiber.

So, if I pull my sample in the transverse direction what is the ratio of stress and strain that is what I am just interested in finding out? Now in case of the longitudinal modulus for longitudinal modulus we assumed that the strain in the fiber and the strain in the matrix and the strain in the composite is same in this we cannot make that assumption because you can visualize in some sort of a that as I apply this load there is matrix there are layers of matrix and fibers. So, you may have a thin layer of matrix than thin layer of fiber thin layer of matrix thin layer of fiber one simplified view of this thing would be like that.

So, there is nothing which ensures that the extension of the matrix in the T direction and extension of fiber in the T direction will be same that is not going to be the same; however, as a first approximation first very crude approximation crude.

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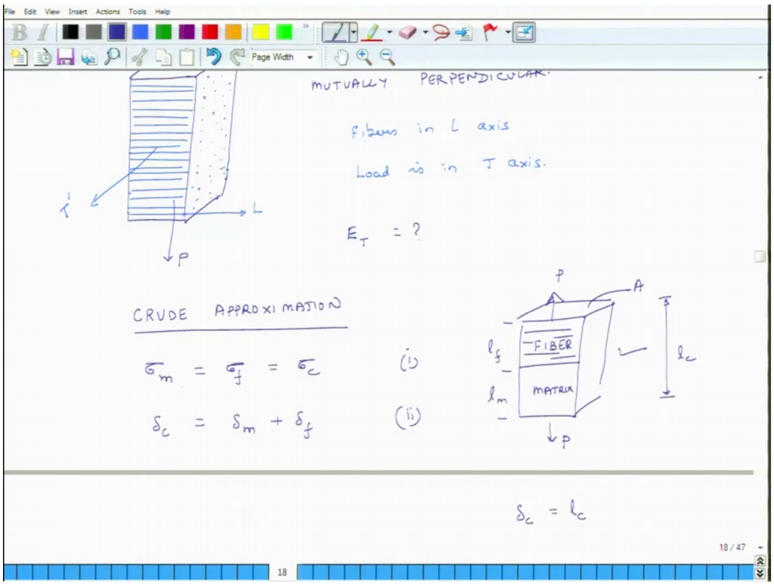

We can have a crude approximation what can we say we can say that stress in matrix is same as stress and fiber same as stress in composite why because we can idealize this entire sample as a slab of matrix excuse me.

So, I can have this as a fiber and matrix and of course, fibers are running like this and if I pull it like this it could be a slab of fiber slab of matrix and what does that mean that the stress and fib what is the stress in if this kind of model is considered, then a stress and fiber the cross sectional area is a then stress and fiber is P over a stress and matrix is also P over a is stress in composite is also P over a in this situation ok.

So, this is a crude approximation we will see why it does not give good results. So, this is one the other thing is that when I pull because of this load the whole composite will extend it will become longer. So, the extension of composite delta c is equal to extension in matrix plus extension in fiber this is the second.

So, what we will do is we will take the second equation and we will divide it.

Now, we know that extension in composite extension in composite is what overall length of the composite what is length of the composite this is length of the composite this is length of the fiber block this is length of matrix. So, extension and composite is length of the composite times strain in composite in the transverse direction.

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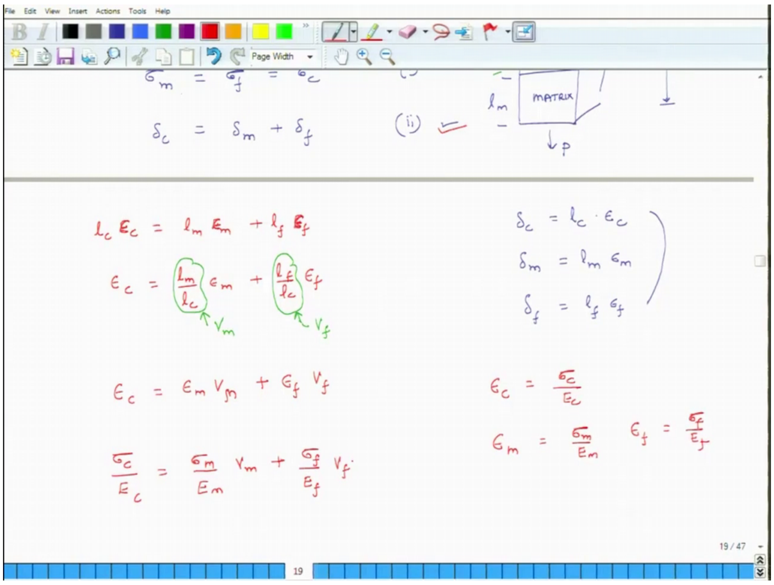

Similarly, extension in matrix is equal to lm times epsilon m and extension of fiber is equal to lf times epsilon f. So, I put all these three equations in my second equation. So, I get lc ec is equal to lm E m plus l f E f and I can say, I am sorry, it is not E, these are strains, I am sorry, these are strains, these are not Young's modulus; what do you like these are strains and if I divide this entire relation by. So, lc. So, I get strain in composite is equal to L m over lc plus lf over lc epsilon f.

Now, this is what volume fraction of the matrix this is volume fraction of fiber why is it volume fraction of fiber and matrix what is the volume of fiber volume of fiber is lf times cross sectional area a volume of matrix is l m times cross sectional a and volume of the whole thing is l c times A. So, A cancels out. So, that is why we have this. So, epsilon c equals epsilon m V m plus epsilon f V f.

And now we know that epsilon c stress in the strain in the composite is what a stress in composite divided by E c epsilon m is a strain and matrix divided by E m and epsilon f is strain and fiber divided by E f. So, I get sigma c by E; E c, excuse me is equal to sigma m by E m V m plus sigma f by E f V f, but we said sigma m equals sigma f equals sigma c. So, because of this.

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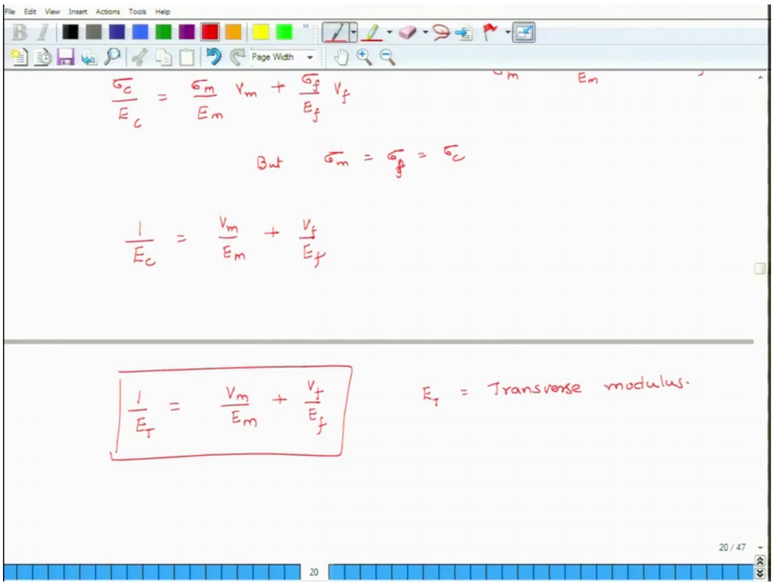

So, but sigma m equals sigma f equals sigma c. So, what I can write it as 1 over E c is equal to V m over E m plus V f over E f.

Now, Ec is what? It is the Young's modulus of the composite in transverse direction. So, I will not use Ec, but I will use the term ET because that tells us the direction of the curve. So, I will say that 1 over E T is equal to V m over E m plus Vf over Ef.

This is my relation for transverse modulus. So, this is our first and as I said earlier it is a very crude approximation, why is it rude, we will see it tomorrow, but this is a good starting point in terms of how to predict the transverse modulus of a unidirectional laminate or a unidirectional composite.

Tomorrow, we will discuss what does this expression mean and what are the limitations of this expression and how can we get better estimates of transverse modulus using some other methods. So, that concludes our discussion tomorrow we will continue this discussion on transverse modulus of unidirectional composites and till then have a great day we will meet once again tomorrow.

Thank you.