

Introduction to Composites
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Lecture – 33

Example Based on Failure of Composite Material

Hello. Welcome to introduction to composites. Today is the third day of the ongoing week which is the sixth week of this course. Over the last 2 days, we have been discussing how to predict the strength of a unidirectional composite when it is loaded in the direction of fibers and what we have found is the fact that a composite can fail in 2 situations, if the volume fraction of the fibers is extremely small and below a certain minimum number minimum volume fraction, then if breakage of the composite will start with that of the fiber then load gets transferred to the matrix and then after some additional load the composite is going to fail.

However, if the composite volume fibers volume fraction is more than this particular minimum value, then the failure of the composite of the fiber and of the matrix material will be at the same point of time and in this case, the expression for the strength of the composite is found through a different expression, we have also in the last class, developed the relation for a critical volume fraction and if the fiber volume fraction exceeds this particular critical number, then the strength of the composite is higher than the strength of the matrix material.

So, with this information we will now proceed to do 2 examples. So, that we learn a little bit more about the failure of unidirectional composites in tension loaded in the length of the fiber.

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EXAMPLE 1

A = 10 cm²

	P	M _s (%)	E (GPa)	σ _u (GPa)
MATRIX	1.3	35 ←	3.5	0.06
FIBER A	2.5	45	70	1.4
FIBER B	1.6	20	6	0.45

- MAX. LOAD WHEN NOTHING BREAKS. ≡ P₁
- MAX. LOAD WHEN ENTIRE COMPOSITE BREAKS ≡ P₀
- SEQUENCE OF FAILURE.

$$P_0 = \frac{l}{\frac{M_m}{E_m} + \frac{M_{fA}}{E_{fA}} + \frac{M_{fB}}{E_{fB}}} = \frac{l}{\frac{0.35}{1.3} + \frac{0.45}{2.5} + \frac{0.2}{1.6}} = 1.74$$

So, the first example for a system which has not one, but 2 different fibers; so, it has one single matrix and 2 different fibers. So, you have a composite and it is having a matrix material and then there are 2 fibers. So, you have matrix then fiber A and fiber B and then we have. So, this table will give you the properties of different materials all these 3 constituents. So, first thing is density.

So, density is 1.3; this specific gravity is 2.5 and 1.6 and then we are giving you the mass fraction not volume fraction, but mass fraction of each of the constituent. So, this is 35 percent, 45 percent and 20 percent and the Young's modulus of this material. So, we assume that all these are materials with linear stress strain curves. So, this is given in GPa. So, Young's modulus for matrix is 3.5 GPa for fiber A is 70 GPa and fiber B is 6 GPa and finally, the strength of the material each of the materials and this is given in MPa is 0, oh excuse me. So, this is also given in GPa.

So, this is 1.06; 1.4 and this is 0.45. So, with this information we have to find out 3 questions answered. So, before that this is a rectangular slab of composites and we are loading it in this direction fibers are like this and this cross sectional area A is 10 cm square. So, this is additional information which is given. So, the first question is. So, as I keep on pulling it at some point of time something will break. So, what is the maximum load when nothing breaks this is the first thing and nothing breaks.

So, if I increase this. So, so load beyond this threshold something will break either the matrix or the fiber or fiber B. So, what is that maximum load second question which I have to answer is. So, let us call it P 1, we will call it P 1; second question is maximum load when entire composite breaks. So, this is we call it P ultimate and the third thing is we have to figure out what which component fails first then next then next and so on and so forth. So, sequence of failure. So, that is another thing we have to figure out.

So, to do this problem first we have to compute the volume fraction for all these components; what we are given is mass fraction and earlier we had developed an expression for volume fraction. So, we have to compute the volume fraction and we also compute the density of the system. So, rho c is this is the expression, we had developed earlier is $\frac{1}{\frac{M_m}{\rho_m} + \frac{M_{fA}}{\rho_{fA}} + \frac{M_{fB}}{\rho_{fB}}}$. So, f A is the mass fraction of fiber A and rho f A is density of fiber A plus m f B by rho f B. So, this equals 0.35. So, this is in percent. So, 0.35 by density 1.3 plus 0.45 by its density 2.5 plus 0.2 by its density 1.6 and that works out to be 1.74 this density is required that is why we are computing.

So, now what is now with this density we can calculate the volume fractions of each component because we know the mass fractions, we know the mass fractions and with this thing we can compute the volume fraction?

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• MAX. LOAD WHEN NOTHING BREAKS. $\equiv P_1 \leftarrow 422 \text{ kN}$
 • MAX. LOAD WHEN ENTIRE COMPOSITE BREAKS $\equiv P_0 \leftarrow 460 \text{ kN}$
 • SEQUENCE OF FAILURE. ✓

$$\rho_c = \frac{1}{\frac{M_m}{\rho_m} + \frac{M_{fA}}{\rho_{fA}} + \frac{M_{fB}}{\rho_{fB}}} = \frac{1}{\frac{0.35}{1.3} + \frac{0.45}{2.5} + \frac{0.2}{1.6}} = 1.74$$

$$V_m = \frac{\rho_c}{\rho_m} \times M_m = \frac{1.74}{1.3} \times 0.35 = 0.47 \quad V_{fA} = 0.31 \quad V_{fB} = 0.22$$

FAILURE SEQUENCE
 BREAKING STRAINS

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So, volume fraction of matrix is what is density of composite divided by density of matrix into its mass fraction and if you it will be worthwhile to develop this expression

once you go back from this lecture. So, this is equal to 1.74 which is what we have calculated divided by density of matrix 1.3 into mass fraction 0.35 and this works out to be 0.47.

Likewise $V_f A$ volume fraction of the A fiber is 0.31 and $V_f B$ is 0.22.

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FAILURE SEQUENCE

BREAKING STRAINS

$$E_{\mu} = \frac{0.06}{3.5} = 1.71\%$$

$$E_{fA} = \frac{14}{70} = 2.0\%$$

$$E_{fB} = 7.5\%$$

SEQUENCE OF FAILURE: MATRIX \rightarrow A \rightarrow B.

So, now we have volume fractions of each component. So, now, we look at failure sequence we look at failure sequence. So, for failure sequence we have to compute. So, what are we doing in the system we are we are taking this composite which has 2 types of fibers a and B and we are increasing the load on the rod we are.

So, we put 10 kilograms 20 kilograms, 30 kilograms so on and so forth right and as we are increasing the load what is happening the length of each of the components is also increasing, right and the length of each components. So, the matrix gets elongated by some amount the fiber gets fiber A fiber B and the elongation of each of these components for a given load is going to be the same because this is what we had assumed when we were developing the relation for $E L$, right.

So, to figure out which component fails first we have to figure out the breaking strains of each of these components whichever components has the least breaking strain it will fail first. So, breaking strains; so, first we compute for matrix ϵ_{mu} what is its strength

its strength is 1.06 GPa and its Young's modulus is 3.5 GPa. So, that works out to be 1.71 percent.

Breaking strength for a fiber is equal to 1.4 GPa divided by 70 and that is equal to 2.0 percent and breaking strength for fiber B you can do the same mathematics is 7.5 percent. So, what; that means, so, because it is a load controlled experiment; what is the what do you mean by load control, we are applying external load we are applying external load because it is a load controlled experiment first thing which is going or an everything is having the same strain the first thing which will break is the matrix in this case. Now when we were discussing in the last lecture and last several lectures we said that matrix take can take a very large amount of strain it fails in the last.

In this case, it just happens that we have chosen a very fancy matrix which has a very low breaking strain this is not the case for regular matrices it just happens that this particular matrix is a very special type of matrix may be it is a ceramic matrix ceramics most of polymer based matrices can take very large strains, but maybe it is a ceramic based matrix. So, ceramic matrices even if you pull them a little bit ceramic is like crockery you pull it even a little bit it will crack ok.

So, these special types of matrices can take very small strain. So, that is why this special type of matrix will fail first then at 2 percent strain fiber A is going to fail and then at 7.5 percent strain fiber B is going to fail. So, the answer is. So, what is the failure sequence. So, sequence of failure will be matrix, then A and then B. So, that is clear of one question which is the sequence of failure.

Next the question is; what is the load when nothing breaks and just if we increase the load a little bit more than that then something is going to break, right. So, what is going to fail first we will have the failure of matrix and that failure happens at 1.71 percent? So, we have to figure out at what is the load level in the system when the strain is 1.71 percent in the system.

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$E_m = \frac{0.06}{3.5} = 1.71\%$
 $E_{fA} = \frac{14}{70} = 2.0\%$
 $E_{fB} = 7.5\%$
 SEQUENCE OF FAILURES: MATRIX \rightarrow A \rightarrow B.
 LOAD WHEN NOTHING BREAKS $\rightarrow P_1$
 $\sigma_c = \sigma_m V_m + \sigma_{fA} V_{fA} + \sigma_{fB} V_{fB}$
 $= 422 \text{ MPa}$
 $P_1 = 422 \times 10^6 \times (10 \times 10^{-4}) = 422 \text{ kN}$

So, the next thing is load when nothing breaks that is the next thing we are trying to figure out. So, so this is the second one. So, if nothing is broken then what do we do we know that the relation in a composite strain stress in a composite in the longitudinal direction is equal to sigma m V m and now this has 2 fibers. So, what is it sigma f stress in fiber A times V fiber A volume fraction of fiber A plus stress in fiber B times volume fraction of fiber B.

So, and then, but we do not know these values. So, we know volume fraction of each of these fibers, but we do not know stress in these components. So, we can, but we can calculate that, what is sigma m it is equal to Young's modulus of matrix times; how much strain level 1.71 percent. So, 0.0171 sigma f A is equal to E f A into 0 0.0171 and sigma f B is equal to E f B into 0 0.0171 and we already know the volume fractions of each of these components the volume fraction of matrix is how much we just calculated it; it is 0.47 for fiber A is 0.31 and for fiber B is 0.22. So, with all these, so and we already know E m s Young's modulus. So, we can calculate all this. So, what we get is 422 MPa, but what is the question we have to calculate the load is not same as stress. So, how do we calculate load.

Student: (Refer Time: 1802).

Multiply by area. So, P 1. So, this we had said was P 1. So, P 1 is what P 1 is 422 into 10 to the power of minus 6; 10 to the power of 6 into area. So, 10 centimeter square and we

have to do it in meter square. So, this works out to be 422 kilo nektons; that is my stress. So, if I keep on increasing the load till it reaches 422, kilo Newton nothing breaks.

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Next Break point:

$$\sigma_c = \sigma_m V_m + \sigma_{fA} V_{fA} + \sigma_{fB} V_{fB}$$

$$= 460 \text{ MPa}$$

$$P_2 = 460 \times 10^6 \times (10 \times 10^{-4}) = 460 \text{ kN} \leftarrow$$

Load required to break fiber B.

$$\sigma_c = \sigma_m V_m + \sigma_{fA} V_{fA} + \sigma_{fB} V_{fB}$$

$$= 99 \text{ MPa}$$

$$P_3 = 99 \text{ kN}$$

$\sigma_{fA} = E_{fA} \times 0.02$
 $\sigma_{fB} = E_{fB} \times 0.02$
 $\sigma_{fB} = E_{fB} \times 0.07$
 $V_{fB} = 0.22$

And if I increase it may be 423 what will happen once the load exceeds P 1; what will happen?

Student: It will crack.

The matrix is going to crack. So, if P exceeds P 1 matrix cracks first it depends how. So, once matrix has cracked all the load which is bond by the matrix it gets shifted to fiber A and fiber B. So, once it cracks all the loads gets shifted to other f A the fibers, but I can still keep on pulling it further, right the next thing which is going to break will be what.

Student: Fiber A.

Fiber A is going to break and that will break at 2 percent. So, next, let us find out next breakage point next break point. So, at the next break point, stress in composite will be stress in matrix times volume fraction of matrix this is the general relation plus stress in fiber A times volume fraction of fiber A plus stress in fiber B times volume fraction of fiber B, but the matrix has already broken if the matrix is broken this thing will be 0. We cannot use this term.

So, this will be 0 because matrix does not exist it takes it takes no load and $\sigma_f A$. So, the next breaking will happen at 2 percent, right at 2 percent what will be the stress in $\sigma_f A E_f A$ times 0.02 and stress in B will be $E_f B$ times 0.02 you do this and you compute that and we already know the volume fractions of fiber A and fiber B we already know volume fractions of fiber A and fiber B. So, they are not changing.

So, once we do the computation this comes out to be 460 MPa; 460 MPa and P_2 is what? P_2 is 460 into 10 to the power of 6 into 10 area. So, this is 460 kilo Newton's; 460 kilo Newton's. So, the next breaking will happen at 460 kilo Newton's. So, I have pulled it at 1.71 percent, matrix breaks I pull it further at 2 percent fiber A breaks, then what happens once fiber breaks its load also get transferred to fiber B and fiber B will break only at what; 7 percent.

So, let us see what happens. So, at load required to break fiber B because this fails in the end at 7 percent. So, in that case σ_c again we will use the same formula $\sigma_m V_m$ plus $\sigma_f A V_f A$ plus $\sigma_f B V_f B$, but this is already 0 at 7 percent, matrix is already broken at 7 percent fiber A is also broken. So, the contribution from these 2 terms is going to be 0 and $\sigma_f B$ will B what; $E_f B$ into 7 percent and $V_f B$ is how much; 22 percent.

So, you do this all this calculation and you find that this is only 99 Newtons; 99 megapascals; what does that mean that once the fiber A, breaks it transfers all the load to fiber B, but just to break fiber B you only needed 99 megapascals, right and all this extra stress has got only shifted to it fiber A is going to break at 460 megapascals let us do one more computation.

Student: 460 kilonewton.

460 kilo; now one say P_3 is equal to 99 kilo Newtons. So, so once fiber B breaks fiber B breaks 460 kilo Newtons is transferred to how much 2 fiber B, I am sorry, once fiber A breaks all once fiber A breaks at breaking point it is having 460 kilo Newtons, right, right all that load gets transferred to fiber B fiber B can only take 99 kilonewtons fiber B can only take 99 kilonewtons at 7 percent load if right at 7 PA.

So, fiber B cannot handle all this extra load which it gets. So, the point what I am trying to make is once matrix breaks it transfers the load to A and B that load can be taken by

fibers, but once fiber B breaks then there is complete failure. So, you have 1.7 percent failure more load 2 percent failure and then all of a sudden 7 percent also strain gets generated and everything fails. So, the question the answer to this first question is. So, what is the maximum load when nothing breaks that was we have calculated 422 kilonewtons and maximum load when the entire composite breaks that is.

Student: 4.

460 kilonewtons; so, the point what I am trying to make is that when we do analysis of cracks and failures in composites it requires a little bit more thinking into if doing it more aware of the physical processes which are happening and using that understanding we can predict failures of different components of the material.

Tomorrow we will do one more example of failure and that will be related to critical volume fractions of a fiber matrix system, but this discussion on failure of fiber matrix system, today, I hope is helpful in help making you understand the complexity involved in predicting failure of composite materials and we will continue this discussion tomorrow, till then have a great day, bye.