Basics of Noise and Its Measurements
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Lecture - 25
Fourier Series Expansions

Hello, welcome to Basics of Noise and its Measurements. Today is the start of the fifth week of this course and over last 4 weeks we have dealt a series with the series of topics and particularly, in the last week we had learnt lot of above lots of stuff about microphones their classification, different categories, what is the sensitivity of a microphone, what are different criteria using which we can select a microphone which meets our needs directivity and so on and so forth.

And what we will discuss today is how to analyze sound, and we will start from a theoretical stand point and next week we will actually start analyzing sound you have what happens is that, you have microphone it gives you data and essentially what you get is, some sort of a function a voltage as a function of time, and then the question is that once you have this data voltage as a function of time; what do you do with it? How do you analyze it? So, it is important to have some grounding in some of the important techniques and methods which are use to analyze these sound signals. Specifically what we will cover today are 3 concepts. The first is, what is a Fourier Series expansion of a function. Then, we will progress on to talk about fourier integrals, and then while we will finally, close today’s discussion with fourier transform.

So, lot of the material which we will cover today will be conceptual and somewhat theoretical in nature, but next week when we discuss this further you will realize the importance of these concepts. So, that is how we are going to proceed.

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In this particular class we will discuss, Fourier Series expansions of some nice periodic functions. So, we will discuss Fourier Series, the context is that suppose you have a periodic function

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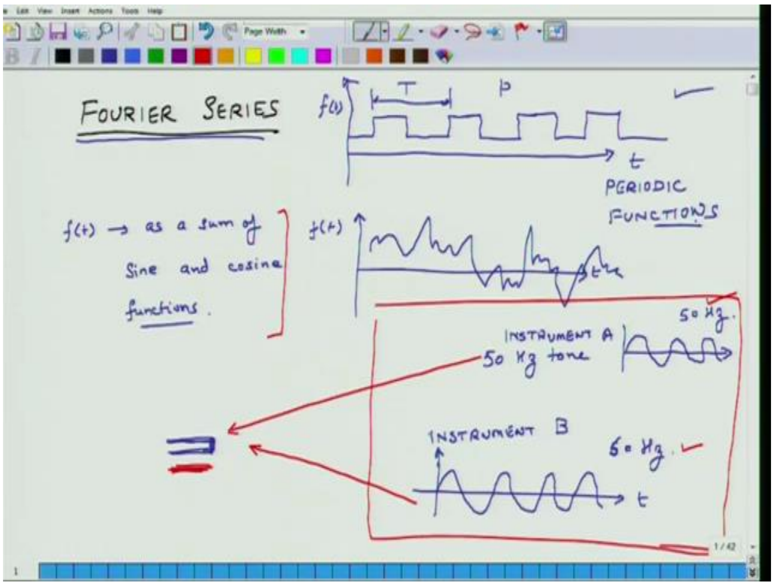

So, let us say this, my x axis and here instead of x, I will call it let us say time. On the y axis, I have some function this of time f (t) an example of this function could be the sound which I am recording through a microphone or the voltage which is being generated by some instrument or whatever.

Now, this function could be a periodic function, for instance, this is a periodic function or it could be a non periodic function. So, example of non periodic function will be this, there is no repetitiveness in this function. So, this whole concept of Fourier Series is applicable to this class of functions, which are periodic functions. Fourier Series is not applicable to non periodic functions, and the time after which the function repeats itself that duration. So, this function repeats itself after, so much of duration this is known as time period or capital t in some literature you will also find it as p, which is the period of repetitiveness.

Before we go further into it you would wonder why, what is it that we are trying to do in this? So, what we are trying to do is, that we are trying to express f (t) as a sum of sine and cosine functions. That is our fundamental goal, that if we have a periodic functions, can I express it as a sum of sine and cosine functions a question will be, why had we interested in doing that, so to address that I will give you prospective. Suppose I have a microphone, and its recording sound and it is positioned in a room, and let us say there is 1 instrument and this instrument is generating a 50 hertz tone.

This just one example, I am talking about. So, this is some instrument, INSTRUMENT A. So, this is some instrument and it is generating a 50 hertz tone and let us say, it is a nice sinusoidal tone. So, this microphone is receiving signals, from this instrument. There could be another instrument and what this instrument is doing, is its generating some other repeating thing. What it is doing is that. So, this is another instrument, INSTRUMENT B, and for purpose of simplicity, let us says that it is generating, so this also generates some noise, and for purposes of simplicity it is generating a 60 hertz tone. So, this 1 is doing 50 hertz and this guy is doing 60 hertz.

And using this microphone I am also recording this noise. I am recording both these noise. Now they could be an interest. So, what I will record will be a sum of this sound and this sound. If I plot these on in on x y graph then what the microphone is going to record is some of those 2 graphs. And that also will be repetitive, and I may be interested in figuring out that what is the nature of this noise, and suppose this is a black box, you know this whole thing is a black box. So, I do not know what is happening in the room, but all I am recording is this thing. I have some knowledge that may be 1 instrument is doing 1 tone, and another instrument is doing 1 tone. If I can do fourier analysis of my signal then, I can resolve the signal which is recorded by microphone into its components, and then I can resolve map these components, to the specific instrument or their specific source is. So, this is 1 motivation, that is 1 particle application why is Fourier Series and some other similar you know representations of time signal in frequency domain important.

So, I can resolve signal into individual harmonic components. This is 1 you know benefit I can have and then if I have a physical system, which is generating a lot of noise and different parts of that system are generating different part types of noise. I can relate these individual harmonics to specific features of the overall system. So, that is 1 benefit which I get out of it.

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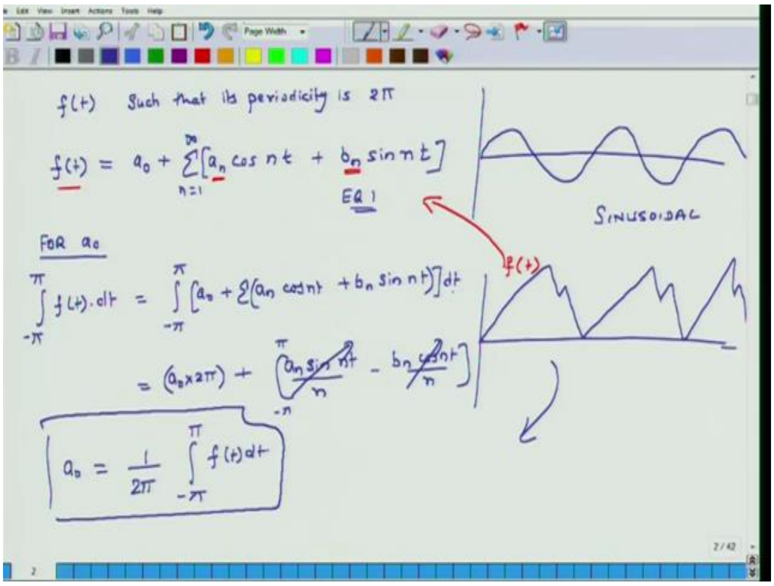

So, I will I will draw another periodic function. So, this is another periodic function, it is a sinusoidal function. The one which I had drawn earlier, this is a square wave type of a function. Another periodic function could be something like this. So, the try I am point what I am trying to make is, that a periodic function need not be only sinusoidal in nature as long as a function repeats itself, it is a periodic function. Sinusoidal functions are a particular type of periodic functions, but those are not the only periodic functions which are there and in Fourier Series our goal is, to express these functions periodic functions as sums of cosines and sins. So, that is what we are going to do. So, what I will do is, that suppose there is a periodic function f of t, such that its periodicity is 2 pi which means that on it repeats are itself after every 2 pi, you know seconds or hours or minutes. So, its periodicity is 2 pi or its period is 2 pi then I can express f of t has a constant, plus a series of sums of cosine functions and sine functions excuse me this should be t.

So, what you have here is, a cosine function and there are infinite number of cosine functions, and each cosine function has a coefficient and similarly each sine function has a coefficient bn and n can change from 1 to 2 to 3 to and it can go on till infinity. So, this is the form in which I want to bring my f (t) 2 this is my f (t) and I want to convert it into this form that is my goal. So, the form is already there if I can figure out what are the values of a naught an and bn, then I have my transformation is complete. So, we will figure out how to calculate the values of a naught an and b n. So, to calculate a 0, what do we do? We multiply both sides. So, let us call this equation 1. So, multiply both sides of equation 1 with d x or d t an integrated between minus pi to pi which is the limits of the which is the period of the system. So, this is what I will do, minus pi to pi f (t) d t equals minus pi to pi a naught plus an cosine nx plus bn sine nt, excuse me, I am somehow struck with x cosine of nt d t. So, if I have to calculate for this thing, all what I am is doing is I am multiplying both sides of this equation by this term d t, I am integrating the whole equation in the limits, minus to pi to pi which correspond to the period of this repetitive function.

So, this tells me that because of this I get a 0 into 2 pi, plus integral of minus pi to pi an sine nt divided by n minus bn cosine nt divided by n and actually even I integrate it I have already integrated it. So, I am removing the symbol of integration, and I am going to calculate these limits in the integral minus by 2 pi. So, when I calculate the value of this thing in the limits, minus pi to pi sine when t is pi and n is always a integer this term is always 0 and when t is pi or minus pi it is an even function. So, when I take the difference this is always 0. So, a 0 equals 1 over 2 pi integral of minus pi to pi f (t) d t this one.

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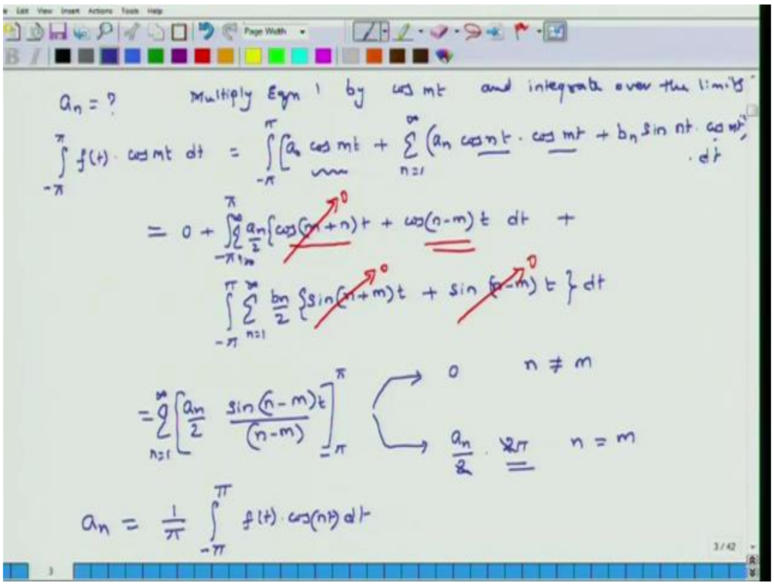

Next, we will compute the value an. So, an equals what? So, here what we do is we multiply equation 1 by cosine of nx and integrate over the limits. So, what I get is minus pi to pi f (t) times cosine again I am this should have been time cosine of nt d t equals integral of minus pi to pi a 0 cosine of how actually this should not be nt it should be mt. cosine of plus summation of n equal 1 to infinite, an cosine of m nt times cos of mt plus bn sine nt times cos of mt and then, I multiplied this whole thing by d t and I integrate it. This is cosine mt and this should be also, this is cosine of mt.

So, what I get is, when I integrate cosine of mt this first term, and then I take the limits in minus pi to pi I get 0, and then I integrate I separate these 2 terms and then and now this is cosine nt times cosine mt, and cosine a times cosine b is cosine a plus b the whole thing divided by 2 plus cosine a minus b the whole thing divided by 2. So, it is an over 2 cosine m plus nt plus cosine of m minus n, actually should not matter, but I will just write it n minus m, t and this whole is being summed from n is equal to 0 to infinite d t plus integral of minus pi to pi bn over 2 n is equal to 1 to infinite and sine nt times cosine of mt is sine of n plus mt plus sine of n minus mt dt.

Now, if I integrate this term cosine m plus nt. I will get sine and in the denominator, I will get m plus n and sine. When t is equal to pi and m and n are integers this will be 0 and also when t is equal to minus pi. So, this value will be 0, we will not talk about this right now.

Then let us look at this sine n plus mt, when I integrate it I will get negative of cosine and in the denominator I will get n plus m and cosine is an even function. So, its value will be whether t is pi or minus pi it will be same. So, when I take the difference this will be 0. For the same rho reason this is also going to be 0. So, what I end up getting is, is an over 2 summation n equals 1 to infinite, and this is sine n minus m over n minus m and I can take this value at minus pi to pi and there is a t here so, t here.

Now, the value of this is equal to 0, when n is not equal to m when n is not equal to m, then n minus m is a non 0 thing, when t is pi then sign of anything times pi is 0. Then this is equal to an over 2 times 2 pi, when n is equal to m. What happens when n is equal to m as t becomes pi and when n is equal to m the numerator is 0 the denominator is 0. So, I have to take the limits of numerator and denominator and I will get the ratio. So, and what I get is that if I you this Lupital's rule then I get this thing as 2 pi.

So, my an is equal to. So, this 2 goes away, 1 over pi minus pi to pi f (t) cosine of nt dt. That is the value of an. So, that covers the topic for this particular class, we will continue this particular topic in the next class also, and what we will do in the next class is we will finish this Fourier Series by computing the value of bn. Then we will make this thing more general in context of if the limits are not minus pi to pi, but rather from some time period t. So, that is what we are going to do. So, thanks a lot for listening to this lecture and look forward to seeing you tomorrow.

Thank you.