Basics of Noise and Its Measurements

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Lecture - 31
Discrete Fourier Transform

Hello, welcome to Basics of Noise and its Measurements. Last week, we were discussing the concept of Fourier Transform. We started with fourier series expansion of well defined functions, then we moved to fourier integrals and then we closed the week by discussing and introducing fourier transforms which are representations of a time series function in frequency domain. What we will do today, at least for the first part of this week is continue the discussion, but one significant difference will be that in the last week when we were discussing fourier transforms these were for continuous functions.

But in reality or in very few think about practical considerations most of the data which we acquire from our data acquisition systems it is not in form of a continuous function, the other word we have is a set of points which are discrete in nature. We have discrete sets of points as function of time, so we have a point that t is equal to 0 we have another point t is equal to t 1 and there is nothing in between then another point t is equal to t 2 and so on and so forth.

This is a discrete set of data. If we have this discrete set of data as a function of time, then how do we map this on to frequency domain? So that is going to be the focus of our discussions today, pretty much. What we will use in that context is something known as DFT or Discrete Fourier Transform. Essentially, we have the same theory which we used for fourier transforms which we discussed last week, but we are mapping that information into discrete space and these transforms are known as DFT or Discrete Fourier Transform, suppose I have a bunch of points.

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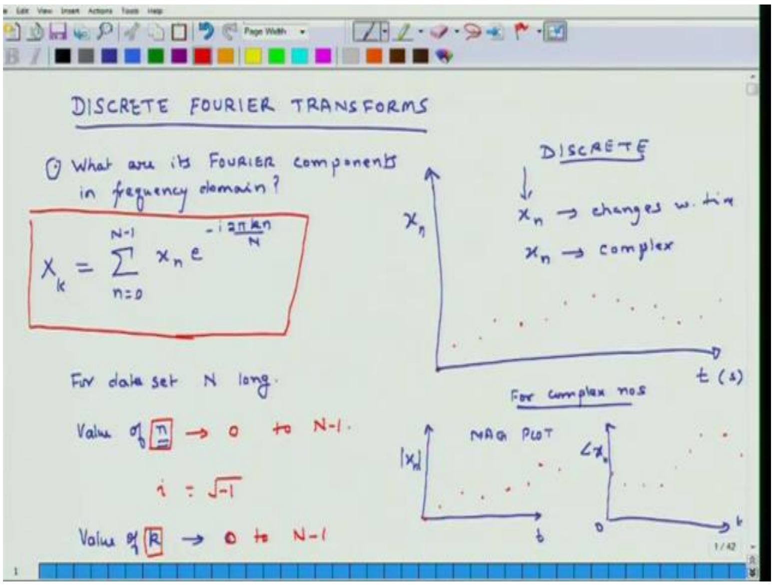

We will discuss Discrete Fourier Transforms. We will not prove whatever relations I am going to talk about, but we will directly state the results and then do a couple of examples to understand this whole concept. Suppose, I have a data acquisition system and its acquiring some data and I am going to plot this data, on the time axis we have time in seconds and on the y axis I plot values of this data and I call it x n.

I get one point which may be in the beginning. I am going to plot all these points in red, so this is in the beginning at t is equal to 0, then at t is equal to 1 second, t is equal to 2 seconds, t is equal to 3 seconds, 4 seconds, 5 seconds. This is x n, and this changes with time and x n is discrete. There is no continuous function between x 0 and x 1 and x 2, so these are discrete points. Then the question is, if I have this type of a discrete function then, what are its fourier components in frequency domain? This is the question, this is one.

Second thing we have to note, x n is a discrete function and also x n can be complex. If it was a real number then all I have to do is plot x n on y axis and on the time axis I plot t and I make this plot. If it was a complex number, how will I plot it? I will actually have two graphs; one graph will be for the magnitude of each of these points, and the other will tell us its phase. In that case, time and then this will be modulus of x n and then I will have a bunch of points. These are the modulus or the magnitude of this function. This is magnitude plot. Then I can either plot its imaginary component or phase, so here is time and this is phase of x n, and the phase could be, so let us say this is 0 and let us say it is in radians and the phase could be anything I mean. So, this is for complex numbers.

So, discrete fourier transform is able to handle complex numbers. The relationship for converting this data into frequency domain is, so x n, where x is lowercase number, lowercase letter or small letter and that is the time data. The frequency data is, capital X or uppercase X k and that equals summation from n equals 0 to N equals capital N minus 1 x n e to the power of minus i 2 pi k n over n, couple of things, remember. This relation is good if there are n points. This equation is good for data set N long, there will be n points. But the value of n lower n, it does not start from 1 it starts from 0. So, it starts from 0 and it goes on to N minus 1.

Total number of points is capital N, but the value of lowercase n, is little n it starts from 0 this is extremely important remember, it starts from 0 and goes on till N minus 1. We end up having same number of points that this capital. And here, i equals square root of minus 1. Value of k, it is also from 0 to N minus 1. If there are capital N points, I can calculate n values of x; x 0, x 1, x 2, x 3, dot dot dot till x N minus 1 where N is uppercase. I calculate the value of x 0, in this equation I add this whole equation over in this n and here in the exponent term I put k is equal to 0. If I have to calculate equal x 1, then it is the same relation I still add up over the whole range n is equal to 0 to N minus 1, but the only thing which I do different is I put k is equal to 1 and so and so forth. I go on till I get n values of x and the index of x starts from 0 and ends to capital N minus 1.

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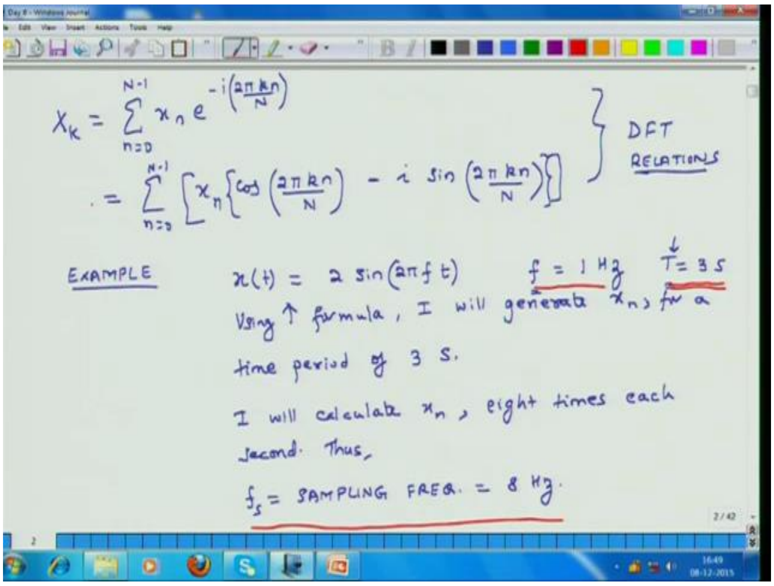

Now, I am going to express this relation in another form. We have seen that X k is x n exponent minus i 2 pi k n over n. This I can also write it as, because e to the power of i times something I can express it is cosine as well as a sin and this entire term which is in brackets is a real number, it is not complex, 2 pi k n divided by capital N so the entire entity is real. So the exponent of e is imaginary, so I can have a cosine and a sin term. This is equal to x n cosine 2 pi k n over N minus i sin 2 pi k n over N. This is another form, you can actually use this. These relations are known as DFT relations, and they help us compute the transform a time series data into frequency domain.

I have still not said that X k corresponds to what frequency, I have not defined that. And I have also not said that X k is identical to the magnitude of the frequency at a particular frequency component, but X k is its representation. I will later change this X k into an actual magnitude component, but that we will see later and also a phase component so that we will see later. Let us do an example.

The example is that let us say, my x is a function of time such that it is equal to 2 sin 2 pi f t. Where f equals, I want to make this problem simple, so I will say it is 1 hertz. And using this formula let us say I will generate points. So, I will get x n for different values of time and I will get discrete points, and then I will see how I convert that discrete point into frequency domain. Then I can keep on generating these points forever, I can generate it for 1 second, 2 second, 3 second, 10 second, 20 second. So, I will generate x n for a time period of 3 seconds, let us say. I call this t, so this is not the period of the signal this is the total duration of time and that is 3 seconds. This is 1 parameter f, this is the duration of time, capital T.

Then the third question which I have to addresses is how frequently I will find the points. I can calculate the value of this function every 0.1 second; I can calculate it every 0.2 second. What I will do is, I will calculate it 8 times each second or this is called f s. Thus, f s is equal to sampling frequency and that equals 8 hertz. This is not the frequency of the signal, it is the number of times I am calculating this value each second. So, I have 3 parameters here, one is the frequency of the signal, second one is the duration over which I am calculating the value of x n, and then the third parameter is sampling frequency which corresponds to number of times I compute the value of x n each second. So, that is what I am going to do. And what I have actually already done this and this is what I did, I actually used some equation computed then I have plotted the value of x t.

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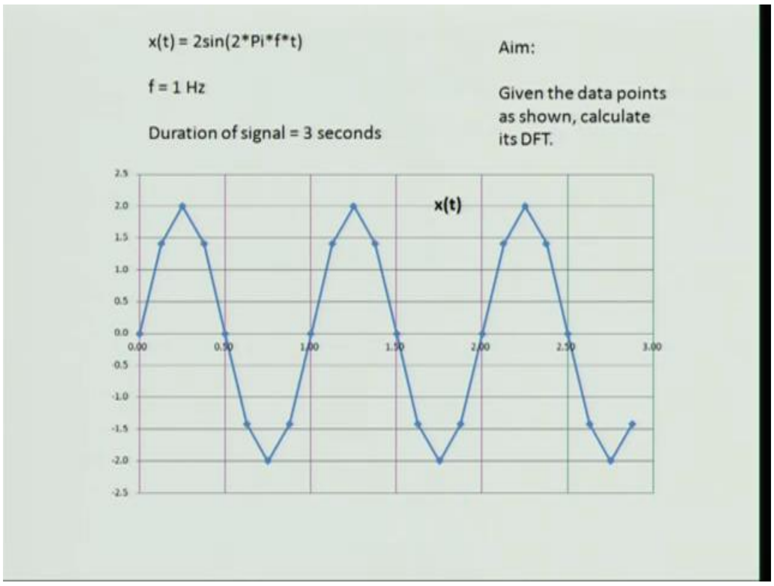

This is the value of x t. So, x t was 2 sin times 2 pi f t. The frequency of this signal I specified at 1 hertz, duration of signal was 3 seconds. X axis is time, so I have computed for 3 seconds. Then I calculated this value 8 times each second. This is 1 second, so one point two time three, so how many steps we have 1, 2, 3, 4, 5, 6, 7, 8, I have broken each second into 8 units. These are the discrete points, each of these diamonds is a discrete point and I have connected them with a straight line.

So my time domain data is not the line in between because this is a discrete function. The time domain data is actually just the point. Now, I have generated these points, and then what I will do is that I will use that relation to calculate its fourier transform that is in frequency space. If we do our mathematics correctly, then from these data, these points only we should be able to infer that the amplitude of the signal is 2 and the frequency of the signal is 1 hertz. So, that is what we should be able to do.

We will do that in the next class and thanks for listening and we will see you tomorrow. Bye.