Basics of Noise and Its Measurements
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Lecture 28
Fourier Integral

Hello, welcome to Basics of Noise and its Measurements. Today, we will continue our discussions from where we left yesterday and what I am essentially going to talk about is concept of Fourier Integral. So, these concepts become relevant specifically for those signals where the time period is extremely large or theoretically it is infinite. In those types of situations, the notion of fourier integral comes very handy, and that is what we are going to discuss in today's class.

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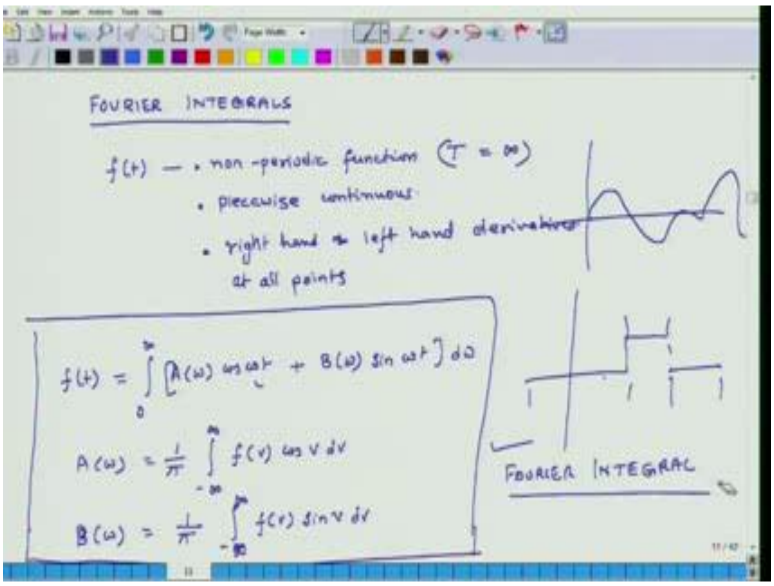

So, Fourier Integrals; let us say that we have a function effects f t, and this is a non periodic function. In other words, its periodicity is infinite. The other thing, so this whole integral is a valid, if there are certain set of conditions which have satisfied, so it can be non-periodic. Then the other thing is, it has to be piecewise continuous. This is a continuous function. This function, is piecewise continuous. It is continuous in this region; it is also continuous in this region. Here, it is not continuous, but in this piece it is continuous in this piece it is continuous. Third thing is, that it should have right hand and left hand derivatives at all points. Mathematical basis for these assumptions, you can go and dig deeper into books on mathematics or engineering mathematics, but if these conditions are satisfied then I am going directly to write the results, then I can express this function f t in terms of cosine and sin and using this format.

Then f t is equal to integral from 0 to infinity. Some function, so please remember here A is not a constant, but it is a function of a parameter w or you can call it omega, cosine omega t. So, A is a function of omega plus another function B which again depends on omega and sin of omega t and we will define what omega is. Then I am integrating this whole thing with respect to omega. What are these functions? A omega equals 1 over pi integral of minus infinity to infinity f v cosine v dv. And, B of omega equals 1 over pi integral minus infinity to infinity f of v sin of v dv. So, this is fourier integral.

I can express a piecewise continuous function, which can be periodic or non period does not matter, should have right hand and left hand derivatives at all points. If there is any function like this, then I can express it in a cosine form or sin form using this fourier integral. I have directly written the results, and the proof of this I will encourage you to go and check to some books on mathematics. We will do an example to make things clearer.

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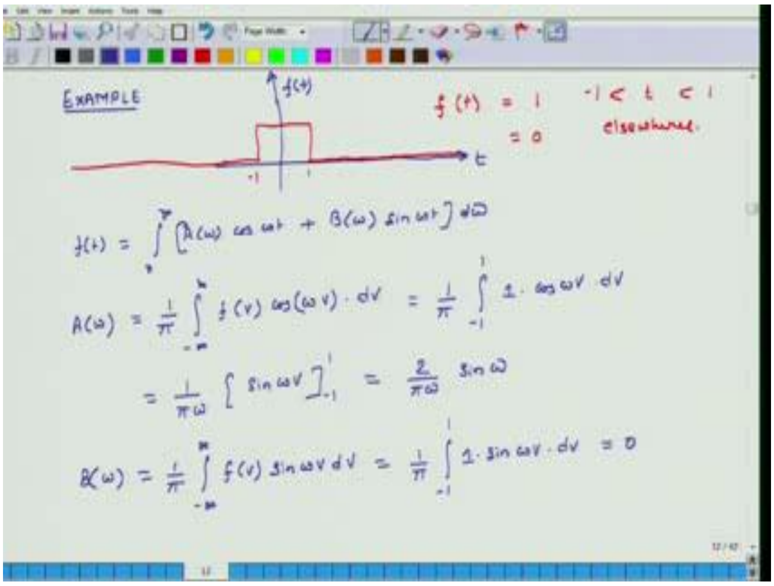

Example: Suppose, you have a function like this. Your function is such that f t equals 1 between the limits, and it is 0 elsewhere. This is 1, this is minus 1, and then interesting thing about this function is that, it is 0 all cross the whole length. So this, unlike other functions like which we handle earlier this is not a periodic function, and it is just having a bump in the region of minus 1 to 1 and at all other places that is 0. Then we see that it is piecewise continuous and also I can compute the derivative of this from right side as well as from left side at all the points. It meets the differentiability requirement, and it is not necessarily period, so it is meeting all our requirement.

So, for such a function I can express this function f t. This is my f t as integral of in 0 to infinity, A as a function of omega cosine of omega t plus B a function of omega cosine of omega t sorry, sin of omega t. And I am going to integrated it with respect to omega. Omega here is a dummy variable, after integration it goes away. After integration only t exits, omega is absorbed into the mathematics. Now, what I will do is, I will calculate what is A omega and B omega?

A omega, we had seen is nothing but 1 over pi integral of minus infinity to infinity f of v cosine omega v dv. When we look at this function f of t, its value is non zero only in the range of minus 1 to 1. So, if I am integrating this function f of v from minus infinity to plus infinity, the value of f of v will be only non zero in minus 1 to 1 at all other places it is going to be 0. So, I can write it as 1 over pi minus 1 to 1, and what is the value of f of v in this range? It is 1 times cosine of omega v times dv. So this equals 1 over pi omega sin omega v minus 1 to 1. This is equal to 2 over pi omega sin of omega, what am I doing? I am just putting the value of v as 1 in first case, so the value of the function just becomes sin omega and then minus sin of minus omega, so its 2 sin omega.

Then, B of omega equals 1 over pi integrate minus infinity to infinity f of v sin omega v dv and that equals 1 over pi minus 1 to 1 1 time sin of omega v dv and that is 0. Because the integral of sin function is cosine which is at even function in the limits 1 to minus 1, the difference between these two values will be 0.

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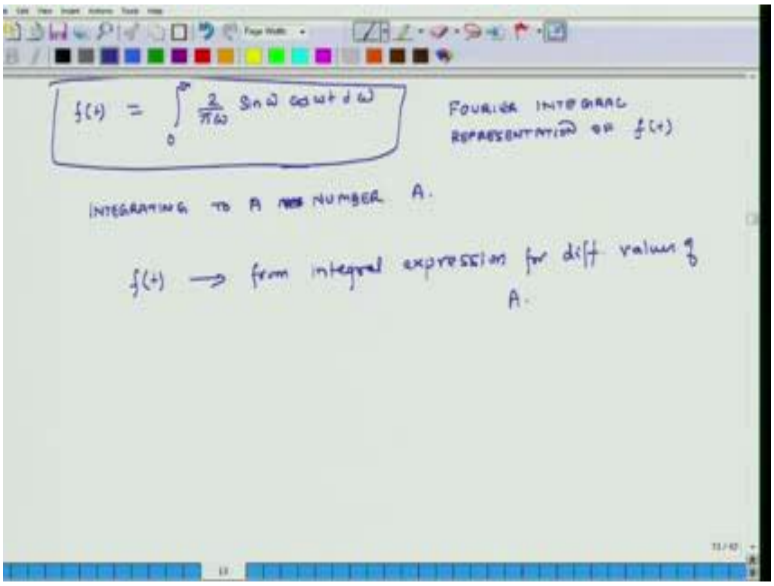

So, f of t, so what is our goal? We have to express f of t in this form. I am going to substitute A omega here. So, f of t is nothing but integral of 0 to infinity, so remember when I am writing the expression for f of t the limits are 0 to infinity not from minus infinity to infinity. And you can explore the mathematical basics by referring some standard books. This is 0 to infinity 2 over pi omega sin omega cos of omega t dw. This is the Fourier Integral Representation of f of t. See, one representation was the graphical format which we saw; this is another representation of the same function but in an integral form. What you do is that, if you take this function and integrate it, and you integrate with respect to omega and you will get some function out of it.

Once you integrate omega will be gone. You will get finally a function of f t, and when you will plot that f t, hopefully it should come out to be same as original function which we had plotted, like this. And you have to integrate in the limits 0 to infinity. So that is important. Now, what you can do is, see integrating to infinity suppose, you get some integrating to infinity may be tricky, so what you can make this is, I can replace this to give a backdrop, When we were doing fourier series, we had represented a function in terms of sins and cosines, and there the number of sin terms and cosine terms theoretically could be infinite. If I take 100 terms, I will get some level of accuracy, if get 200 terms I will get more level of accuracy, if I get 20000 terms I will get more accurate representation of the same function in terms of sins and cosines.

Similarly, if I do integration up to infinity I will get a good result, précised result, but if I truncate it and I bring that number to say some finite number A, then I will get some approximate result but it may be good enough. But, there are issues with that approximation. I will show you. What I am trying to say is that, if f t, I get from this integral expression and for different values of A. Theoretically, it has to be infinite but I cannot, suppose I gone to just limit my exercise to some finite number, especially in when I am doing numerical integration I cannot integrate in finite times. So, I have to do finite number of times. When I am doing finite number of times there are some issues with this integration approach, and I wanted to show you some results.

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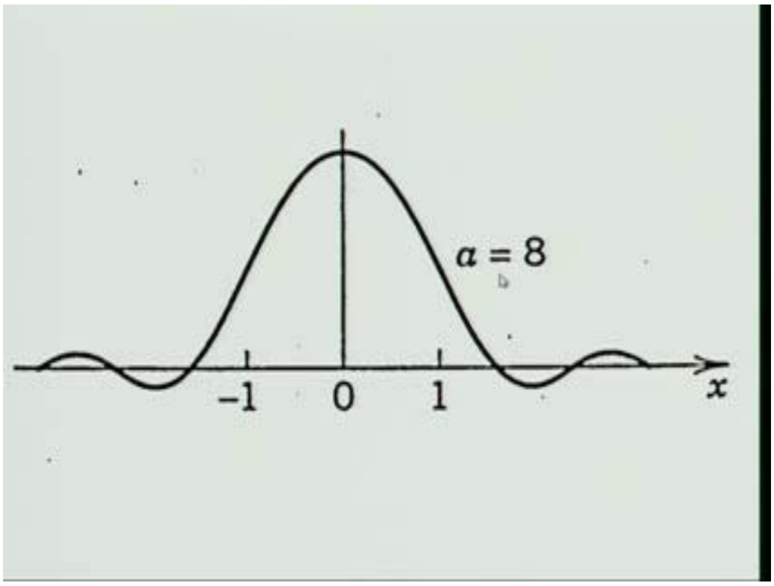

What we have done is, we have use the same relation and the same function and then I have integrated the expression such that, that a the limit was 0 to 8 and what you see is that you got a peak point here, it is not flat as we would have expected it, and here you get a bump at the edges. You got a bump here, you got a bump here, and beyond these bumps it is fairly close to 0. Let us see what happens if I increase the value of 8 and I make it 16.

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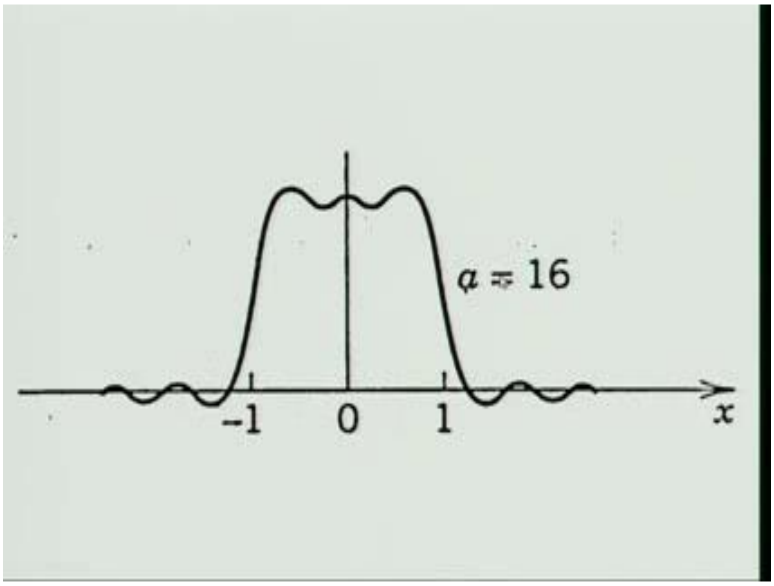

This is what I get. Again, it got a little flatter but there you have this bump here, and then its flattening out beyond 1 and it is also flattening out beyond 1. But remember this bumps are here, there is a bump here and there is a bump here. Ideally it should be perfectly flat.

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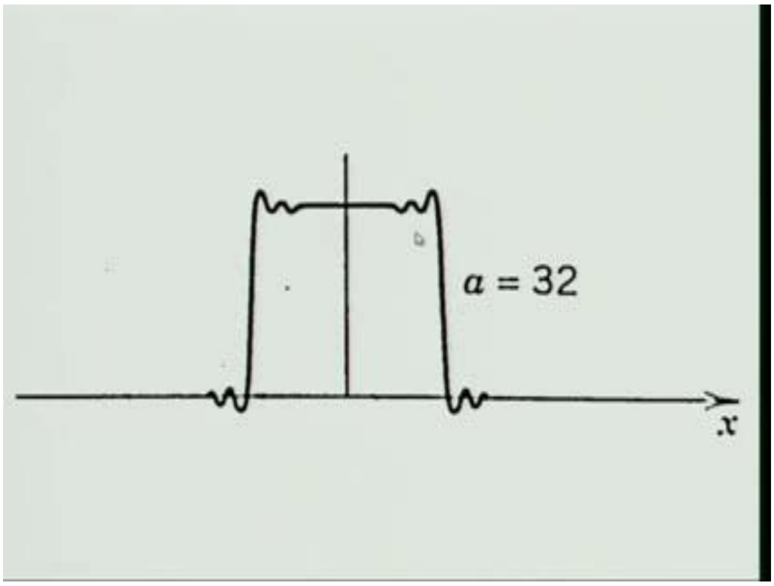

Then you look at this thing. So once again you get a bump here, you get another bump here, in this zone it is pretty close to 1. Here also it is pretty close to 1, here also it is pretty close to 1. But, at the boundaries were derivative of the function was not well defined you take a right hand derivative left hand derivative there will have different values. You have this bump. These grass correspond to three values; a equals 8, 16, and 32, and these bumps are not going away. It does not matter if you make this 64 or 120 or 20000 or 1 lack, these bumps will become sharper, the length of this width will become more and more close to 1, but these bump will not go away. Same thing about this, this bump will not go away. This is because our function was not having unique derivatives at the point of the jump, where it was jumping. If you compute the derivative from one side it is something, if you compute it from other side something different.

So, this effect does not go away if you just increase the number of terms. It is not because you did not have sufficient number of terms, it just because it is just because the mathematics of this approaches that these bumps are preserved. This effect is known as Gibbs Effect. You will see the same thing for fourier series also, and you will see the same issue is in for fourier integral approaches also. Gibbs effect is there in fourier series and also it is present a fourier integral. So, this is something to remember about.

That covers the treatment of whatever I wanted to cover in this particular class. In the next class, we will move on to the final goal, which is fourier transform for functions, in the fifth class and sixth class will discuss fourier transforms in detail and that is how we are going to close this topic.

Thank you very much and we will see you again tomorrow.