Introduction to Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 31
Failure of Unidirectional Lamina

Hello; welcome to introduction to composites. This is the start of the sixth week of this course and what we plan to do over this period is have a detailed discussion on how to predict different values and different properties of unidirectional composites.

Last week, we had started this theme and we had covered specifically, how to predict the density of a unidirectional composite material. And then, we had just started discussing prediction of longitudinal modulus of composites and we closed last week by having a discussion of how do fibers and matrices fail because that discussion will constitute the basis of our initial discussion on this week because what we plan to start from today is discussion a detailed discussion on how do you need unidirectionally composites fail especially when they are loaded in tension along the length of the fiber.

So, that is what we plan to discuss today.

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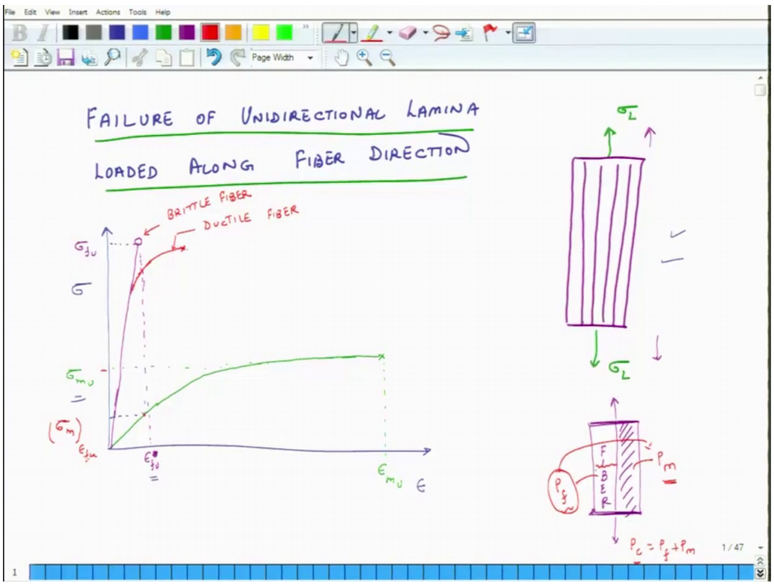

So, what we will discuss is failure of unidirectional lamina loaded along fiber direction. So, what does that mean what it means is that suppose I have a unidirectional composite and the fibers are aligned along the length of the composite and suppose I apply an external load sigma L why do I call it sigma L because it is in the longitudinal direction we had defined longitudinal transverse and transverse prime directions in the last week.

So, I am applying the external load in the direction of the fibers and the because of that load the stress in the composite is sigma L, then I want to discuss how does this lamina or this composite which is unidirectional in nature it fails. So, just a very quick overview in the last class, we had discussed in detail about the failure of how do fibers fail, how do matrices fail. So, we will very quickly review that and then extend that understanding to failure of this type of a composite.

So, if I have a piece of matrix material, initially it is linearly elastic and then it starts becoming somewhat plastic and slowly the slope becomes almost 0 and then it fails at a particular point. So, this strain at which the matrix is failing, I can call it epsilon m u ultimate strain in the matrix at which it fails and the corresponding stress at which it fails I can call it sigma m u, it is the ultimate strength at which this matrix is failing.

For fibers we had discussed two different types of fibers one was an elastic fia brittle failure and the other one was a ductile failure. So, if the fiber is brittle in nature typically these fibers are much much more stiff than the matrix than the matrix. So, if the fiber is brittle, then let us say I keep on pulling it. It takes on more and more load and at a certain stress level. It just cracks and the fiber fails catastrophically. So, this is the case of a brittle fiber and this value would be sigma f u which is the ultimate fiber strength.

Now, if the fiber is not brittle, but ductile in nature then at a certain point it starts behaving plastically and at some point it again fails this is for a ductile fiber and this curve is for a brittle fiber is for a brittle fiber, it is for a brittle fiber. Now in case of a composite, how would this kind of a composite fail, it will depend on several things, but first we will draw a graph for it and then, we will develop some mathematics around it.

So, if there is a composite material which has fibers all in the same direction and if I keep on pulling it, typically, what will happen if I pull that composite, essentially, what is going to happen is that the composite is going to become longer, right and we note that typically the failure strain of fibers. So, what is the failure strain of fiber for the brittle fiber for the brittle fiber for brittle fiber this is epsilon f u; this is the failure strain.

So, we see that for brittle fiber as well as for the ductile fiber the failure strain of matrix is much more than the failure strain for the fiber does not matter whether it is a brittle or a ductile fiber the typically matrix becomes very elongated and only then it actually fails. So, with that understanding, when I am pulling this material in the length direction, what will happen the fiber starts becoming longer and the matrix also starts becoming longer, but once the strain and the fiber reaches its breaking point the fiber will fail the fiber will fail and. So, consider this.

So, let us say, this is matrix and this is fiber hmm and I am pulling it like this and whereas, I pull it the strains in the fiber and matrix because they are going to be the same because I am pulling it by the same amount and as I keep on pulling it at some point what will first fail is the fiber because it breaks at a smaller strain point. So, whereas the fiber breaks all this material is no longer taking the load? So, at the breaking point let us say fiber takes P f total amount of load which is being taken by the fiber P f and the load b which is being taken by the matrix is P m and if I add these two f these two things together then P composite is equal to P f plus P m, right.

So, initially as I keep on pulling it there is a stress in the fiber and load in the fiber and load in the matrix material and there are some of these loads is equal to load in the composite P c, but at some strain level the fiber breaks and what happens, but I am still applying the same amount of load suppose I have applied two hundred kilograms of load on a piece of material that two hundred kilograms is still there that is P c. So, where does that all that load go this P f all of a sudden once the fiber fails it becomes 0 all that extra load is shared by P m. Now, it depends if the cross sectional area of matrix is very large and the cross sectional area of fiber is very small then the amount of P f will be very small, right.

Suppose, I have only one single fiber in the composite only one single fiber then the amount of load which will be carried by the fiber will be extremely small negligible and that load will get transferred to the matrix and matrix will be able to bear it. So, then I can still apply more and more load and then once the matrix fiber matrix breaking strain is reached then the matrix fails. So, that is one case and that is the case when the amount of fiber in the material is extremely small.

If the amount of fiber in the material is reasonably large and we will find what is the meaning of that reasonably large term, then what will happen the magnitude of P f will be significantly large and all this load signify gets transferred to the matrix and all of a sudden matrix has to now bear a very high amount of load, but the breaking stress in the matrix is not very high it is sigma m u which is much more much less than sigma f u right.

So, because it cannot now all of a sudden handle such an extra load matrix also fails immediately. So, we have two types of situations if fiber volume is very small volume fraction of fiber is small I keep on pulling it fiber breaks whatever is the extra load that is not large enough, it gets transferred to matrix and the amount of that extra load which is now transferred to matrix is not large. So, matrix can handle this extra load and I keep on pulling. So, the composite does not break.

So, now, I have to pull the matrix even more and then the matrix breaks the second situation is that if the volume fraction of fibers is large then once the fiber breaks the amount of load which is transferred to matrix is significantly large and at that point of time matrix cannot handle that extra load. So, matrix also breaks. So, the entire composite fails.

So, these are the 2 failure scenarios for composite. So, now, we will develop the relation for both these scenarios.

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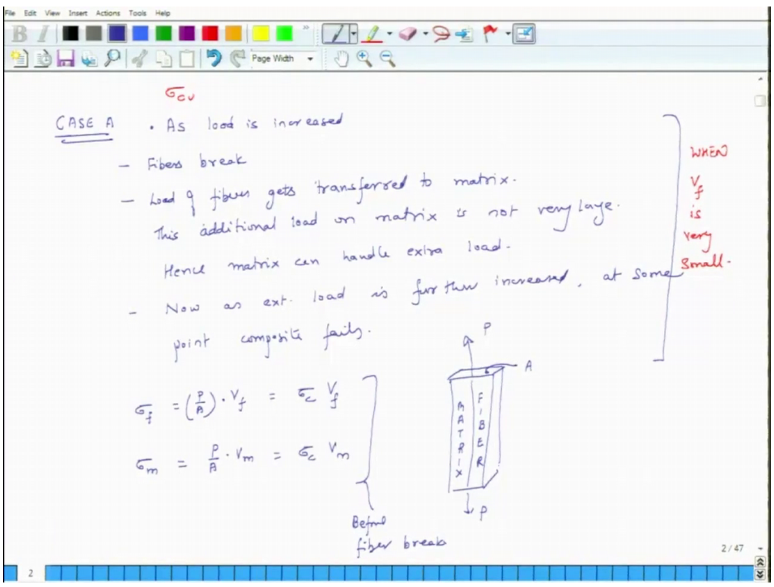

So, the first one is case A; what is the in this case here what is the situation as load is increased first thing is fibers break and then once the fibers breaks what happens load of fibers gets transferred to matrix this load this additional load on matrix is not very large hence matrix can handle extra load. So, when matrix can. So, now, as external load is further increased as external load is further increased what happens at some point composite fails.

This is the first case and this case happens when V f is very small because the V f is large, then the load transfer will be reasonably high and matrix will not be able to handle it. So, in this case, we will find; what is the value of sigma c u which is the ultimate tensile strength of the composite ultimate tensile strength of the composite. So, again we will draw picture.

So, let us say this is fiber and this is matrix and I am applying a load of P. So, what is and let us say the cross sectional area is cross sectional area is A. So, sigma f will be what it will be P by a into volume fraction of fiber. So, P by A is what? Sigma c the stress in the composite times volume fraction and sigma m is again P by a into volume fraction of matrix is sigma c V m. Now these are the relations before breaking before fiber break these are the relations before the fiber breaks.

At the point once fiber breaks what happens.

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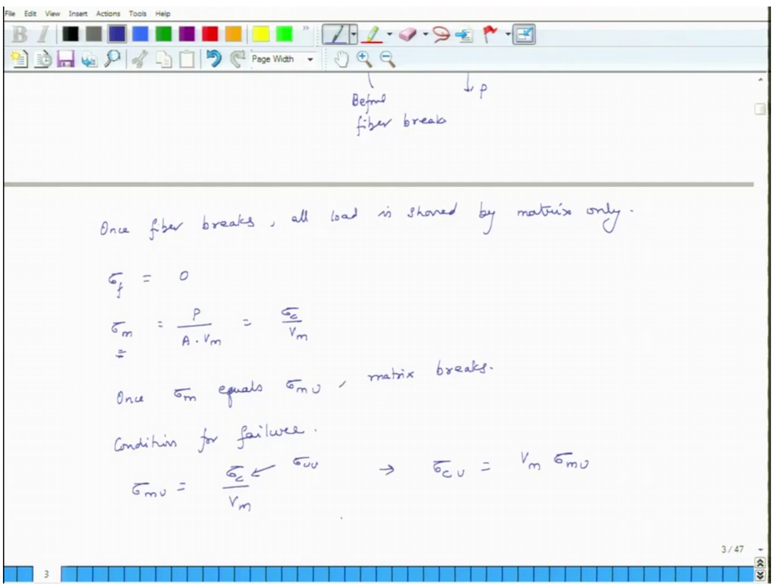

Once fiber breaks, all the load is being shared by matrix only all load is shared by matrix all the load is shared by the matrix. So, in that case sigma f the stress and fiber is what is 0 and the stress in matrix is what? What is the stress in matrix? Well total load is external load is force and what is the area of matrix area times V m right this is the area of the matrix.

So, this is sigma c by V m. So, once fiber breaks all the load is shared by matrix only. So, this is the stress in the matrix. Now as I keep on increasing my sigma c. Now as I keep on increasing my sigma c sigma m also increases and once this sigma m reaches sigma m u this sigma m u. So, once sigma m equals sigma m u matrix breaks. So, at this point, fiber has broken and matrix has broken. So, entire composite has failed. So, condition for failure is sigma m u equals sigma c by V m, right sigma m u is equal to sigma.

And at that point and that is what I call sigma c u that is what I call sigma c u. So, this is actually sigma c u ultimate strength of the composite. So, I can write sigma c u equals V m times sigma m u and I know that V m is what? 1 minus V f.

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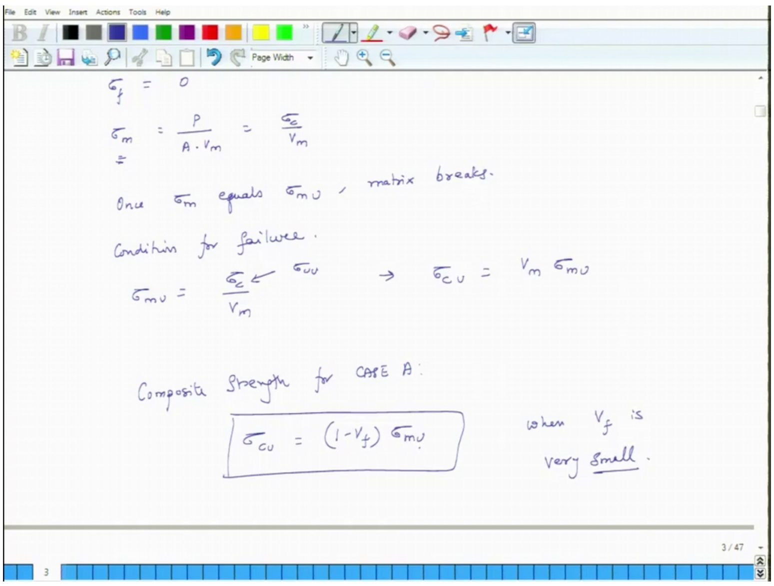

So, the strength of composite for case A, what is the strength of composite case A say sigma c u equals one minus V f sigma m u and this is when V f is very small.

And we will define how small it has to be later, but and when V f is extremely small fibers break and at that time all the extra load from fibers get transferred to matrix can still absorb that extra load. So, now, I can keep on loading it further and finally, the matrix will fail and the matrix will fail when the stress in the composite sigma c u equals 1 minus V f times ultimate tensile strength of the matrix material ultimate strength of the matrix material in tension ok.

So, this is case A. So, we will now do case B.

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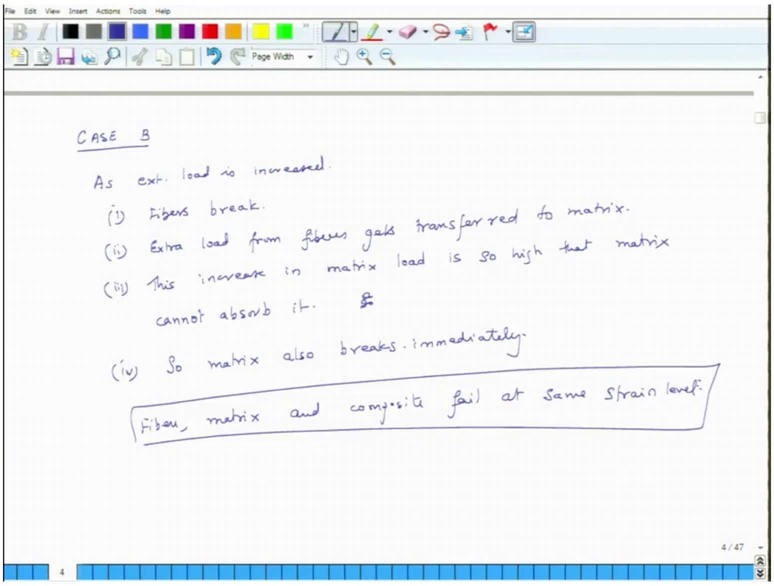

In case B. So, in case A, what did we say? In case A, we said that as load is increased, initially the fibers are going to break load of the fibers gets transferred to matrix and this increase in the load of matrix is not sufficiently high. So, matrix can handle extra load. So, matrix takes the external load. So, I can still further increase my external load and as I increase it beyond the threshold the matrix also fails in case B that is not the situation. So, in case B as external load is increased one fibers break extra load from fibers gets transferred to matrix

Third this increase in matrix load is. So, high that matrix cannot absorb it, it cannot absorb it, so, fourth. So, matrix also breaks immediately. So, we can say that fibers matrix and composite fail at same load this is the condition fibers matrix and composite fail at the same load or I will say at the same strain level I will not call it load, it can create confusion the fail at the same strain level ok.

So, look at this. So, in this case the fibers the matrix and the composite they fail at the same strain level and what is the strain level that the strain level that is epsilon f u. Now, this epsilon f u could be a little higher for ductile fibers or then for brittle fibers, but it is very small, but they will fail at the same strain level which is epsilon f u. F u means fiber strain ultimate breaking strain in the fibers at the breaking strain in the fiber; what is the stress in the matrix that stress; if I look at this picture is this level, right. So, at this strain level epsilon f u the strain stress in the matrix is what this stress is sigma m at epsilon f u level, right.

So, it corresponds to this point on the green curve. So, add this stress. So, add the stress level at the strain level of f epsilon f u; what is the stress in the fiber? In the fiber, the stress is sigma f u and in the matrix the stress is sigma m add this strain level which is epsilon f u understood. So, now, let us look at this picture.

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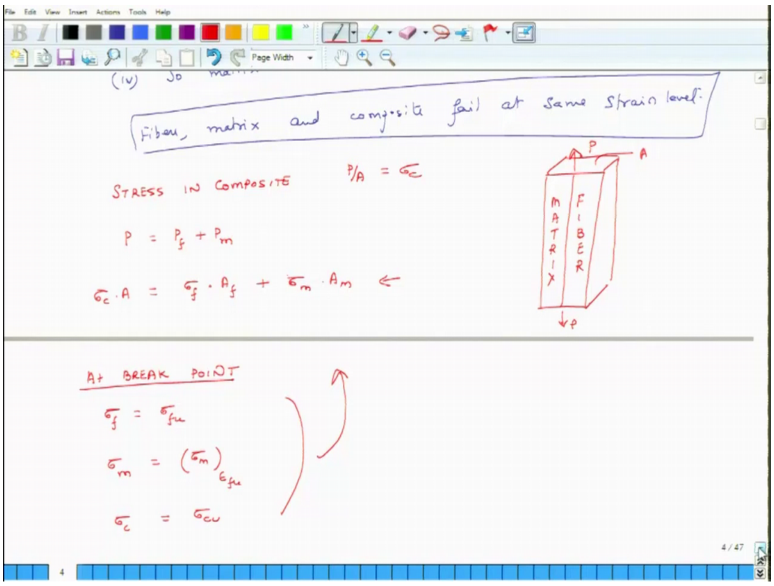

So, here this is my fiber and let us say this is my matrix and I am pulling it with a load P I am pulling it with a load P. So, what is the stress in composite? Stress in composite is P by A. So, that is equal to sigma c right, sigma c.

Now, if I keep on pulling it again fibers fail and immediately, all that load gets transferred to matrix and then matrix also fails. So, at that situation at that situation, what is the; so, just a little prior to breakage, just at the point of breakage P which is the external load will be load shared by fibers plus load shared by matrix, right.

Now, what is the load shared by fiber this will be equal to sigma f times A f plus load shared by matrix is what sigma m times A m and this is equal to sigma c times A. Now this is the general relation before anything is breaking at break point at breakpoint, what is the value of sigma f? Sigma f will be sigma f u. What is the value of sigma m? Sigma m will be the stress in the matrix at the strain level sigma f u epsilon f u, right, breaking strain and what is the value of composite stress it will be sigma c u because the matrix the fiber and the composite all three are failing at the same point. So, I put all these things in above expression.

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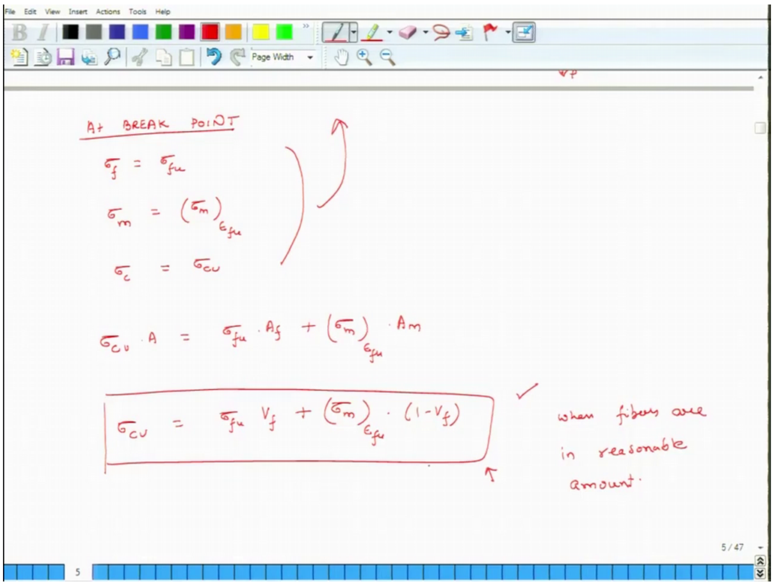

So, I get sigma c u times A is equal to sigma f u times A f plus sigma m evaluated at breaking strain of the fiber times A m.

And if I divide this entire thing; so, what is A? A is the cross sectional area of the composite. This is the cross sectional area of composite, A m is the cross sectional area of the fiber alone A f of the matrix and A f is cross section area fiber. So, if I divide this entire thing by A, I get sigma C u is equal to sigma f u V f plus sigma m at epsilon f u breaking strain of the fiber times V m, but V m is what if there is no air in the system, if there are no voids, then V m is nothing, but 1 minus V f.

So, this is when the situation. So, this is the strength of the composite for case B, this is the strength of the composite for case B and when is this case happening when fibers are in reasonable amount when fibers are in reasonable amount. So, when fibers are reasonable and this is my strength of the material then fibers are extremely small. So, that when they crack the stress transfer to matrix is not large enough to cause it to break then the strength of the composite is given by this relation.

So, we will extend this discussion tomorrow as well and we will conclude our discussion for today here itself and tomorrow, we will try to understand what is the means meaning of very small how small this volume fraction of fibers should be to for the composite to fail in this state and if the fibers are reasonably large in numbers then when does it at what level of volume fraction does this type of failure happen. So, that is what we will discuss tomorrow, till then have a great day, bye.