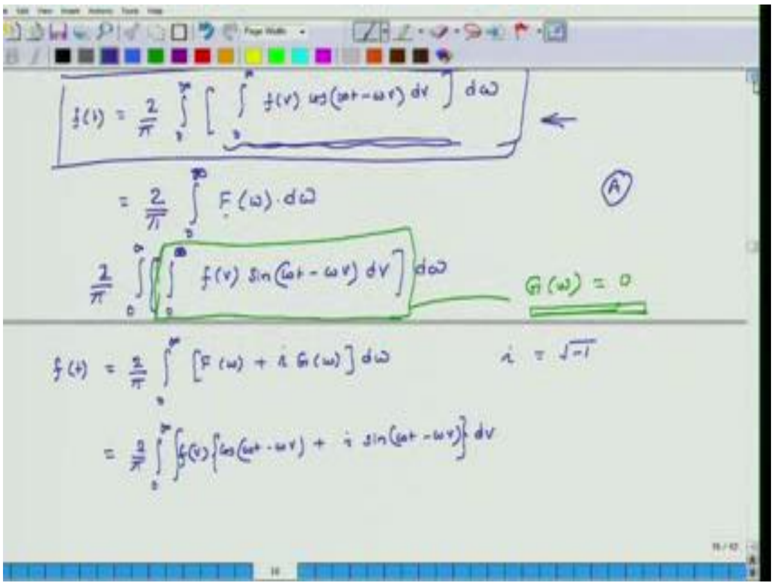
Basics of Noise and Its Measurements
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian institute of Technology, Kanpur

Lecture – 30
Fourier Transform

Hello again, welcome to Basics of Noise and its Measurements. We have been discussing starting last class this concept of Fourier Transform, and we have not yet completed that treatment. So, what we will do in this class is complete the discussion on fourier transform, develop some explicit relations for fourier transform and then see what they mean from our physical standpoint, what is the physical interpretation of these transforms. So, that is what we planned to do in this particular lecture and let us see how it goes.

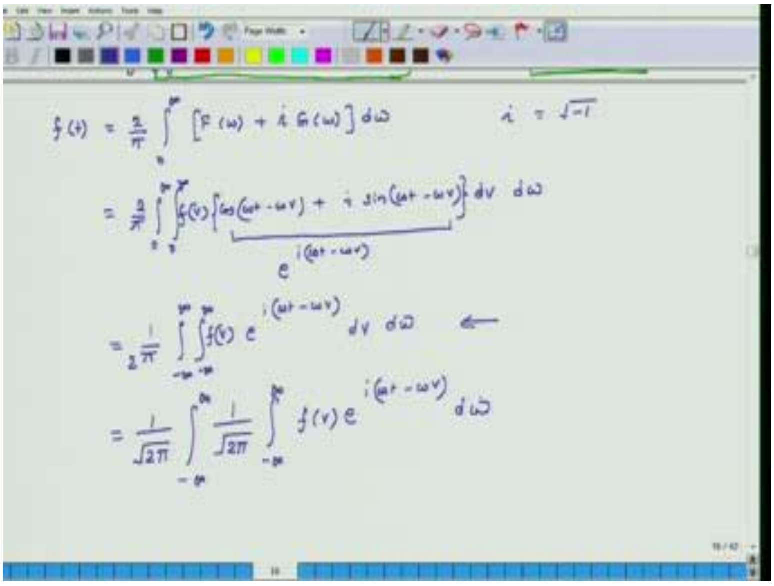
(Refer Slide Time: 00:46)



In the last class, we had developed this relation that f of t. So, f of t is a time series function, which this particular function does not have to have a finite time period and that can be expressed in this somewhat longish format, and where it is equal to 2 over pi integral 0 to infinity integral of f v cosine omega t minus omega v dv time dw and this entire integral thing I am calling it as f of omega v omega. Let us call this equation A. So, I will guide the expression and then I will define the G w. This expression is 2 over pi 0 to infinity 0 to infinity, and here I am having f of v and instead of cosine I am having a sin term omega t minus omega v dv d omega. What I am going to define is, that this entire thing is G w. So, G w definition is pretty much same as that of f w, but instead of cosine function I have a sin function. And here, I am going to straight away say that this is equal to 0, because this is an odd function and if you to our earlier discussion you will find that if you do the requisite math you will find that this thing is 0.

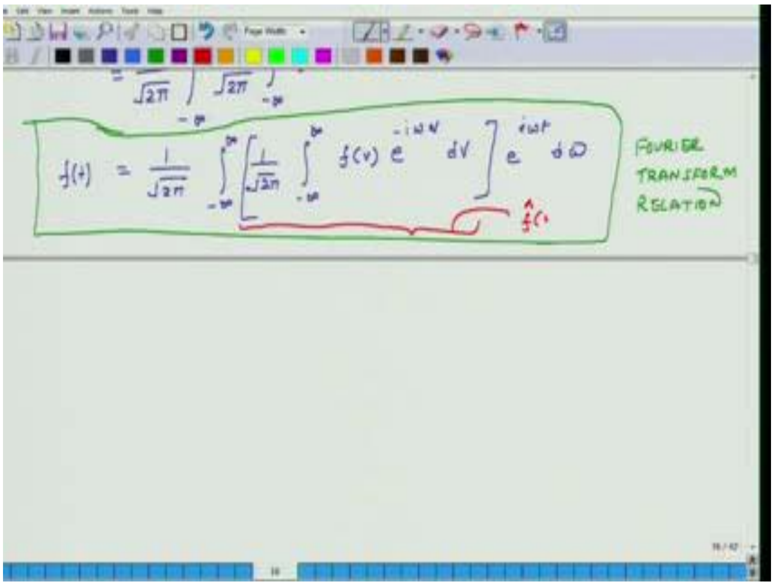
So, if this entire thing is 0, then if I add this to f of t I still get f of t. My expression for f of t equals 2 over pi 0 to infinity F of w plus I will write G of w and I am going to integrate it with respect of, actually not w I will call it omega because a lot times we use for angular frequency. This expression is valid. Now what I will do is, if I multiply by it some constant where i equals square root of minus 1 then still this expression is valid. So this is equal to 2 over pi 0 to infinity f of v cosine omega t minus omega v plus i sin omega t minus omega v, this entire thing has to be multiplied by dv and it has to be integrated, if you go back so this has to be integrated with dv and then there is d omega.

(Refer Slide Time: 05:19)



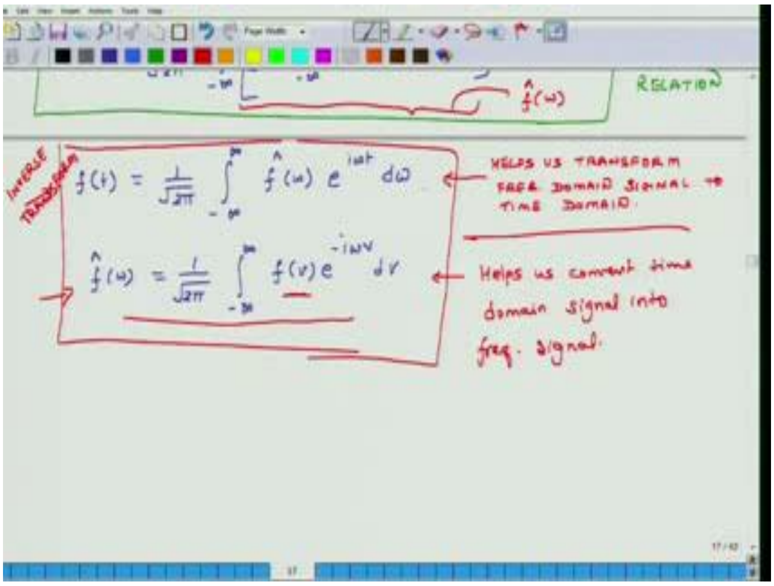
This expression I can write it as, e to the power of i omega t minus omega v. What I get is, 2 over pi 0 to infinity f of v e to the power of i omega t minus omega v dv d omega, and this also be integrated. What I can do is I can change these limits. So I can make this 1 and here I have minus infinity and I still get the same thing. Now, we are pretty close to the fourier transform, and I can change the limit again so these limits were 0 to infinity. If I make this minus infinity to infinity, then I have to make it 1 over 2 pi. Now, what I do is, I again further transform this relation and I make this 1 over square root of 2 pi integral minus infinity to infinity, and then again inside the integral I multiplied by square root 2 pi minus infinity to infinity f of v e i omega t minus omega v d omega.

(Refer Slide Time: 07:00)



I further modify it minus infinity to infinity 1 over square root of 2 pi, minus infinity to infinity f of v, and what I do at this stage is that I here separate omega t and omega v. This is i omega, so I had missed here this was dv and d omega. So, e omega t and e omega v, if v dv, and then e omega t d omega. So, this is the fourier transform. We will actually explain what all this is about fourier transform relation, and I can call this entire expression which in parenthesis, as f hat of omega. So, f hat is a different function than f.

(Refer Slide Time: 09:00)



I can express f t as 1 over square root of 2 pi integral minus infinity to infinity f of omega d omega hat. Where, e is to the power i omega t d omega. Where, f hat is 1 over square root of 2 pi minus infinity to infinity. So what does this first relation tell us? Actually, we will first look at the second relation. That if I have a function of time, v could be time. If I have a time function of time I can convert it into the frequency domain using this relation for f hat. This helps us convert time domain signal into frequency domain signal. The first relation, if I have a frequency domain signal, then I can back convert it into the time domain signals so that is why, it known as inverse transform. This helps us convert transform frequency domain signal to time domain. So this one is also known as inverse transform and this guy is actually a fourier transform or direct fourier transform.

If you have time series signal and again this is not the digital signal so we are not having discrete points, if I have a continuous function of time and that function is not periodic and I still fond to figure out what are its frequency components, then I use this second formula and using this I can calculate its frequency components. Suppose, I have full frequency components and I want to make a time signal out of it then I use this first formula which is the inverse transform and using that I can calculate or generate a time signal out of it. From time, I can go to frequency from frequency I can go to time and I can do this back and forth. So this is what this fourier transform helps us do.

The second thing I wanted to explain you is that, there is this exponent to the e to the power of i omega v and e to the power of i omega t these are here. What that tells us is that when you do this calculation you will have complex part, imaginary part and real part. That information will help you compute something more than just these numbers. For instance, when you are in the frequency domain and if you have a real part and imaginary part you should be able to using that information able to compute the phase information about a frequency component. Suppose, there is a frequency component and you have an imaginary fraction of it and real fraction of it, take the ratio of those two and that will give you the phase information, and you take squares and add them up that will give you the magnitude. You are able to calculate phase information also using this approach.

This is the overview of fourier transform and what we will do in the next week is that we will actually do some examples, but not in context of some analog signals. All this information is valid only if signals are continuous or at least you know piecewise continuous. But when we take data from actual instruments we do not get continuous data, we get data at different points, discrete points. So, how do you convert that discrete data, which is a function of time into frequency domain is something we will discuss and actually we will do some examples in the next class and learn that. So with that, I want to close my lecture for today and also this week's lectures and I look forward to seeing you tomorrow.

Thank you very much. Bye.