Basics of Noise and Its Measurements
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Lecture - 43
Short Time DFT

Hello. Welcome to Basics of Noise and its Measurement. This is the last week of this particular MOOC course and I hope that you have had a good time and time over which you have acquired some expertise and knowledge about noise, sound, some of the fundamentals associated with acoustics and also, some of the details, as they relate to the technology of noise related measurements.

This week, what we are going to do is, cover a couple of different topics and we will start by continuing the discussion which we had in the last week. In the last week and the week before that, we were discussing about Fourier transforms and their applications and this week, at least in first 2 - 3 lectures, what we are going to cover is some of the techniques related to Fourier transforms and they are known as short time Fourier transforms or short time digital Fourier transforms and also spectrograms. So, that is what we are going to start this week with.

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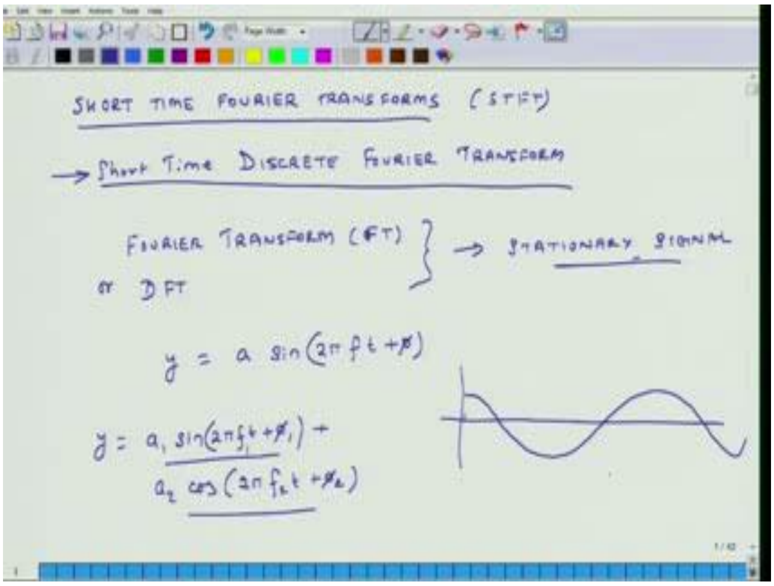

So, this is short time DFT and in this context. So, the topic we are going to cover is Short Time Fourier Transforms. And, because we are going to use discreet form of this Fourier transforms, so this is STFT. So, what we will actually discuss is Short Time Discrete Fourier Transform; that is what we are going to discuss. And, before we start discussing this particular method, just wanted to give you some background as to why there is a need for this kind of a method.

So, the Fourier transform, that is FT or its discrete version – DFT, they do pretty good job if the signal is stationary. So, they do a good job, if the signal is stationary. So, you may wonder, what is a stationary signal? So, I will give you an example of a stationary signal: y equals a sin 2 pi f t plus phi. This is a stationary signal. Why is it stationary? If I plot it, it is a phase; but this signal keeps on going forever and the nature of the signal, the fundamental nature of this signal, it does not change over time. So, it is a stationary signal.

Another example: y is equal to a 1 sin 2 pi f t plus phi 1. So, this is first frequency, plus a 2 cosine 2 pi f 2 t plus phi 2; this is also stationary signal, because both these components of this signal y, they are not changing over a period of time; the nature of this signal. Value of the signal of course, it changes from time to time, but the nature of the signal it does not change from time to time. So, as time changes, the nature of the signal does not change.

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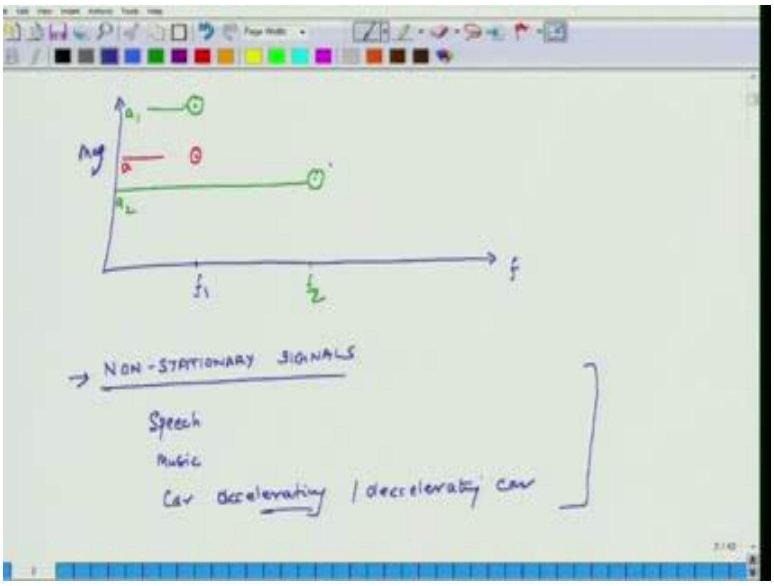

To make it more explicit, so this is my frequency axis and this is my magnitude. If I do a Fourier transform of this signal, what is the amplitude of this or the magnitude? Amplitude is a. So, if this is f 1 and let us say this frequency is f 1 t, then on the Fourier, the Fourier transform of it, it will depict it as one single point and this value will be a. And, the value of a or this graph, is not going to change; maybe if I compute the value of this Fourier transforms of f 1 from 0 to 5 second, I will get this; from 5 to 10 seconds, I will again get this and so on and so forth. So, this figure does not change with time; does not change with time.

The same thing is true of the other signal. So, here, I have a 1 and a 2; these are the 2 Fourier components. So, if I plot them on the same graph, so a 1 may be some other number a 1 and this may be a 2, corresponding to frequencies f 1 and f 2. So, this was a; this is some other number a 2 and this is some other number a 1. So, that is the property of a stationary signal, that its nature it does not change over a period of time. I do not, it does not happen, that may be in the first 5 seconds, I am only having some number of frequencies and after 10 seconds, some new frequencies start coming up.

And, also the magnitudes of the individual frequencies and the phase, they also do not change with time; again if it is Fourier or in the frequency representation of the signal it does not evolve over a period of time. But then, there are non-stationary signals. Here, if I take the FFT of the signal over first 5 seconds, I will get one picture; then, may be from 5 seconds to 10 seconds, I will get another picture; then, from 10 seconds to 15 seconds, I will get a third graph, and all this graphs need not be identical. If they are identical, it will be a stationary signal. So, we are talking about stationary signals, examples of stationary signals, which may occur - see in day to day life.

Speech is a non-stationary signal, right. I am talking. So, when I am talking, every word or every letter, it comes in a different sequence, you know; it comes in a different sequence, i t c o m e s. So, the predictability of this sequence is difficult. So, comes in a different sequence. Music - most of the music, unless it is all repetitive music, and most of the music can be depicted as non-stationary signals.

Another example; if I have a car and it is accelerating it is accelerating. So, what happens? You will hear initial frequencies, may be at of lesser values; but as the car speeds up, zoom. So, the frequencies go up with time, it is only when the car assumes a constant velocity, then the sound signal emitted by the car becomes stationary. Otherwise, as it is accelerating, it is generating a noise signal which is non-stationary in nature.

So, an accelerating car or a decelerating car, it is also non-stationary signal. When a train starts moving, the sound is non-stationary. When a plane lands or takes off, it is non-stationary. Only when its operation becomes steady, then things start behaving, producing stationary signals. So, a lot of signals which we encounter in our practical lives, they may be non-stationary in nature.

And then, the question is that, how do I get their frequency content? So, we have to be careful that, when I ask this question about the frequency content, we have to be cognizant that this frequency content will change over a period of time. And, our regular Fourier transform, it just gives us one single picture. So, it does not tell us, how the thing is evolving over time.

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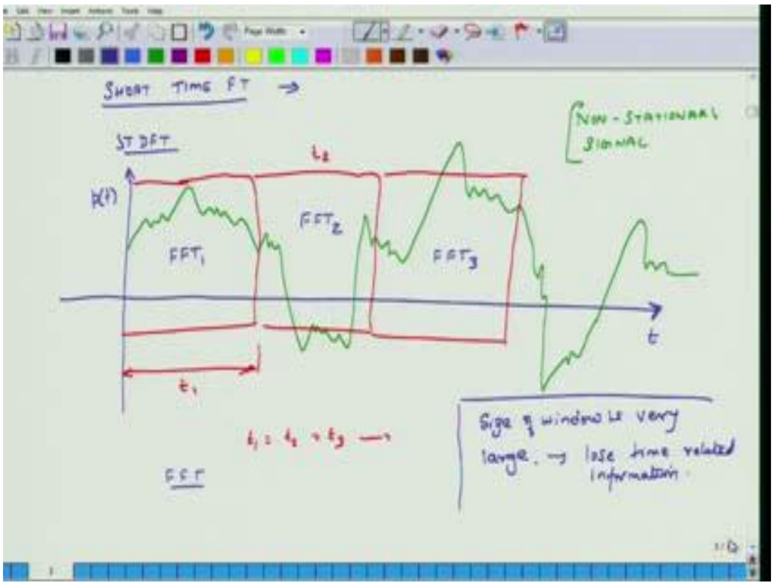

So, to address that problem, we have this tool which is basically a modified version of Fourier transform and it is called Short Time FT. And so, this is in the analog world, but if I want to do the same thing for discrete points, so this is for continuous function. If I want to do it for discrete points, then it is Short Time - STDFT, Short Time Discrete Fourier Transform.

So, what do we do in this short time Fourier transforms? So, on the x axis, I have time; on the y axis I have pressure as a function of time and I want to see how its frequency content, and each frequency amplitude, they change with time. So, suppose my signal is something like this. So, there is no predictability in it, right; there is no predictability, it is a non-stationary signal. This is a non- stationary signal. So, the way I do it is that I will say, what I will do is; So, this is the first level of analysis and then, we will refine our approach, as we move further.

So, our first level is that, I will break this into windows. So I will break my signal into a window of this size. So, let us say this is my t 1 seconds long, and my next window will be again t 1 seconds long, so I break it up in this next window. So, again, I mean, this red box should be large enough, so that it covers this green part here also.

Then, I make my third box, again t 1 seconds long. So, t 1, equals t 2, equals t 3; these boxes are of equal time intervals. So, I break my signal into several boxes and then, for each box or window I will conduct the Fourier transform. So, I conduct my FFT or DFT. So, what I get is, I get the first FFT plot will be for this box; I will call it FFT 1. The second FFT plot will be for this box; I will call it FFT 2. The third FFT plot will be for this box; I will get FFT 3 and so on and so forth.

So, then, what I have done is, I have broken my entire signal into small portions, spanning over a short time; that is why we call it short time digital, discrete Fourier transform; spanning over a short period of time. And then, for that short period time, I do a FFT and I get a plot for amplitude and plot for phase. So, then, I have some information about each time segment. So, that is the basic concept that, you break the entire signal into these small chunks and then, for each chunk you conduct fast Fourier transform, from that you extract information about, so each frequency. So, the point is that; so now, here is the issue.

So, if size of window is very large. Suppose, this t 1 is very long, then it will not help me understand how the signal is evolving over time period, over the time, right; because, I have taken a long duration of time. So, if the size of the window is very large, then I lose time related information.

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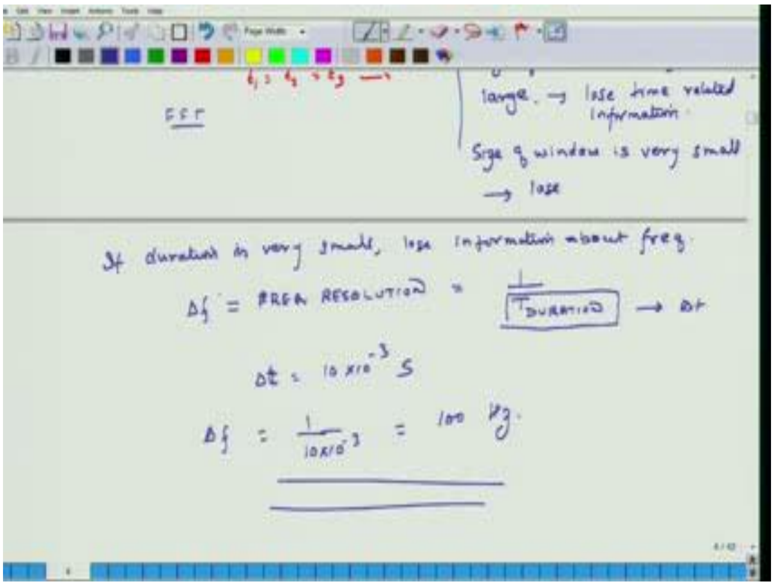

And, what happens if the size is very small? And, if the size of window is very small, then I lose; we had seen earlier that, if I have very small size, which means that is the duration of time over which I am doing the FFT is very small, what does that mean? My frequency resolution, what was frequency resolution? We had discussed it; it is 1 over the total duration of time, right. So, what I will get? My solution will be the frequency resolution will be 1 over that duration; if that duration is very small, then maybe I will get my first point at 100 hertz, second point at 500 hertz, if the duration is very small; you understand.

So, if duration is very small, I lose information about frequency. Why is that? My delta f, that is frequency resolution we had calculated and shown, that it was nothing but 1 over time or duration. So, this number is very small. See, suppose, my window is 10 milliseconds long. Suppose, this is delta t, so suppose delta t is equal to 10 into 10 to the power of minus 3 seconds, then my frequency points will be 1 over 10 into 10 to the power of minus 3. So, that will be 100 hertz. My first point will be at let us say, 100, then 200, then 300. So, I will not have any information about all the intermediate frequencies. So, this is a trick, this is a challenge that how do I manage both these contradictions.

So, if I want to increase my frequency resolution, if I want to increase my frequency resolution, I have to make the size of the window long, but if I make the size of the window long, then I lose the time information. So, this is the whole problem. So, this is like your physics, we have this Heisenberg's uncertainty principle where the particles' momentum and position cannot be precisely determined at the same time, to infinite level of precision.

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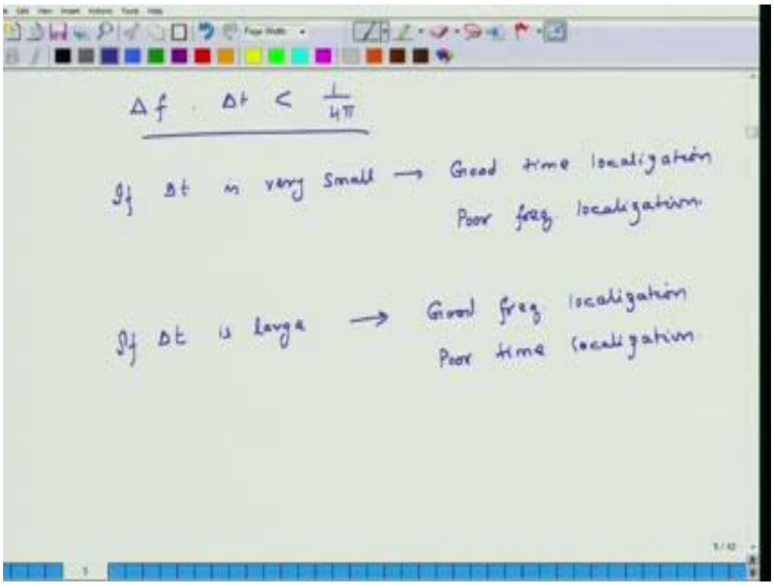

So, something similar we have in this thing. So, there is a rule that delta f times delta t should be less than 1 over, it should be less than 1 over 4 pi. So, I cannot arbitrarily choose any value of delta t and delta f, because if I take extremely small value of time, I lose information of frequency; if I choose extremely large value of time, then I lose information on time.

So, there is some terminology here. So and I just want you to make conversant with that terminology. So, if delta t is very small, then people say that we get good time localization because we get good information about how things are changing with respect to time. But we get poor frequency localization; we get poor frequency localization.

On the converse, if delta t is large, we get good frequency localization, but poor time localization. So, this is a problem and when we do this short time discrete Fourier transform, we should be aware of this problem and we should work within these boundaries; I mean, we will still get more information; we will get more information from, if we just did FFT of the whole signal. But, we should be aware that whatever we are seeing will not be the exact reality, because there is a limit, mathematical limit which is being imposed, and because of that limit we are getting only limited amount of information.

The second thing, the other concept I wanted to discuss here was about sliding, window sliding. Actually, that improves some of the steps which we are discussing. So, in this picture, what I had shown was that, you break the entire - what do you say, entire signal into chunks. So, you have chunk 1, then you have chunk 2; and this chunk 1 and chunk 2 are adjacent to each other; there is no overlap of these 2 chunks. So, when you get FFT 1 and when you get FFT 2, the signal for FFT 1 time signal may be significantly different than signal for, time signal for FFT 2. So, you may see sudden changes in frequency spectra of signal 1 and signal 2, because signals have changed significantly. But in reality that may not be happening, things may be evolving slowly or smoothly.

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So, to address that particular problem, what you do is - you do this sliding of windows.

So, what do you do there? So, what you do here is, suppose this is your signal. What you do is you take the first window, this is window 1. Then, the second window is not just adjacent to the first window; rather, your second window is basically, your first window has moved out a little bit further. So, then you take, you get a second window. Then, you take a third window and that moves out a little further, rightwards. So, this is your, green is your, this purple is your third window; green is your second window and so on and so forth.

And, in this process, you get relatively smoother transitions from one window to other window. You get relatively smoother transitions in this way. So, that is this whole version of sliding. So, these are some of the important concepts in context of Short Time Digital Fourier Transform.

I would like to close this lecture at this point of time and in the next lecture, we will actually see some of the results of this Short Time Fourier Transform, and if we have time we will also introduce another thing called a spectrogram. So, that is all I wanted to discuss today.

Thank you very much and we will meet you tomorrow.