**Introduction to Composites**
**Prof. Nachiketa Tiwari**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Kanpur**

**Lecture – 34**
**Example Based on Minimum and Critical Voulme Fraction**

Hello. Welcome to introduction to composites. Today is the fourth day of the ongoing week. Today, we will do one more example which will help you understand the importance of minimum volume fraction and critical volume fraction and also how they are actually calculated from data related to different matrix and fiber materials.

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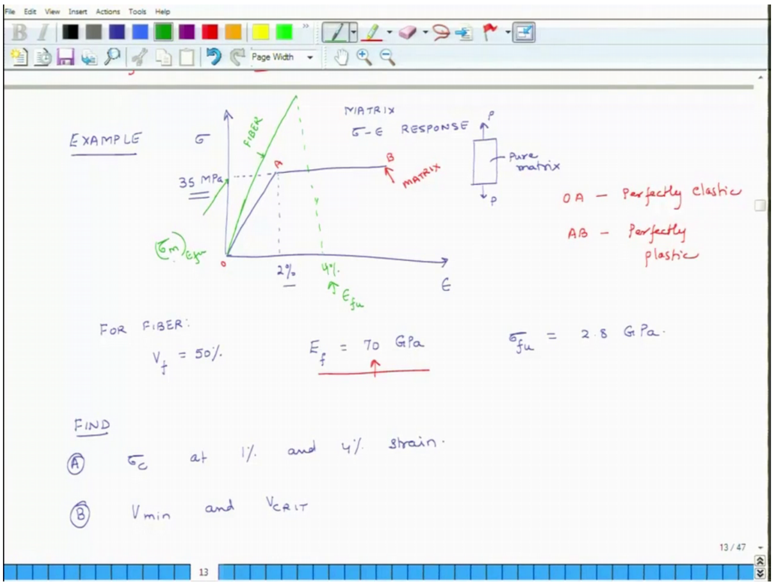


So, the example is related to fiber volume fiber matrix system and what we are going to do is we are going to first see how the matrix behaves. So, this graph is about matrix stress strain response. This is about stress strain response of a matrix.

So, how do you den generate this curve you take a piece of material which is only matrix and you apply some force. This is pure matrix and you keep on measuring strain and stress in the system and you make a curve out of it. So, on the x axis, we plot a strain and on the y axis, we are plotting stress in the matrix material and this matrix material behaves something like this. So, this region; so, initially, it is perfectly plastic a perfectly elastic. So, this matrix material has stress strain response has 2 curves O A is perfectly elastic. It is perfectly elastic material. So, if I leave the strain at a point within the region

O A, it will get back to its original position which is 0 position and then after A, it becomes perfectly plastic; what does that mean that if I apply a strain which is more than the strain level at A, then it never gets back to its original configuration.

It holds that configuration. So, this breaking point in the curve is 2 percent strain. So, if the strain in the matrix is less than 10 percent and you release that strain matrix goes back to its original undefined position. If you deform the matrix beyond 2 percent maybe 3 percent, then it does not change its shape. Once the strain and external stresses relieved it just stays there.

So, that is there. So, and the corresponding stress level in the matrix is 35 MPa. Now I am using this matrix and a fiber to make a composite. So, these are the properties of the matrix as given in the graph for fiber. What do we have volume fraction of the fiber is 50 percent and volume fraction and Young's modulus of the fiber is 70 GPa and breaking strain; strain breaking stress of the fiber strength of the fiber is 2.8 GPa.

So, the question is find A; what is the stress in the composite stress in composite at 1 percent and 4 percent strain; what is the stress in the composite at 1 percent and 4 percent strain B. What are the values for V min and V CR IT? This is what we have to find. So, we have the properties of fiber we have properties of matrix, we have the volume fraction of fiber and with this data which is provided we have to compute the stress in the composite when I pull it by 1 percent and stress in composite when I pull it further. So, that the strain in composite is 4 percent and we also have to find V min and V CR IT.

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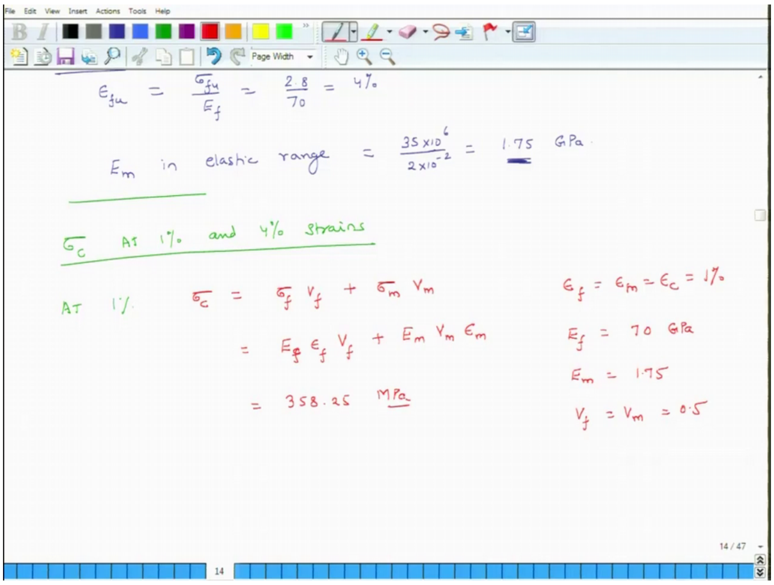

So, let us find this first we find epsilon f u. So, we this is the solution because to find V min and V CR IT we have to find what is the breaking strain in the fiber and corresponding to that breaking strain what is the stress in the matrix we have to find that. So, to do that first we have to compute breaking strain of the fiber. So, the fiber breaks at. So, this is equal to sigma f u divided by E f and this is equal to 2.8 by 70 that is equal to 4 percent.

So, the fiber breaks at 4 percent strain fiber breaks at 4 percent another thing we have to compute is E m Young's modulus of the matrix. So, Young's modulus of the matrix in elastic range, yeah, let us find in elastic range. So, this is equal to. So, how do we find the Young's modulus? This is O A is the elastic range at point A 35 percent; 35 MPa stress strain is 2 percent. So, this is equal to 35 into 10 to the power of 6 by 2 into 10 to the power of minus 2. So, that comes out as 1.75 GPa and above that it becomes plastic.

So, the reason I am saying that is. So, this is 1.75 GPa; what does that mean the slope of this line 1 zee O A is 1.75, right, slope of O A is 1.75 and if I also had to plot fiber data then slope of that line would be how much 70 because this is 70 right the slope of O A reflects the Young's modulus of the material and that is 1.75.

So, if I have to put fiber data on this. So, this is matrix and if I also want to plot fiber data on this it will be a very steep line it will be very steep line. So, let us call this fiber and this fiber is going to break at how much strain 4 percent strain, we have computed that right at 4 percent strain this fiber is going to break. So, this is epsilon f u and when the fiber breaks at 4 percent what is the stress in the matrix 35 MPa. So, this is say this point is also sigma m epsilon f u understood. So, this is very important to understand because we had to figure out what is sigma f at the breaking strain of the fiber. So, now, we have figured these things out. So, the next steps are not all that complicated.

So, we will find sigma c at 1 percent and 4 percent strains. So, first we will do it at 1 percent; at 1 percent.

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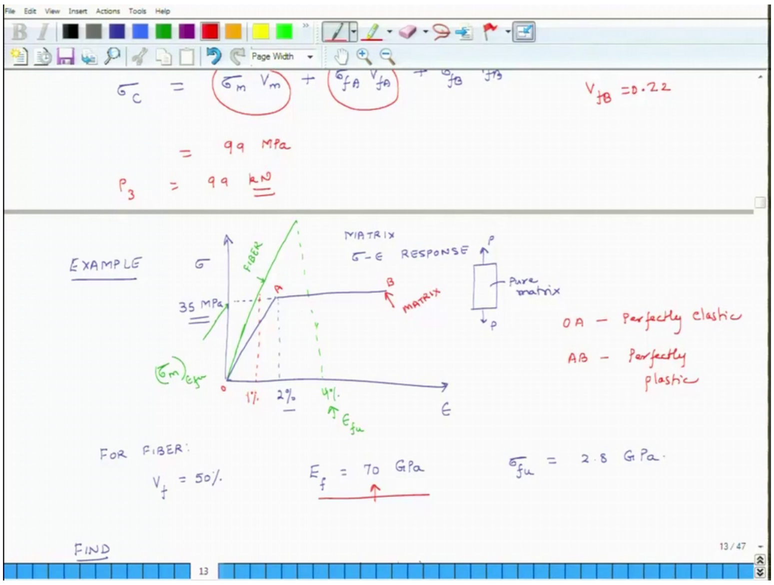


Now, look at this curve again at 1 percent. This is 1 percent; at 1 percent, the matrix will not break nor will the fiber nothing is going to break. So, at 1 percent, sigma c is equal to sigma f V f plus sigma m V m and this is equal to E f epsilon f V f plus E m V m epsilon m and we know that epsilon f is equal to epsilon m is equal to epsilon c is equal to 1 percent and we also know E f; E f is how much E f is.

Student: 70 GPa.

70 GPa E m; E m, we are still in the linear range right. So, E m is how much 1.75; V f is equal to V m is equal to 0.5; 50 percent. So, we put all this information and what we get is we comp compute it to be 358.25 megapascals at 1 percent strain.

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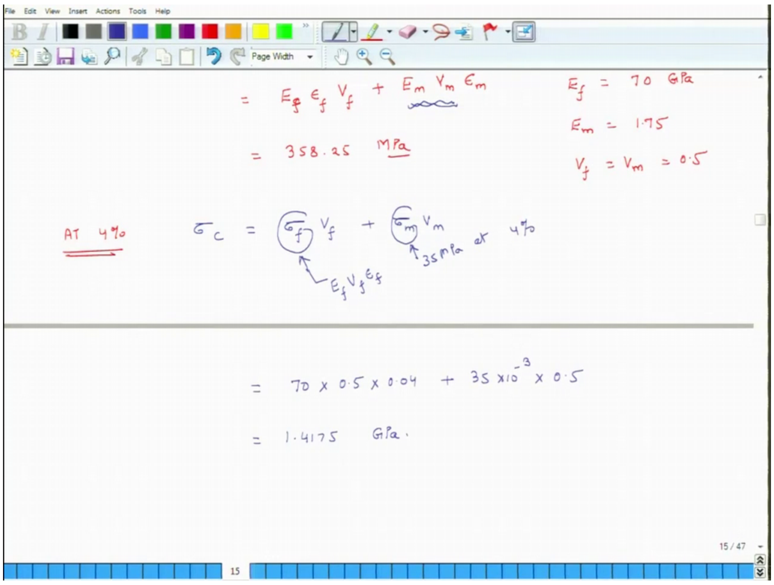


Now, let us look at 4 percent strain. So, sigma c; see this relation sigma c is equal to as long as things are not breaking this relation does not change. So, sigma c is equal to sigma V f plus sigma m V m, this relation does not change as long nothing has broken we can always use this relation when the we are pulling composite in the longitudinal direction we can always use it this thing we cannot use sigma m is equal to E m; V m if unless the material is perfectly linear this thing can only be used for linear materials. Now in 4 percent when the strain is 4 percent fiber is still behaving linearly, but the matrix is not.

Student: Linear.

Linear, but at 4 percent what is the stress in matrix 35 MPa from this graph we know. So, this value is 35 MPa at 4 percent and sigma f is how much E f V f epsilon f. So, this is equal to 70 GPa into volume fraction 0.5 into strain 0.04 plus stress in matrix 35 into 10 to the power of minus 3 because we are putting everything in GPa into volume fraction 0.5. So, this is equal to 1.4175 GPa.

So, even though matrix has become flat it is helping load fiber take more and more load. So, now, we have answered these 2 questions first part we have done next we have to find V min and V CR IT and that is actually very straight forward V min and V CR IT.

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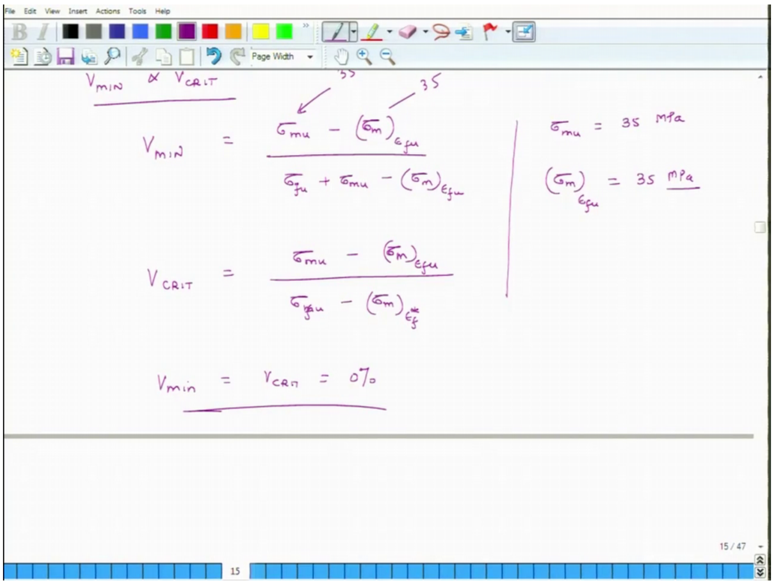

Now, let us write down the relations. So, what is V min; V min is equal to sigma m u minus sigma m evaluated at breaking strain of fiber divided by sigma f u plus sigma m u minus sigma m breaking strain of fiber and V CR IT is equal to sigma mu minus sigma m breaking strain evaluated sig breaking strain of fiber divided by sigma m u minus sigma m. This is sigma f u. Now if you look at what is sigma m u from your graph at what stage does sigma m u fail?

Student: 35 MPa.

It fails at 35 MPa; second question; what is the value of stress in the composite at breaking strain of fiber the fiber breaks at how much strain?

Student: 4.

Fiber breaks at 4 percent strain, we have calculated it at 4 percent strain; what is the stress in fiber in the matrix.

Student: 35 MPa.

Is 35 MPa; so, the numerator; so, this is 35; this is 35 numerator of V critical and V minimum is both 0. So, if that is the case then V min is equal to V CR IT is equal to 0 percent. So, in this special case the minimum volume fraction and the critical volume fraction this comes out to be 0. So, even if you put any volume amount of fiber that will ensure that the strength of the composite is going to be more than the strength of the matrix and also the failure of the composite will be governed by.

Student: case B situation.

The case B situation. So, that is the thing. So, this concludes our discussion for today tomorrow we will move on to next topic. So far, we have been only discussing failure of a unidirectional composite in the when it is loaded in the longitudinal direction tomorrow we will see what happens when it is loaded in the transverse direction. So, that concludes our discussion and we will once again meet tomorrow.

Thank you.