Basics of Noise and Its Measurements

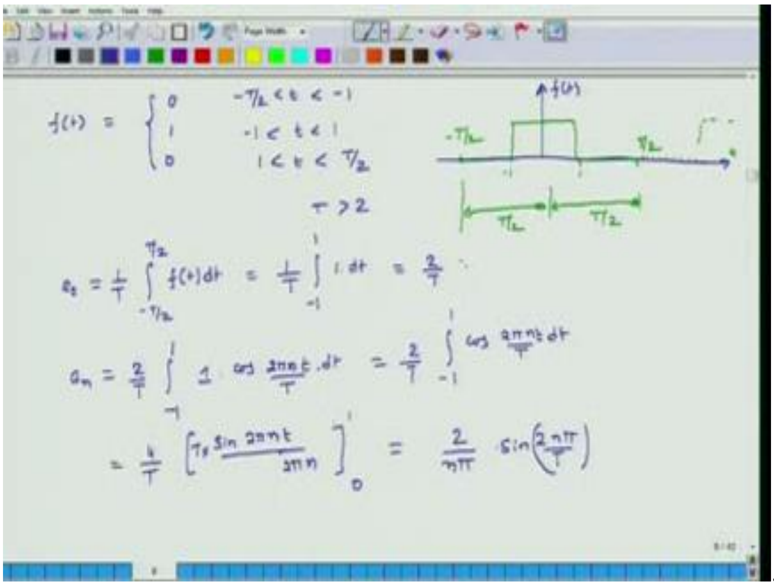
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 27
Fourier Integral

Hello, welcome to Basics of Noise and its Measurements. This is week 5 of this particular course, and today is specifically, we are on the third day of fifth week of this course. And what we are going to discuss today will be introduction to Fourier Integrals, and why do we need these entities, so will discuss fourier integrals. But before we discuss that, I just wanted to give you a back drop as why is it need to.

So, we have discuss that the approach of using fourier series expansions of any functions. They work when the function is periodic, when it is periodic; it repeats itself after a finite duration of time. Now, one question would be what happens if this periodicity or the duration of this periodicity it becomes very large and actually it becomes infinite, then what happens. That is what we will do in today's class and then in next class we will actually explore fourier integrals. What we will do is, we will extend the method of fourier series in context that what is happening as T becomes extremely large, and we will do that using an example.

(Refer Slide Time: 01:48)

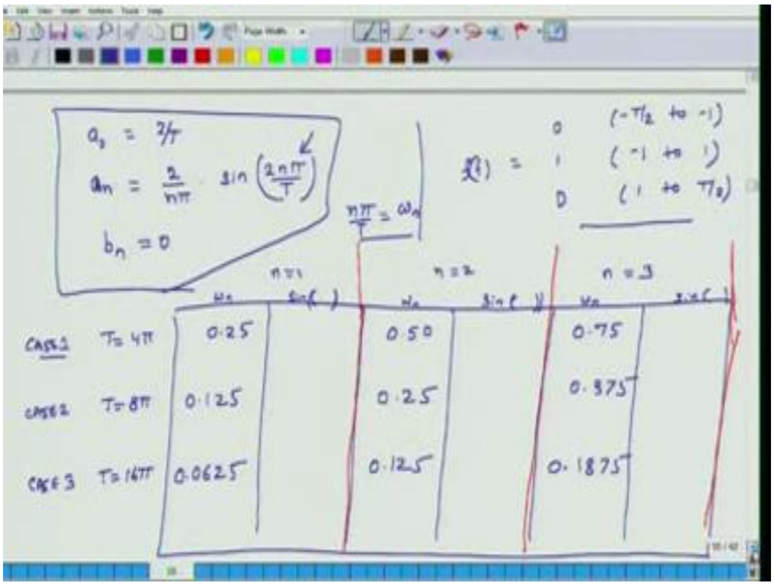


I will consider a function f t and it is such that it can have 3 values; 0, 1 and 0. It is 0 in the limit at times when t is more than that half of the time period, and it is less then minus 1. The value is 1, when t is between 1 and minus 1, and the value is t over is 0, when the time is anywhere between t over 2 to 1. First, let us plot this thing. This is my x axis, this is f of t, and the value is 1 between minus 1 and 1, this is my time axis and then the value becomes 0. Let us say this is the time period. This whole thing is t over 2, and this thing is also t over 0. So this is t over 2, this is minus t over 2 and the function repeats. So if it has to repeat then the again after it becomes positive. It repeats after a time period of T.

And we will assume here, in this context that t is at least its more then 2. So what we will do is that we will compute the values of 0 a n and b n for this function and its fairly straight forward process. Then we will make certain observations that what happens to these values of a n, b n and a 0 as t increases and how close they become to each other, that is what we are going to look at. So, a 0 is equal to 1 over T minus T over 2 to T over 2 f t dt, and that equals 1 over T, f t is 0 at all locations in over the time period accept between minus 1 and 1 and the value is 1 in this range, so it is 1 times dt so equals 2 over T.

Then a n equals 2 over T minus T over 0 to T over 2 f t cosine of nt dt I am sorry, this should be 2 pi n t over T dt. As I explained earlier, because if the value is 0 at other positions so all have to do is I have integrate this only in the limits of minus 1 to plus 1 and the value of f t in these limits is 1, So what I get is 2 over T integral of minus 1 to 1 cosine 2 pi n over T dt, which equals 2 over T sin 2 pi n t over T into 2 pi n in the limits minus 1 to 1. I can also simplify this as, I can remove this and I can make it 0 but what I have do here is I have to multiply this by another factor of 0 so this becomes 4. So when I do, all these math essentially what I get is 2 over n pi, what this should have been is T into this. So, T T cancels out, so its 2 over n pi times sin, 2 n pi over T. So first thing is that a 0 is 2 over T, so we will write down the results.

(Refer Slide Time: 07:47)



A 0 is equal to 2 over T, a n equals 2 over n pi times sin 2 n pi over T. And if you do the math, b n actually comes to be 0. We will not go into that but please do it, make sure that it comes out as 0. Now, what we are going to do is, we are going to make some observations and we will look at the amplitude spectrum of this function. Our original function is, is equal to 0 1 0 for the limits minus T over 2 to minus 1, minus 1 to 1, and 1 to T over 2.

I will do one more thing. What I will do is I will make a table. We will look at different cases of values of T, we will look at these constants how they change and how closely of for way become from each other as T changes. The first cases we will look at is, the T equals 4 pi, why did I choose this 4 pi? Because, if I put 4 pi in the denominator this pi goes away so my mathematics becomes easy that is only reason. But all what I am going to do is, I am going to increase T. This is case 1: T is equal to 4 pi. Case 2; T equals 8 pi I am going to double my T. Case 3: T equals 16 pi.

The second thing I am going to look at is this parameter. So I will call n pi over T as omega n, some number omega n. In this whole thing, T can vary and n can vary ok so in the vertical direction I am varying T and in the horizontal direction I will vary n. So, n is equal to 1, n is equal 2, n is equal to 3, and here we will look at omega n. And then we also look at this sin of omega, this sin term. Actually, we will not very too much about the sin term but it is the omega and which is important. Here also, we will look at omega n and sin, and then omega n and sin. Just differentiate, this is n equals 1, this is n equals 2, and this is n equals 3.

When T equals 4 pi, time period is 4 pi and n equals 1, I can calculate n pi over T and what I get is that I get this value is 0.25 and pi over T is 0.25. When n is equal to 2 and T is equal to 4 pi I get this value is 0.50. And when n is equal to 3 and T is equal to 4 pi, I get this as 0.75. Next, let us look at this 8 pi row. So when T equals 8 pi and n equals 1, then this omega becomes 0.125, here it becomes 0.25, and here it becomes 0.375. And then when T is 16 pi, then it becomes 0.0625. And in the column corresponding to n is equal 2, it becomes 0.125, and here it becomes 0.1875.

So, let us look at this table more carefully. Now, that I know omega and I can also calculate the sign values of thinks in the bracket so you can fill up these columns. But the point what I am trying to make is somewhat different. What this tells me is, that when time period is 4 pi, on the time axis the first fourior component is corresponds to omega n is equal to 0.25. The second fourier component corresponds to omega n is equal to 0.50, and the third fourior component corresponds to omega n equals 0.75. So, we get successive fourior components and each fourior component is separated by a spacing of 0.25 radiance.

When T is equal to 8 pi, time period has doubled then this spacing goes down in this case the spacing was 0.25, I will not write it here but anyway in the first row the spacing was 0.25 success points were coming every 0.25 radiance, now they will come at 0.125 radiance. As I doubled I time period, the spacing between individual fourier components of which represent the original a function, they are now closer, which means that the fourier components have become more dense on the x axis, on the time axis. It is important to understand.

When I increase my time period to 16 pi, I have increase it by another factor of 2, then first omega n is at 0.0625, second omega n is at 0.125, so again, the spacing shrunk by factor of 2 one more time. Now, the fourior components which correspond to the original function f of t, they have shrunk, they are even closer on the x axis to each other. So, what this shows is that as I keep on increasing the overall time period, I will have closer and closer fourior components if I measure the distance between these two fourior components and the x axis, then the distance between there will be reduce progressively as T grows up. When T becomes infinitive theoretically, then the distance between two for a components will be theoretically 0, which means that there will be to represent a function which has an infinite time period, I will need how will in frequencies infinite frequencies to represent them on the frequency spectrum. If I have to generate their spectrum then I will need infinite frequencies to represent them.

So that is what this exercise shows us. That if I use this fourier analysis approach and I make T theoretically infinite, I will need infinite number of frequencies to handle such a function. If I have a function which has a finite time period and it repeats after finite time period, then I have finite number frequencies. The higher the time period the more closely spaced are the frequencies, the number of frequencies is anyway extremely large n goes from one to infinity, but when the time period becomes infinite the distance or the separation between two adjacent and frequencies shrinks to 0 that is what the point is.

This is very important take away, and the context of this is let in a lot of situations are signals which we want to analyze they are not periodic, for instance I am talking to you so I do not talk the same thing every 5 seconds or 10 second these not a periodic signal. And if I want to do a fourier analysis of this, then I will need very large number of frequencies and two adjacent frequencies will be separated by almost 0 distances, to represent them. And that is the context, and in that context we will introduce the next topic that is called Fourier Integral. That is what we are going to discuss in the next lecture.

Thank you very much and we will see you tomorrow.

Thanks.