Noise Management & Its Control

Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 13
Transfer Functions

Hello. Welcome to Noise Management and its Control. This is the third week of this course. And what we plan to do this over this week is; basically we will be developing the one dimensional wave equation. But before we talk about one dimensional wave equation, I wanted to cover one more specific topic which is related to transfer functions.

So, in this lecture we are going to talk about transfer functions and starting from the next lecture and onwards we will be working on the sound propagation and the equation for sound propagation which is also known as one d wave equation.

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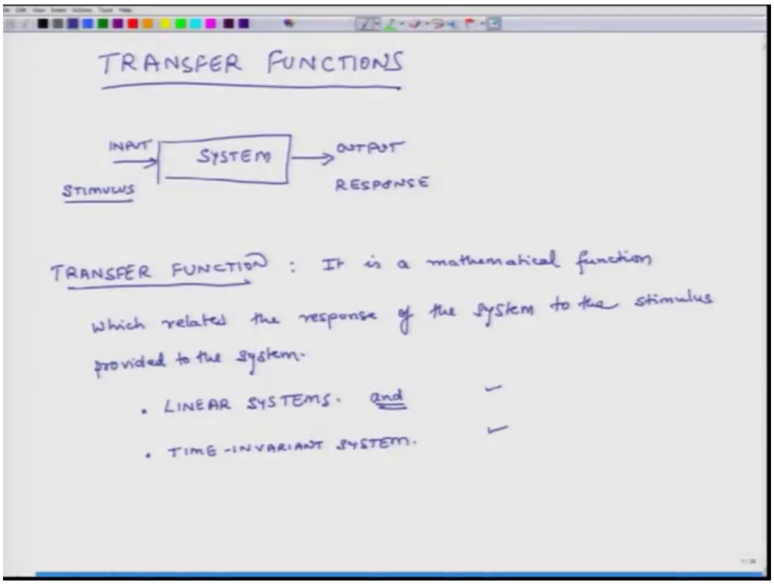

So, right now we will discuss transfer functions. Now earlier, we had discussed that we have a system and we can have an input and then there is an output. So, the input is because of some stimulus and as a consequence of this input or this is stimulus we have the system response and we measure the response. So, the output is in form of some response. So, for example, there is a cantilever simply very simple system I apply some force that is my input and the response will be that the band beam or the cantilever is going to deflect.

So, the response is deflection I can have it just the other way round also I can have an input as a displacement and then I have to see and I can measure how much force is required to produce the deflection. So, that is the response which is the response of the system. So, this is the context in which we will discuss this notion of transfer function. So, what is a transfer function? So, first we will define it and this definition. So, this is it is a function; it is a mathematical function which relates the response of the system to the stimulus provided to the system provided to the system and at least in context of this course we will talk about transfer functions. So, this is its mathematical function which relates response of the system to the stimulus provided to the system and this we will do it.

So, this transfer function notion is really useful only for linear systems and we had discussed; what is a linear system in what are the attributes of linear systems in the last class which is in the last week. So, the systems we will consider are only going to be linear in nature and also. So, it will be end and simultaneously they will be time invariant systems. So, what is the time in variant system where the physical attributes of the system do not change with time for instance a system which is an example of a the system which is not time invariant would be you have a rocket which will we used in fire crackers and we ignite the rocket in the rocket goes up in the air as it is going out it is ejecting mass out of it in form of gases.

So, the mass of the system as the system is moving upwards is changing. So, that kind of a system is not time invariant because the physical some of the physical attributes of the system are changing with time. So, we will talk about transfer functions only in context of linear systems and systems which are linear in nature and simultaneously they are time invariant in nature. So, this is important to understand. So, transfer function is a mathematical relationship which relates the response of the system to the stimulus provided to the system.

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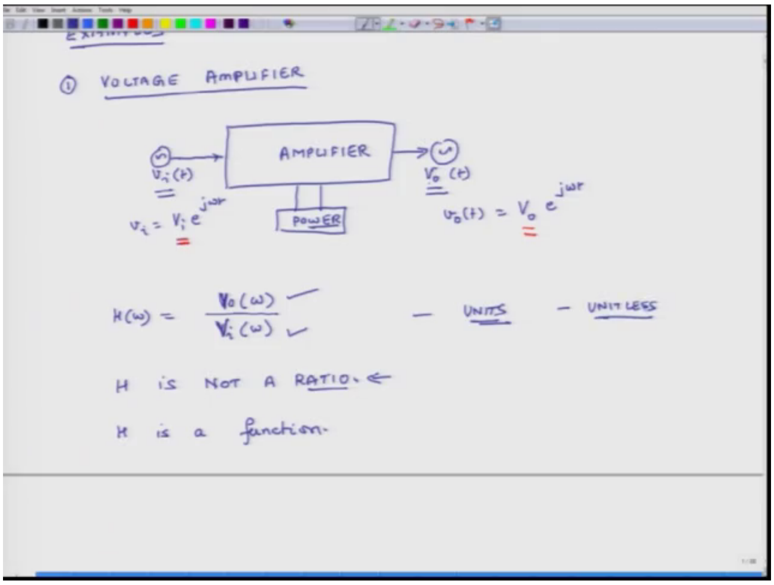

So, we will do some examples. First example could be a voltage amplifier; voltage amplifier. So, what does a voltage amplifier do? It runs on some power source. So, it requires some energy to make it work. So, it runs on some power source. So, it could be a battery or some twenty volt dc supply and then what do I do in an amplifier I exited with some input and let us say it is V i and then the amplifier amplifies this V i and it provides an output. So, it is V o and where does the energy come from to amplify this input it comes from the power source. So, the input to the system is V i and the response of the system is V naught now these are both they could be functions of time.

So, in this case the relationship between V naught and V i the relationship between V naught and V i will be what it will be called the transfer function. So, let us say V i is equal to some complex amplitude V i capital E j omega t and because the system is linear what does that mean that V naught will be some complex amplitude times e j omega t and there could be a phase difference between input and output, but that phase difference gets expressed in what component this complex amplitude part, because we had seen this earlier the phase part component gets into the complex amplitude of V i and V naught. So, the transfer function for the system I can express it as H and that is equal to V naught over V i because e j omega t gets cancelled. Now V naught could change with omega and. So, is V I; V i could change with omega. So, these are complex amplitudes.

So, V naught and V i both could depend on omega which is the excitation frequency. So, H which is the transfer function would be itself;, it can change with omega. So, remember this H is not a ratio because ratio is what it is of 2 numbers a over b, it is a ratio rather H is a function, it is not just a single number H is a function which can change with omega. So, its function and it is known as transfer function. So, this is important to understand lot of times we by error we say that is the ratio between input and output, but no; the numerator is a function the denominator is a function. So, it is a function of 2 other functions.

So, in this case; what are the units unit of numerator is voltage unit of denominator is voltage. So, here unit is unit less.

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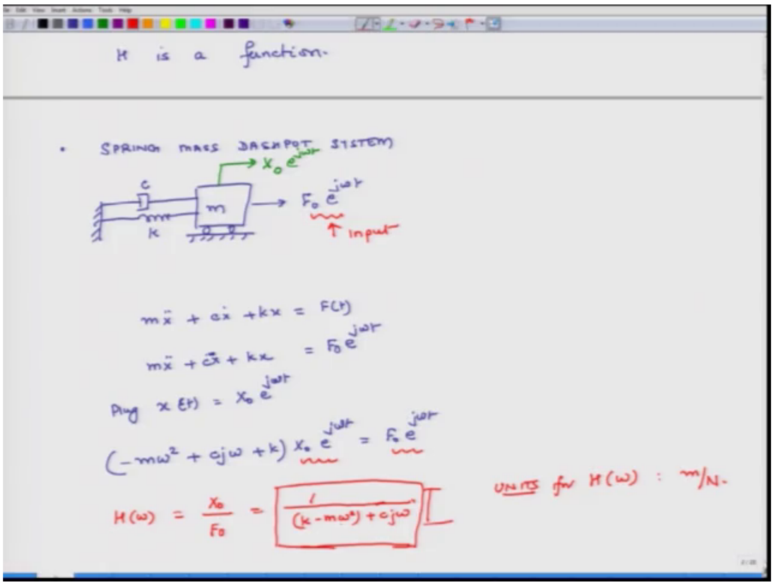

So, in this case, there are no units for the transfer function, but transfer function need not be unit less in all the cases. So, we will consider another example.

So, consider a spring mass dashpot system. So, we have done this example earlier this is my spring this is a dash part both are connected to a mass which can move on a frictionless surface and what I am doing is I am exciting it by some force e j omega t and what I am. So, this is the stiffness of the spring is k the damping constant is c and what I am. So, this is my excitation input and what I am interested in figuring out is how much is the displacement, and because this is a linear system I can say that the displacement will be of a similar form as F naught.

So, it will be some constant amplitude times e j omega t. Now the governing relation for this equation is mx double dot plus c x dot plus k x is equal to F t and this is equal to F naught e j omega t. And now what we do is we plug x t as x naught e j omega t in this equation. So, what we get if we do all the math is minus m omega square plus c j omega plus k x naught e j omega t is equal to F naught e j omega t.

So, this is my response this is my excitation if I come you know take them one other then I get the transfer function is equal to x naught. So, e j omega t gets cancelled from both sides and this is what one over k minus m omega square plus c j omega. So, once again this is not a single number it is varying with omega. So, again it is not a transfer ratio, it is a transfer function that is why we call it a transfer function and what are the units in this case the unit of x naught is meters F naught is Newton. So, the unit of transfer function is meters per Newton.

Now look at this relation the transfer function does not depend on F and neither does it depend on x naught, because what do you have in transfer function, it is 1 over k minus m omega square plus c j omega k is a purely, it purely depends on the system m which is the mass it again purely depends on the system, and c again purely depends on the system. So, transfer function at least for linear systems.

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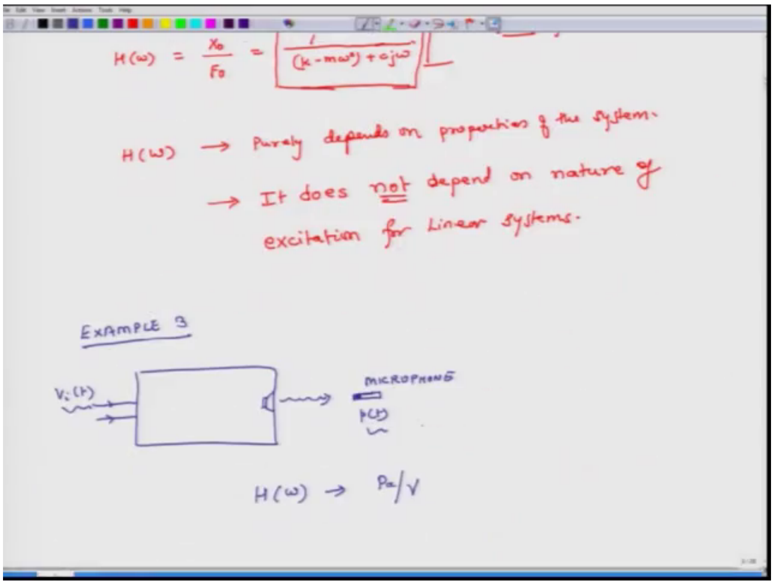

It purely depends on properties of the system it purely depends on properties of the system this is important to understand it does not it does not depend it does not depend on nature of excitation for linear systems, it is important to understand. So, if you know the transfer function does not matter whatever is the excitation you multiply that by transfer function in you get the response of the system another example; example 3. So, you can have a sound system some complicated sound system and you are having a loudspeaker here and I am producing, I am giving some input I am giving some input into this. And of course, there is a power supply which provides energy and because of this the sound system is producing sound and here I have a microphone.

And it is measuring p of t. So, what is happening in this case first thing is that voltage is getting converted into some motion of the loudspeaker and that motion is getting converted into sound waves and the loud sound waves are being measured by microphone is pressure if this is a linear system then the nature of V i and nature of p it will be the same if it is the linear system. So, in this case H of omega what will be the unit it will be Pascals per volte because the response I am measuring in terms of pressure excitation is coming in form of voltage. So, the point is the transfer function could be unit less as we saw in case of a linear voltage amplifier or it could have any other units in this case its meters per Newton.

In this case, it is Pascals per volt, but regardless of whatever the system is; the transfer function is essentially; it purely depends on the properties of the system when the system is linear, it does not depend on the nature of excitation for linear systems.

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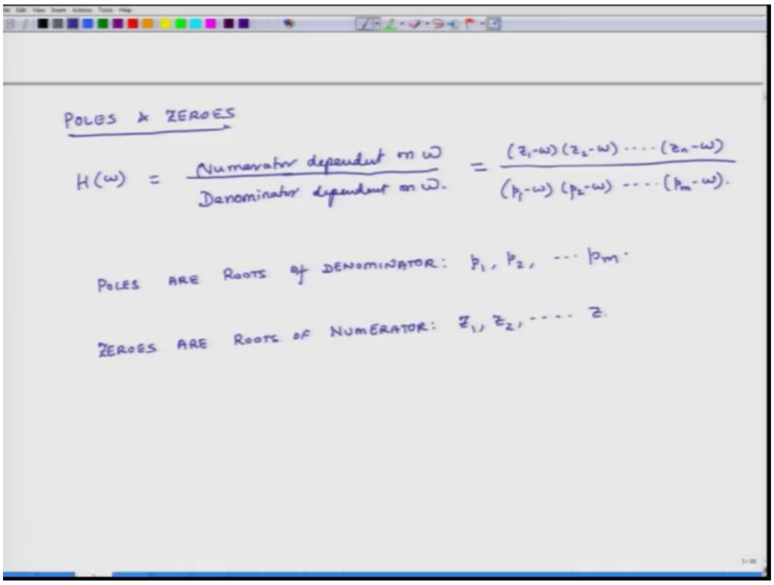

So, associated with transfer functions are terms known as poles and zeroes. So, any transfer functions; now go back to this particular transfer function. So, this will consider as an illustration. So, it has nu numerator number one and in the denominator; it has k minus m omega square plus c j omega, right. So, any transfer function will have a numerator dependent on omega and a denominator dependent on omega. In the case, we saw earlier the numerator was one and the denominator was k minus m omega square plus c j omega and if it is an algebraic expression dependent on omega I can factories.

So, if it is an algebraic expression, I can factories it into lots of linear factors. So, it could be z 1 minus omega z 2 minus omega. So, it is a polynomial of n-th degree, then how many factors it will have n factors. So, z n minus omega and similarly if the denominator is a polynomial of m-th degree, then it will be p 1 minus omega where p 1 p 2 are constants p 2 minus omega pm minus omega. So, poles are roots of denominator are roots of the denominator.

So, in this case we can factories the denominator as p 1 minus omega plus times p 2 minus omega then all these poles are p 1, p 2 till p m and zeros are roots of numerator and these are zee 1 zee 2 and so on. So, forth till zee n. So, if I have any complicated transfer function and if I get my mathematics correct, I can always factorize it in terms of a bunch of linear poles bunch of zeroes and bunch of poles and once I have factorize them I can buy variable. So, I can separate them as sum of individual poles and zeros. So, this is another thing I wanted to talk about.

So, that concludes our discussion on transfer functions and with that we close the discussion for today. And what we plan to do tomorrow onwards is we will actually develop the one dimensional wave equation which will help us understand how sound waves travel in medium and how does pressure move from point a to point b to point c and so on and so forth. So, that is what we plan to do in the remaining part of this week.

And with this, I close the class for today and we look forward to seeing you tomorrow. Bye.