Basics of Noise and Its Measurements
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Lecture – 34
Discrete Fourier Transform Duration – Sampling Frequency – Resolution

Hello, welcome to Basics of Noise and its Measurements. This week we have been discussing Discrete Fourier Transform and what we will do today is continue that discussion further. Till so far what we have learnt is, how to use the DFT method to compute frequency spectrum of a discrete time signal. What we will do today is, a couple of things, but first thing we will discuss is that what happens to our solutions or the frequency representation, if the duration of the time changes.

So, how does the goodness of my results get influenced by the duration of the time signal? Second is how the sampling frequency affect the accuracy, and the third things what is the resolution of my solution or the frequency representation and how does it get influenced by different parameters.

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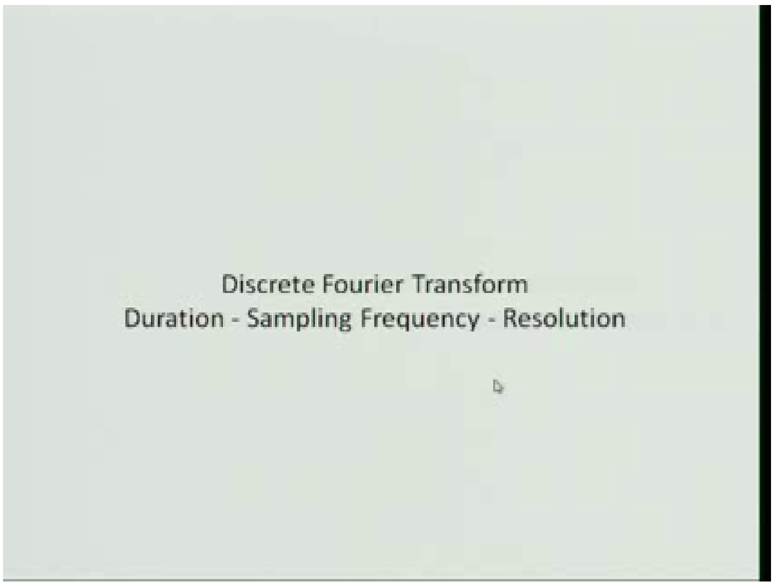

So, that is what we are going to discuss today. So, duration, the influence of duration - sampling frequency and how does resolution get influence by various parameters which go into discrete fourier transform of a time series signal.

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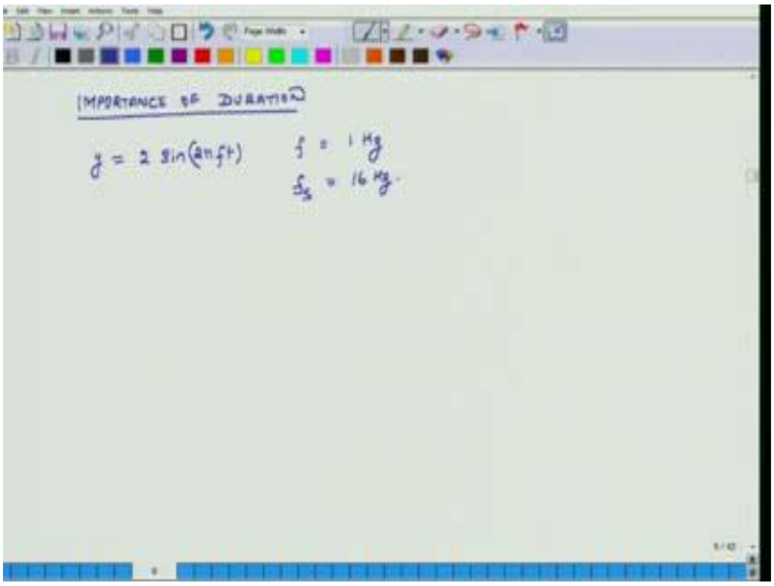

So, the first parameter we will look at is why importance of duration. So, what we will do is. We will look at we first generate a time series signal and we will generate it for different durations and then what we have done is we have taken its a fourier transform, discrete fourier transform using some industry standard tools and we will look at those solution and see what we make out of that. So, the function we will use is distinct y equals 2 of sin 2 pi ft and f we set it at, 1 hertz and the sampling frequency was not changing and we were doing 16s samples each seconds sampling frequency was 16 hertz.

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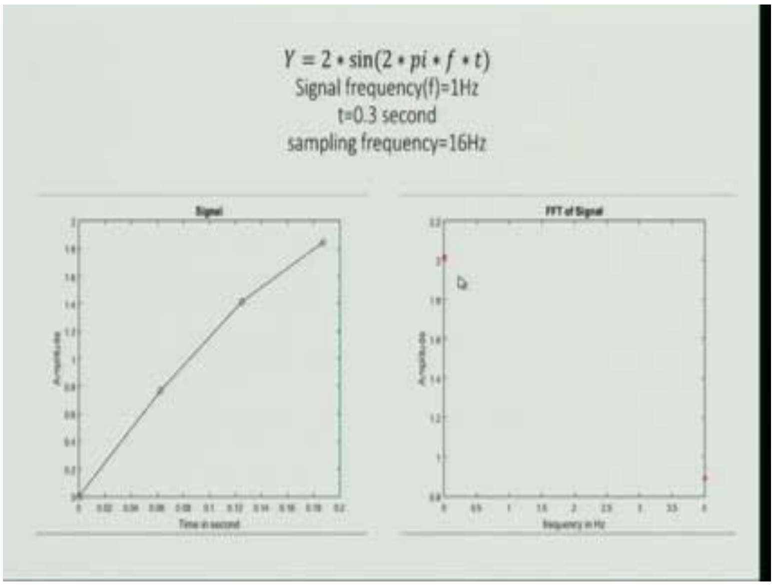

Let us look at some of the results. So, this is the signal this is the time signal on the left and this is the function 2 sin 2 pi ft, f is 1 hertz and I ran this for its says 0.3 second, but I actually ran it for a little work 0.18 seconds. So, I had 1, 2, 3, 4 samples. So, this should not be 0.3 seconds, it is different it was about 0.19 seconds and I got 4 points. So, I have 4 points of this signal, 1 hertz signal and then I did an fft and what I see is that, I can at 0 hertz I get peak and the amplitude is slightly over 2 and then I get another peak at another non 0 value at 4 hertz.

All other values I got as 0. So, I got these 2 values 0 and 4 and, if you look at the accuracy of this result it is totally wrong. So, what that tells means see my sampling frequency was 16 hertz it was fairly high. So, I should be able to capture 1 hertz tone. So, there is no problem with sampling frequency. So, what are is the problem may be the duration, maybe I put the duration too short because those are the only 2 parameters. If I have this is my raw data and if I am not getting correct answer may be the signal is too short.

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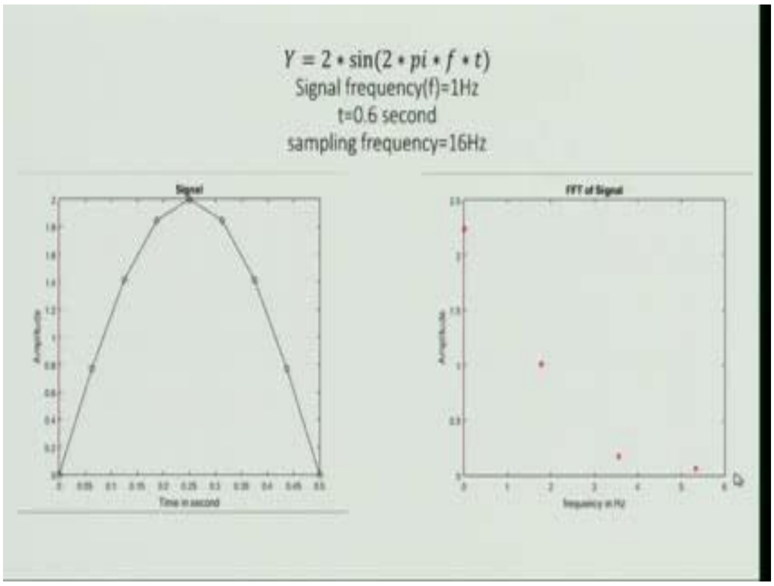

So, then I said, what I do the increase the signal length and again this says duration is 0.6 seconds. It should have been 0.5 seconds and over 0.5 seconds I have this type of a shape. I have this type of a shape and then I again do the fft of this or a dft of this signal and what I get is. So, here I had 2 points now I have 1, 2, 3, 4 points, I have a peak at 0 hertz, another peak at 1.5 hertz or maybe 1.7 hertz, another peak at maybe 3.6 hertz another peak at a little over 5 hertz. The real solution should have been all these values should be actually 0 and I should have had only 1 peak at 1 hertz and its values should have been 2. So, what this means is, that this signal is still too short.

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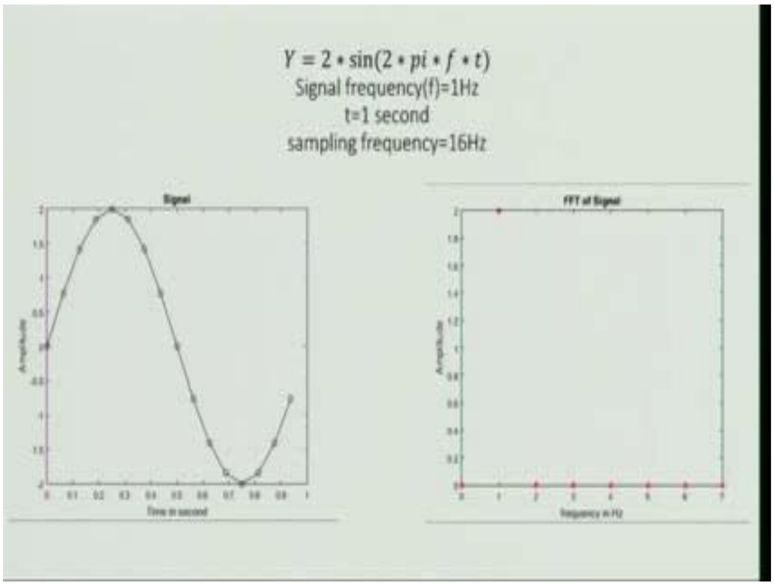

So, then I increase the signal to little over slightly less than 1 second, and I have 16 points and if you take this signal and you cut and paste it again it is repeating signal. So, I have 16 points and sampling frequency as still as a 16 hertz. So, I have not changed this sampling frequency over the entire analysis and what I get is a perfect result. All values are 0 except at 1 hertz and the value of at 1 hertz is indeed 2 which is the actual case. So, first thing what this tells means that if I have a complete sinusoidal cycle. And if I take is 50, I will get the right results, but in real signals we do not know whether we have complete affect sinusoidal cycle or not, because we do not what frequencies are. So, then what happens? So, we have to have fairly large duration.

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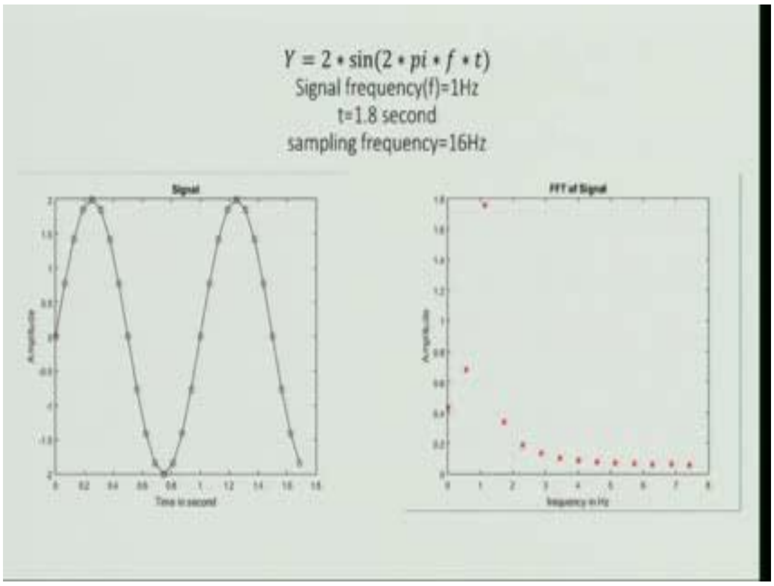

So, this is t is again I deliberately put it as 1.7 again, I do not know why these numbers are incorrect. So, I deliberately put it at 1 second. So, that it is not a complete cycle and if I it is not a complete cycle, but its 1 hertz is 1 complete cycle and this is maybe 1.7 cycles long and when you do in a 50 you get some what moderately or semi moderately good results. What you see indeed is that there is a peak at maybe not 1 exactly, but 1.1 hertz and all other values here in excess 4 hertz are very small, but you still have a appreciable values at 0 and may be 0.7 hertz and may be 1.7 hertz and these. These may be very small, but I can certainly not ignore these 4, 5 values.

But, this values is pretty high 1.8 it is getting close to 2, but it is still not there, see what happen? What is happening is that the energy at 2 hertz it is getting distributed around 2 hertz energy at 1 hertz, which is reflected by its amplitude 2 is getting distributed around 1 hertz. So, that is what is happening. So, what that tells means that 1 point this not sec 8, but 1.7 second, which is still not probably the right duration.

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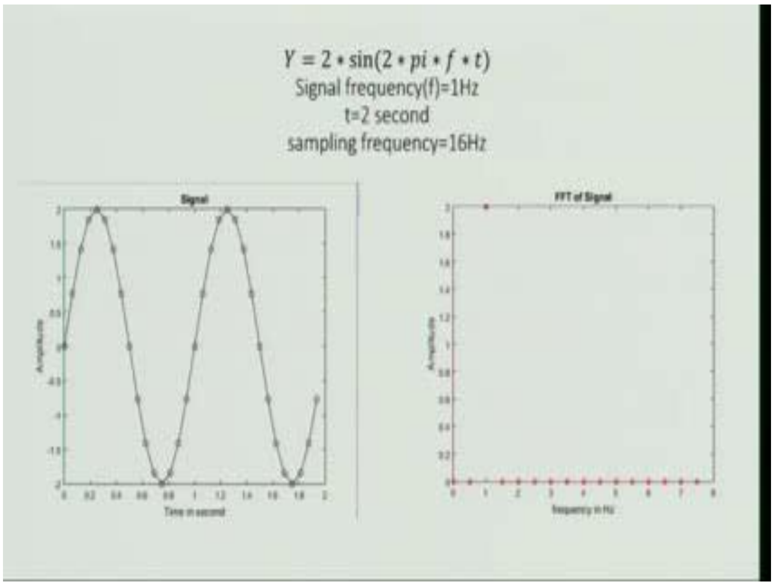

So, again I make it 2 seconds, 2 is I have exactly 32. 16 hertz is the sampling frequency. If you count the number of points its 32 and when you do this you get a perfect result. Again, because your cycle is if I just again take this thing and paste it here I have a complete cycle. Once again, its rain forces if I have a complete repeating pattern then I will get good results, but we do not know, whether are repeating pattern.

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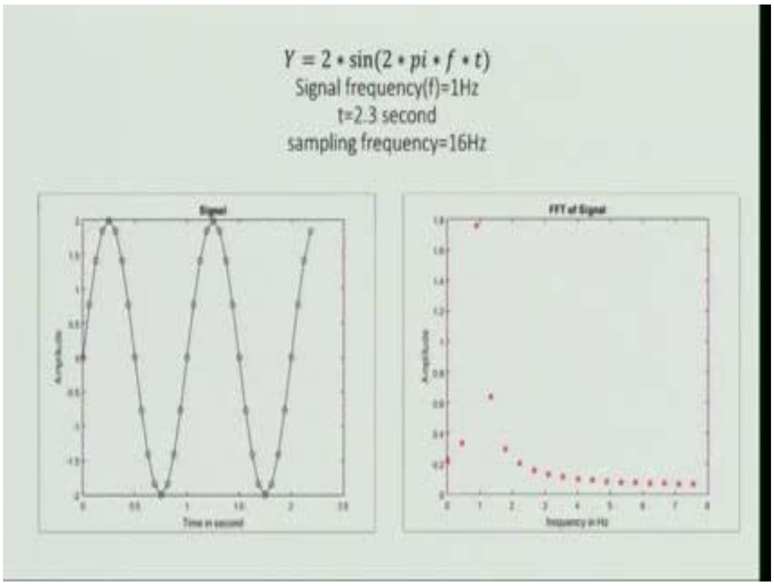

So, I have to increase the; this is 2.3, and you see that compare to that 1.7 is not significantly different.

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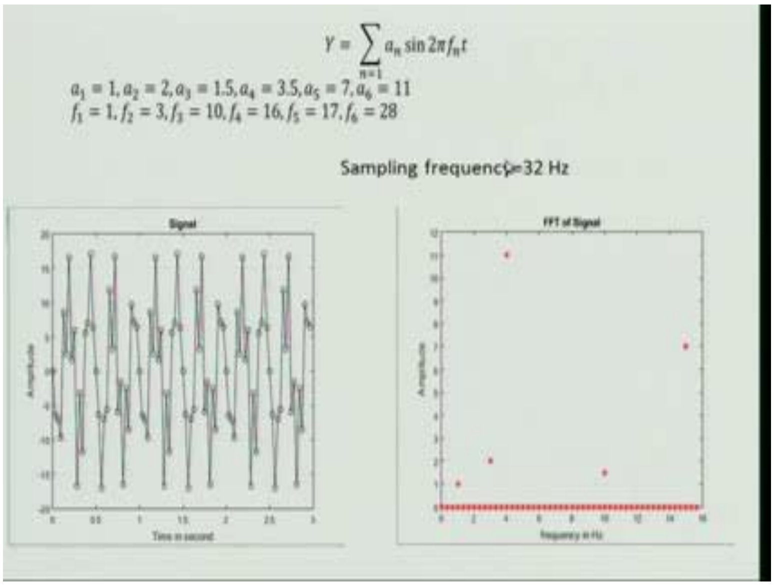

But, it may have slightly improved 4 will again as accepted, it will be good results. So, what this shows is that, we have to fairly large may be what we should have done in this is that we could have taken may be 5 or much longer duration. Maybe 5.5 or 6.i'th seconds and what you would have seen is, that when the duration becomes long then this values gets better and better. So, that is the first moral of the story, that your duration should be reasonably long.

It is also related to another factor that the number of points which you are sampling it should be ideally the power of 2 because that also helps you get good results. So, maybe we will do that numerical experiment and show the results in the next class, but that is another thing. So, this is 1, the second experiment or the second study which we are going to talk about is, about the importance of sampling frequencies.

So, in the first case it was importance of duration, the second is about importance of sampling frequency. So, what we have done is constructed a signal y equals an sin 2 pi fnt and n is. So, basically we have generated 6 sin waves a 1 sin 2 pi f 1 t, a 2 sin 2 pi f 2 t and so on and so forth and these 6 sin waves are such that the amplitude of the first sin wave is 1, second is 2, third is 1.45, forth is 3.5 and so on and so forth.

The frequencies is of first wave is 1, 3, 10, 16, 17 and 28. So, we have picked up some numbers and if you look at our results, this is our wave form, and the overall duration of the signal is slightly less than 3 seconds. So, it is not exactly sitting on this vertical line slightly less then, but you have a lot of point. The number of points it is 32 times 3 because this duration is 3 seconds. So, it will be what, 96 points. When I look at the fa this dft of this, my values are somewhat at 1 hertz. So, this is 1 my amplitude should be 1 and it is indeed 1. At 3 hertz it should be 2 and it is indeed 2 then the next frequency, I do not know why I am getting this at 4 hertz. So, this is some of, you know error.

So, this is inaccuracy which is coming up in dft. Then I have another which is at ten hertz I should have 1.5. So, ten it is indeed 1.5 and what else. So, f 3 where and f 4 was 16, and I get some peak at 16, it should be 3.5, but I get totally off it is giving me 7 and because sampling frequency is 32 it cannot give any values more than 16 hertz, because of that Nyquist limit. So, I do not get results beyond 16 hertz. So, what does that mean? That maybe I can still keep the duration same 3 second, but I have to increase this sampling frequency.

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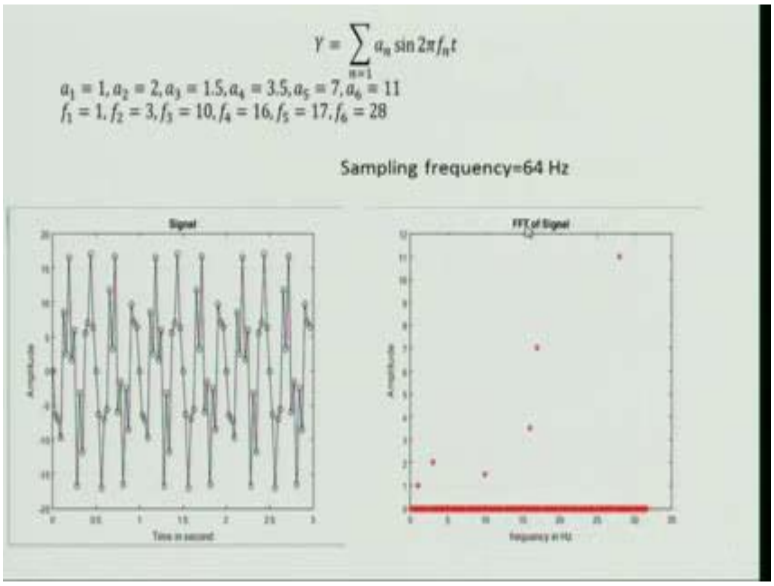

Let us see some my sampling frequency what I did was I made it 64 hertz. So, I doubled it. So, once I double it theoretically, I will get all the frequencies up to 32 hertz and what we see is, first frequencies is at 1 amplitude, 1 which is good. Second frequency is at 3 and amplitude is 2 which is good. Third frequency is at ten hertz amplitude is 1.5 which is good, forth frequency is at 16 hertz amplitude is 3.5 again which is good, fifth frequency is 17 hertz and amplitude is 7 which is good, but then you look at the next frequency, and you get something around 28 or 27 and its amplitude is also 11. So, you are getting good results at this.

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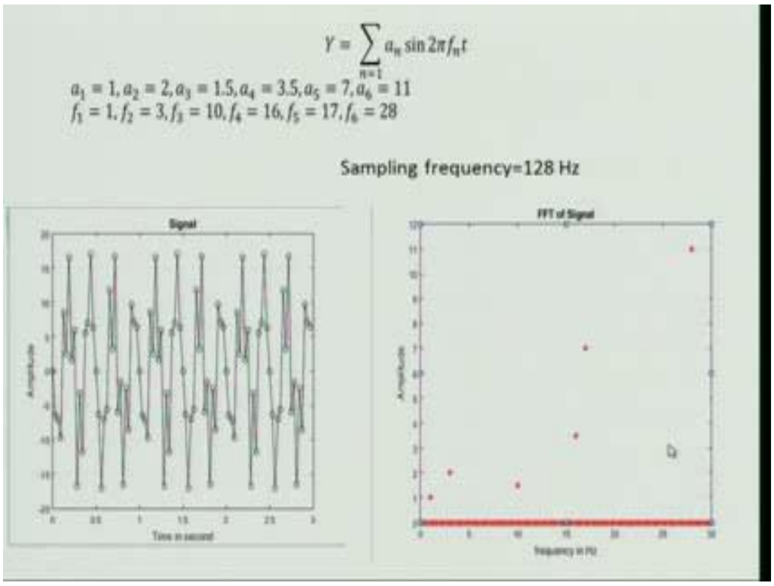

Then you do at 1; 28 and you get probably even more accurate results. So, the point what I am trying to make is that if you have your sampling frequency just at fs that does not that will give you results still fs over 2, but that does not guarantee that the results at in the neighbourhood of fs over 2 will be this size an exact you know. So, even though theoretically you will get some value at fs over 2, but its value may not be good enough.

So, you have to increase, because this is what we see you know here, fs was 32 and I am getting some spurious result as 16 hertz at 16 hertz. I am not getting anything and some spurious result I am getting, which has amplitude, the 7. So, ideally I should have got in at 16 hertz the amplitude should have been 3.5, but I am not getting that. So, what that tells means that even. So, even though I am getting data, see I am getting data till 16 hertz, but that does not mean that the data in the neighbourhood of fs over 2 will be accurate enough, that is the moral of the story. So, couple of important points about these two things, which we discussed that if my duration.

I have to have my duration reasonably long, and if the duration is not reasonably long, and if I accept some frequencies some low frequencies to be present in my signal, which means, that their time period is large and if the duration is not large. Then I may not get accurate results because I need to have at least 1 or 2 cycles of that low frequency information to be completed and if my duration is not large enough time duration then the results at low frequency and may not be good, this is the first thing. Second thing is, if we do not want low frequency compounds, they may be that may be we are interested only in frequency let us say from 30 hertz to 5000 hertz and we are not interested in 30 any information below 30 hertz it could be a situation.

Let us say in that case and we also do not want to have very long durations of signals because it takes more processing time. So, in that case what we should do is we should use these filters, high past filters to eliminate the signal from all low frequencies information and then we subjected to dft. So, that low frequency information is gone, and that will not corrupt our results. It will not create confusion for our results and then we just use information to; then we use the dft. The second point is for the high frequency end to ensure that we get information of high frequency end, we have to first definitely ensure that our Nyquist criteria satisfy, but then we should not limit itself to Nyquist criteria, but we should go our sampling frequency should be may be 3, 4 times more than the Nyquist criteria.

Then we will get reasonable information about high frequency content. Similarly, also if in the high frequency range, if in the high frequency range we are not interested in say some super high frequency information. Then what we can do is that we can acquire the signal and eliminate the high frequency content using low past filters and then do the fft of that. So, that is another interpretation of it.

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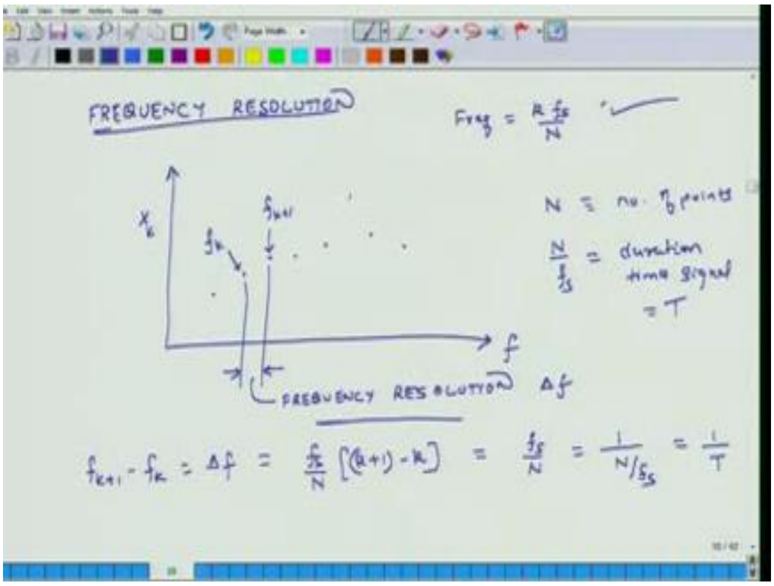

The last point I wanted to discuss was about Frequency Resolution. So, whenever you do a dft, on the x axis you plot frequency on the y axis you plot xk, and then you have different points. The spacing between these 2 points is called Frequency Resolution, or delta f, let us compute this. So, we know that frequency is equal to k times fs over n right. So, if this frequency is f 1 this frequency is f 2. Then f 2 minus f one or I can make it more general fk.

So, this is fk and this is fk plus 1 then fk plus 1 minus fk is the resolution delta f fs over n k plus 1 minus k I am just using this relation for this thing. So, this is nothing, but fs over n or I can express this as 1 over n over fs. Now, n is the total numbers of points which we are considering for our analysis, fs is the sampling frequency. So, n over fs is nothing, but duration of time signal, is equal to t. So, this is equal to 1 over t. So, what does that mean? That if I acquire data for 1 second my frequency resolution will be 1 hertz, if I acquire data for 2 seconds my frequency resolution will be 1 over 2 half hertz, if I acquire resolution for 3 seconds my frequency resolution is going to be 1 third hertz and so on and so forth. If I acquire data for half a second my frequency resolution will be 2 hertz. So, if I narrow the duration of my data acquisition, the frequency intervals and the frequency domain will become larger.

If I acquire data for more period of time, my frequency interval becomes smaller. So, that actually also tells us that at low frequency, we want finer frequency resolution because we have to go from 1 hertz, 2 hertz. Between 1 and 1.3 hertz, there is a 30 percent difference bit. If the same difference is at 1000 hertz, 1000 and 1000.3 that difference is negligible.

So, if I need more accuracy at low frequencies I have to increase the duration of the signal. So, this is again something we discussed earlier, but it also flows from this information. So, this is the last point which I wanted to cover into this lecture and I once thank you for listening patiently to all these discussion and we will continue this discussion in the next class as well.

Thank you.