Introduction to Composites

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Lecture – 28
Composite density as a function of mass fraction and volume fraction

Hello welcome to Introduction to Composites. Today we plan to develop relations for density in terms of volume fractions and mass fractions and densities of fiber and matrix materials for unidirectional composites. So, we have already defined terms like volume fraction and mass fractions and what we want to do next is compute density as a function of. So, we will compute density as a function of mass fraction and volume fraction.

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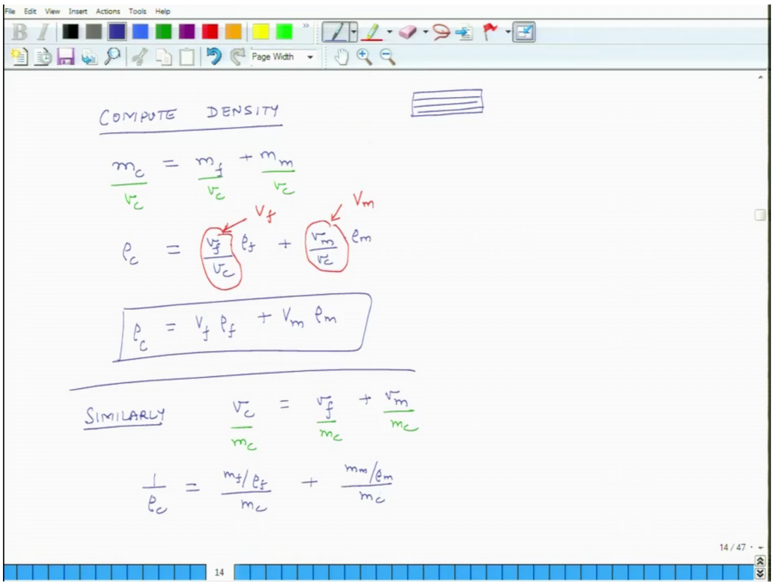

So, we have already stated that if there is a composite material; unidirectional composite material. So, it has fiber and matrix then it the total mass of the composite is m c that equals m f, plus m m, where m m is mass of matrix, f is mass of fiber.

So, what do we do is we now divide this entire equation by the volume of the composite. So, we divide it by v c; the entire thing by v c, so, what do we get? So, what we get is. So, mass of composite divided by volume of composite is what density of composite material? So, this equals now what is mass of fiber? Mass of fiber is volume of fiber time’s density of fiber and then I am dividing this by v c.

Similarly, mass of matrix is what? Volume of matrix times, density of matrix. And this I am getting dividing it by v c. Now we know that v f over v c is what? Volume fraction of fiber and V m over V c is volume fraction of matrix. So, I can write a relation that density of the composite equals V f times, rho f plus, volume fraction of matrix times rho m. So, this is another relation.

So, here I have expressed density of the composite in terms of volume fractions of fiber and matrix materials. Let us write a similar relation for a in terms of mass fractions. So, the other relation so similarly, we can also write that the total volume of the composite, equals volume of fiber plus volume of matrix.

So, we are again right now we are ignoring that this is here in the system. And here we divide the entire thing by mass of the composite. So, on the left hand side what I get is 1 over rho c rho c is the density of composite and on this side. What is volume fraction this volume of fiber? Volume of fiber is what it is mass of fiber divided by density of fiber and the entire thing is divided by m c.

Similarly, volume of matrix is m m divided by rho m and the entire thing is divided by m c.

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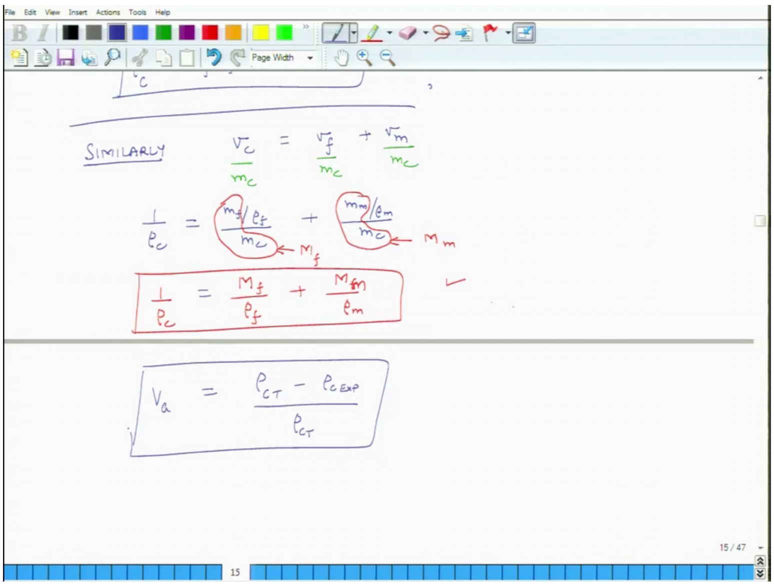

So, I get this term m f over m c is what mass fraction of fiber. And this is mass fraction of matrix. So, what I get is mass fraction of fiber divided by rho f plus, mass fraction of I am sorry mass fraction of matrix divided by rho m equals one over rho c. So, this is the other relation I get.

So, I get these 2 relations for the density in terms of volume fraction. So, if I know volume fractions, I can comp use the first relation to compute the density of composite. If I know mass fractions then I can use the second relation to compute the density of composite.

Now, in both these expressions we did not account for the volume of air. Now this is how you can theoretically compute the overall density of the composite, but then suppose you want to verify, whether it is correct or not. So, how do you do it essentially you have to conduct an experiment and what people do is that they take a small furnace, they measure the mass of the furnace when there is no composite, in it then the place the composite, in it and especially if it has glass fiber or if even if it has graphite fiber, they heat the composite to very high degree centigrade. So, whatever matrix material is there it just gets burnt and only the fiber glass fiber is left.

So, from that they figure out what is the content of glass fiber in the material in the material. So, that gives them an idea of the volume fraction as well as the mass fraction of the glass fiber, because you know the volume and mass of the original composite. So, from that you can figure out what is the volume fraction of fiber in the system?

Now, suppose there is your experimental results and the theoretical results as of the for density as computed from here they do not agree; that means, that there is significant amount of air in the system. So, how do you figure out the from volume fraction of air? So, volume fraction of air equals your theoretical density of the composite will always be a little higher, because it does not account for air right.

So, it will be always higher. So, density of composite theoretical, minus density of composite experimental and how do you experimentally compute the density of composite, you take a piece of composite measure it is find it is weight and find it is volume by dipping it in water divide the 2 and you will get the density of composite. And then if you divide this entire thing by rho c t that will help you understand or assess the volume fraction of air.

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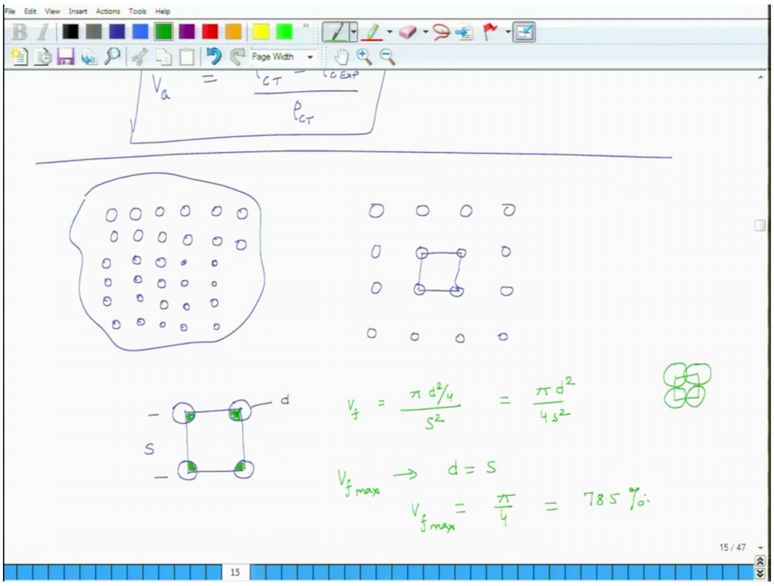

Now, typically you should have an idea I mean what kind of volume fractions of fibers we are talking about. So, if I have a piece of composite the fibers could be oriented in all sorts of formations. So, this is like a rectangular formation because the 4 fa every 4 fibers they are oriented in some sort of a square pattern.

So, if this kind of a pattern is there we can actually figure out what would be the volume fraction for such a system? So, think about it suppose the fibers are oriented in a regular formation like this and this pattern keeps on getting repeated forever.

So, the way you can compute the volume fraction based on this arrangement is you take a. So, they are arranged in a square pattern. If you take a square pattern and then this square pattern in this square pattern, you compute the area of fiber and the area of overall material and whatever is the ratio that is the fiber volume fraction. So, let us look at it.

So, you have 4 fibers let us say the fibers and we assume that all the fibers have same diameter otherwise comp the things get more complicated. So, let us say that the diameter of the fiber is d. And the size of the square is S. Then what is the amount of fiber contained in each square, it is these 4 1 4th circles.

So, if you add up all these grain green areas it becomes one whole circle. So, volume fraction is what pi d square by 4 that is the amount of fiber contained in each square divided by the overall material volume what is that s square. So, that equals pi d square by 4 S square. And what is the maximum possible value of V f if fibers are arranged in such an order, then V f max will be when these circles touch each other when these circles touch each other.

So, V f max will occur when d equals S, that is when circles touch each other, then this is our rectangle or square then d equals S. So, in that case V f max equals pi over 4 and that equals 78.5 percent.

So, if you arrange the fibers in this kind of a configuration this is the maximum possible volume fraction of fibers you can get, but there may be smarter ways to arrange fibers in even more tightly packed systems for instance.

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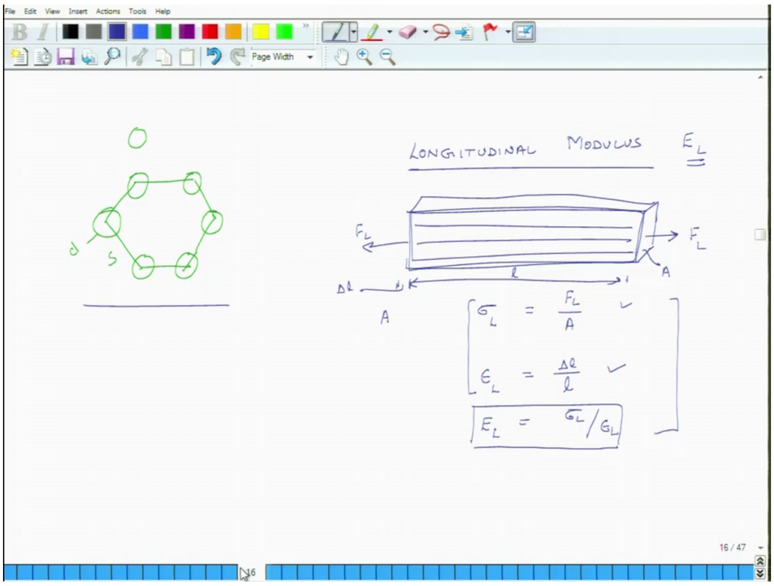

You can arrange them as you know in a (Refer Time: 12:47) that is the tightest possible arrangement.

And in this case let us say the side of hexagon is s this diameter is d and then again you can do the mathematics as what and when fibers touch either you can figure it out. So, this is another thing I wanted to share with you.

So, this is the overview of a density and this is how we go around computing density of composite materials, which are unidirectional in orientation. Now we will start discussing a prediction of other material properties. So, what we will start discussing is computation of longitudinal modulus of a unidirectional composite laminate.

So, what does that mean? So, all the fibers are oriented in the length direction it is a single layer of composite and what we are interested in is that if I pull it in this direction and let us say. So, what is this direction this is L? So, if I am pulling it in L direction.

So, I am applying a load F L and if the cross sectional area is a suppose this is my cross sectional area of the sample, then what is the stress I am applying, then stress will be sigma and this stress is being applied in the L direction.

So, sigma L equals F L over this is the cross sectional area A. So, that is my sigma A you know F L over A. So, I can calculate how much stress I am applying in the overall system? And what will be the strain in the system? So, when I pull it extends a little bit. So, this is the original length and then it extends by some distance delta L. So, what is the strain in the length direction is delta L divided by L. And this is the strain in which direction L direction.

So, then so experimentally this is how I can compute E. So, then E which is the young's modulus, in the length direction is equal to sigma L, divided by epsilon L. So, this is the physical meaning of longitudinal met modulus of this material longitudinal met modulus of this material of a unidirectional composite. And physically the way I am going to measure it is I take a piece of it, pull it in the length direction, find sigma L, find epsilon L take the ration and I will be able to compute the modulus in the length direction. And that modulus I call E L E L. So, this is how we find out experimentally?

But it is what that means, is that to find it out experimentally you have to make a sample, you have to cut it to the right size, you have to find the machine, you have to do the pulling and all that and all that requires a lot of effort. So, the question is that if can I mathematically calculate E L.

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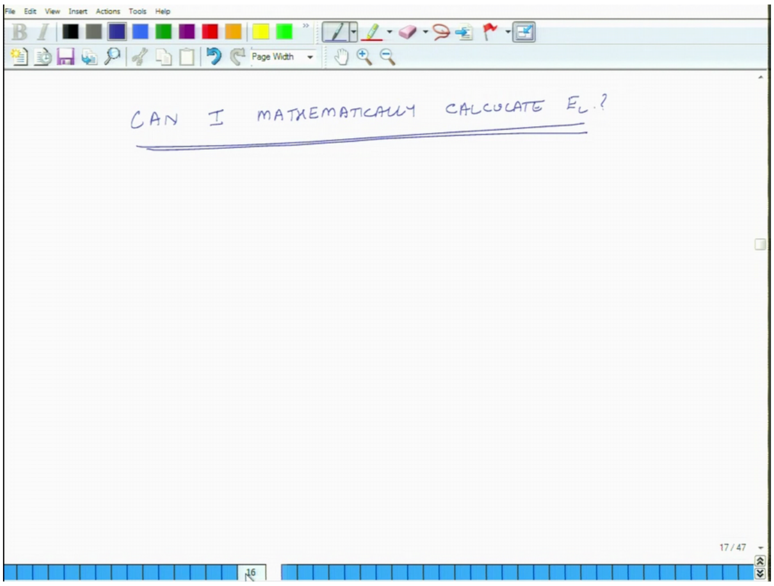

I do not want to let say do experiments suppose, I do not want to do experiments can I get an idea of what this value of E L would be? So, this is our goal this is our goal can I mathematically compute the value of E L.

If I know all the material properties of the fiber as well as the matrix and that is what we will do tomorrow. So, we will compute the mathematically the comp stra value of E L and also the strength of the material, in the tensile direction longitudinal tensile direction; which is one of these 5 strengths which we had discussed.

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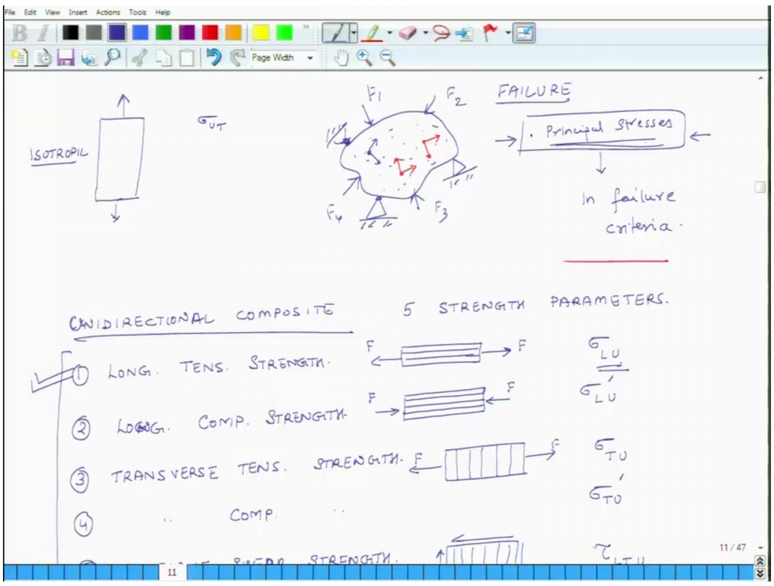

So, we will figure out what is this thing sigma L U of the material. So, this is what we plan to do over the remaining course and I hope you have an enjoyed wonderful time learning all this good stuff and we will meet once again tomorrow and then till then have a great time bye.