Basics of Noise and Its Measurements
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Lecture – 26
Finding bn

Hello again, welcome to Basics of Noise and its Measurement. This week we are discussing some of the signal processing techniques in contexts of noise measurements. What we are specifically doing or what we did in the last lecture was, cover this whole topic related to Fourier series expansions of periodic functions. We will continue that discussions in this lecture and probably also partly in the next lecture.

So, in the last class what I had discussed was, that how can we compute constants a 0, and for a periodic function, whose time period is 2 pi seconds or 2 pi radiance. So now, we will continue this discussion and we had figured out how to find a0 and an, next what we are going to do is, figure out the value of bn.

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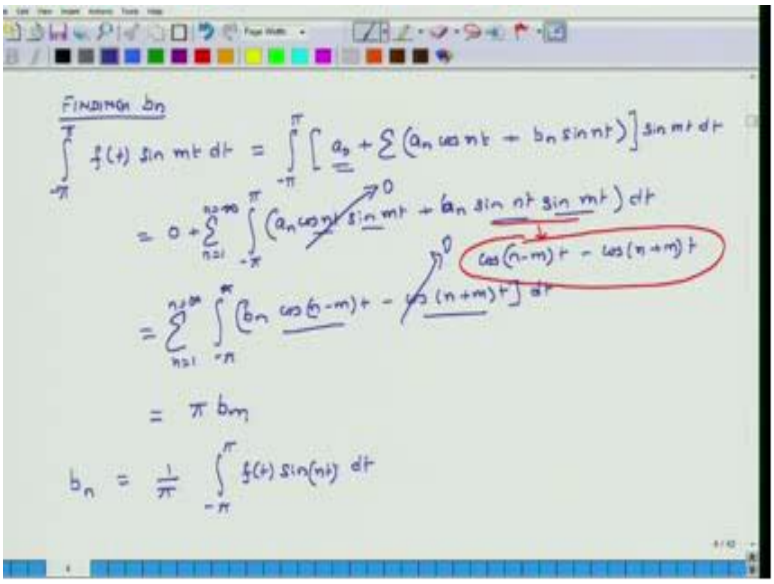

So, finding bn, if we have to find bn, what we have to do is, again I take the original equation which is f (t) equals a0 plus an cosine of nt plus bn cosine of nt and multiply both sides by instead of cosine function, sin function and also dt and then integrate the whole thing.

So, that is what we are going to do. I am going to rewrite that equation and integrate it after multiplying it by cosine the sin function. So, that is my relation and the contribution from this a0 term when I multiply a0 by sin of nt integrated I get a cosine term and the value of that cosine term, in the at both the limits because it is an even function are same. So, when I take the difference I get 0. So, the contribution from the a0 term is 0. Then I have this summation time n equals 1 to an equals infinity integral minus pi to pi, an cosine nt sin mt plus bn sin nt and times sin of mt dt.

Now, in the last lecture we had seen, that the integral of this cosine times sin cosine and t times sin and t in the limits minus pi to pi is 0; and the same is term is appearing here. So, I do not have to redo that whole mathematics, but I can straight away say that is 0 for the same reasons it was 0 in the last class in the last relation. Then the sin nt times sin nt is nothing but, cosine n minus m t minus cosine of n plus m t.

This is what this thing is. So, I can re write this whole thing has t. So, once again, when I integrate this I get again 2 sin terms and the integration of this term. Where in the limits minus pi to pi is again 0 for the reasons which I had discussed earlier, and from here I get nothing, but pi times bn because, all other terms in this series are 0. Excuse me is b m. So, b m equals 1 over pi integral minus pi to pi f (t) sin mt dt or since I was trying to find bn. So, I will just erase it and replace it by n nt, dt.

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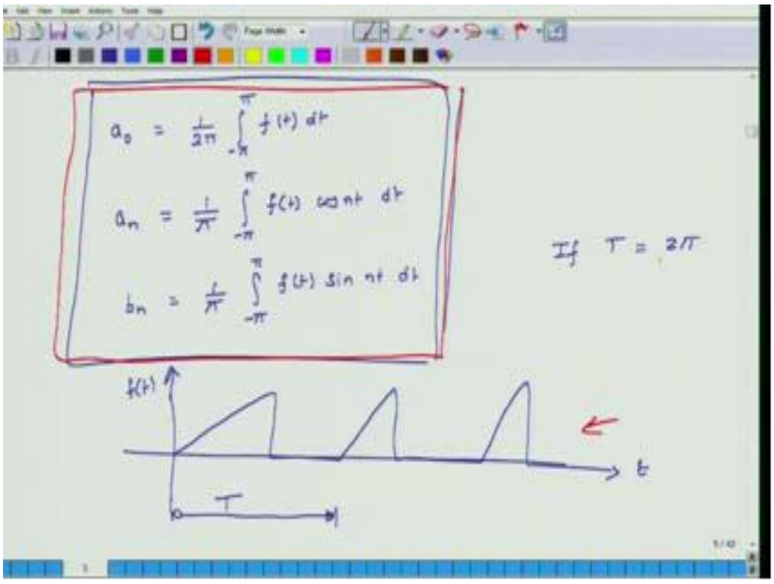

So, to summarize a0 equals 1 over 2 pi minus pi to pi f (t) dt an equals 1 over pi minus pi to pi integral f (t) cosine nt dt and bn is equal to 1 over pi minus pi to pi f (t) sin nt dt. So, if I have a function, something like this and if the time period let us say this is time x and this is the value of f (t) and at the time period is such. So, its time period is what this is the time period and if the time period is if time period equals 2 pi then I can use these relations, to compute a0, an and bn. Then plug them back into Fourier expansion and express this whole function, as a some of sins and cosines, but these relations are good only if the time period is 2 pi. If the time period is not 2 pi, then we have to slightly modify the whole thing. So, what I will do is rather than, going into details, what I will do is actually, give you the whole relation.

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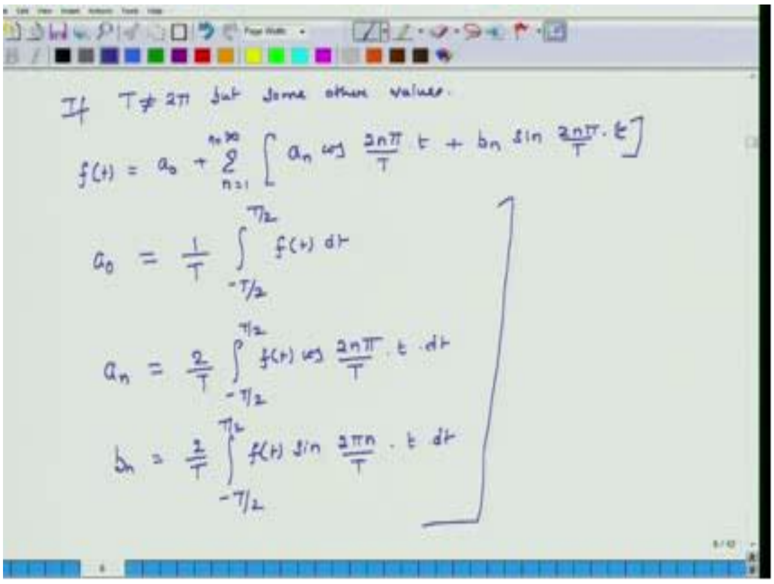

So, if time period is not equal to 2 pi, but some other value, then first thing is then I can express f (t) as a naught plus, against some of sins and cosines, an cosine of 2 n pi over t times t plus bn sin of 2 n pi over t times t. I am going to directly write the results for a0 and for this situation, the value of a naught can be computed by this formula. So, a naught is 1 over t integral of f (t) over dt in the limits minus t o to 2 plus t o 2. an is equal to 2 over t, integral minus t over 2 to t over 2 f (t) cosine 2 n pi over t t dt and bn is equal to 2 over t again integrating between these 2 limits; f (t) sin 2 pi n over period times time times dt. So, these are the relations and they work for all periodic functions, regardless whatever the time period is. So, what will do is will do an example.

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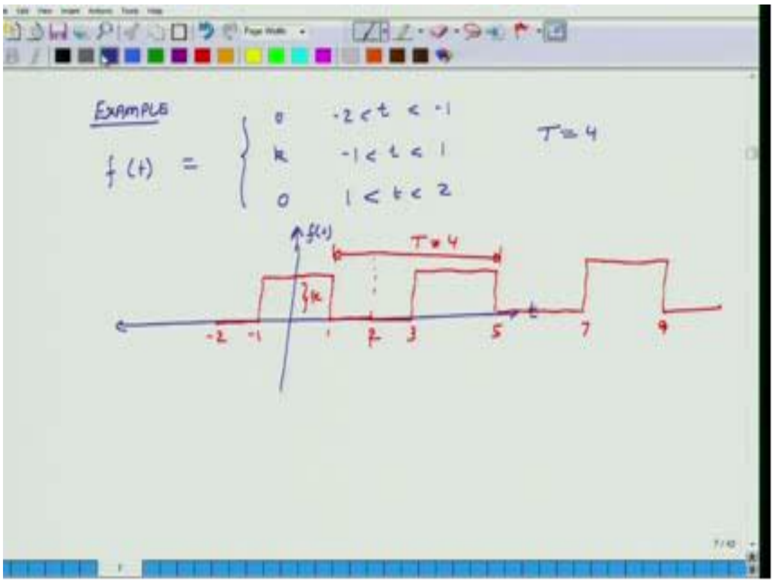

I have a function f (t) such that it has the value of 0 if time is between minus 1 and minus 2. It has the value of k, if time is between 1 and minus 1, and it has the value 0 if, time is between 2 and 1. Then we are also saying that the overall time period is 4 seconds. So, how does this function look like? That is my f (t), this is my time axis. So, it is the value is k, between minus 1 and 1 k is a constant. So, this is 1, this is minus 1, this is k. Then between 1 and 2, it is 0. So, this is 1 and this is 2 it is 0 here, and then between minus 1 and minus 2 also it is 0. So, this is my function, and then it repeats itself and it repeats itself after every 4 seconds, after every 4 seconds.

So, basically, the function is like this, and so on so forth. This is my time period. Let us say this is 4, 3, 5, 7, 9 and so on so forth. What we will do is, we will actually compute values of a naught an and bn for this thing.

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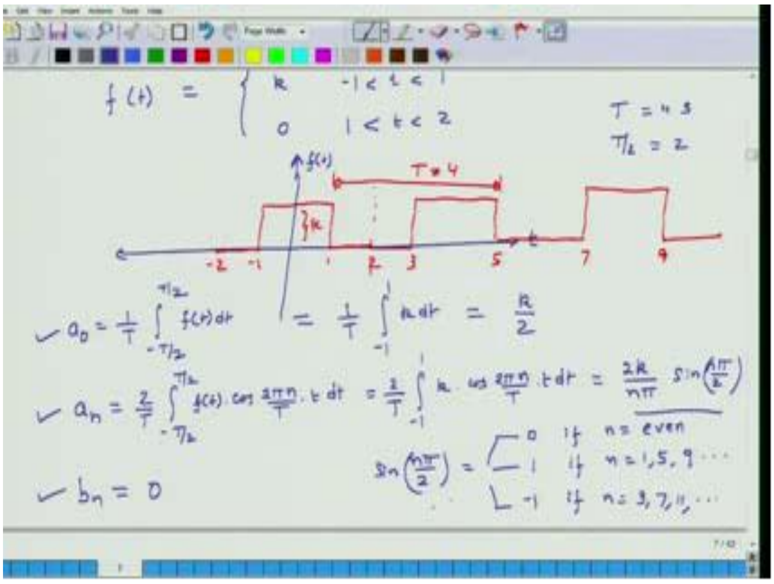

So, a naught is equal to 1 over t minus t over 2 to t over 2, f (t) dt. Now if you look at this function, t is equal to 4 seconds. So, t over 2 equals 2 and the value of this function is non 0 only in the limit minus 1 to 1, for all other; during the other part of the time period it is 0. So, what I can do is I can write it has 1 over t minus 1 to 1 and the value in this limit minus 1 to 1 is k dt. So, this comes to be k over 2.

Next is an and that equals 2 over t integral minus t over 2 to t over 2 f (t) times cosine of 2 pi n over time period times, time dt and this is again if I integrated from minus 2 to plus 2, which is on the limits between minus 2 to minus 1 the value is 0. I can ignore it between 1 to 2, the value is 0, so I have to only integrate in in the reason 1 to minus 1. So, what I get is minus 1 to 1, and the value of f (t) in this range is k times cosine of 2 pi n over t times t dt. If I do the math I will write this directly, and this is equal to 2 k over and pi sin n pi over 2. You can do this math, if you do the integration and do all the things correctly this is what you will get.

And if you find the values of this whole thing, what you will find that is sin of n pi over 2 is equal to 0, if n is even. It is equal to 1; if n equals 1, 5, 9 and so on and so forth. Equals minus 1 if n equals 3, 7, 1 and so on and so forth. So, that is the value of a0 that is the value of an. So, I have to multiply these values for different values of n and get the actual values of an and finally, bn if you calculate and you do this math, you will find that here instead of cosine you will get a sin function and when you integrate it you will get a cosine and cosine be an even function, its value will be same at 1 and minus 1. So, the difference will be 0. So, bn will come out to be 0.

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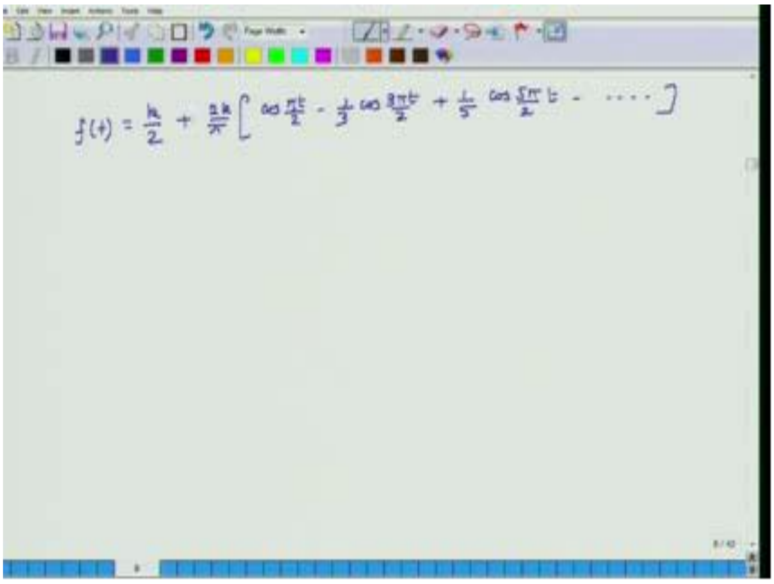

So, my over all function; will be something like this, k over 2 plus 2 k over pi cosine pi t over 2 minus 1 by 3 cosine 3 pi t over 2 plus 1 over 5 cosine 5 pi over 2 minus and then this series keeps on going forever. If I put in a lot of terms of this series and add them up, our overall function will look very close to the function, which we had plotted which is this one. So, that is for fourier series for you, and that completes are treatment of fourier series expansion and in the next class what we will do is we will start discussing fourier integrals, then learn a little bit more about those.

Thank you very much and we will see tomorrow. Bye.