Basics of Noise and Its Measurements
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Lecture - 17
1-D Tubes with Imperfect Terminations

Hello, welcome to Basics of noise and its Measurements. Over this week, we have been discussing transmission line equations and we have also introduced in the last lecture this notion of specific acoustic impedance.

So, what we are going to do today is discuss another type of one dimensional tube, and in this particular tube in state of having a closed end or an open end which are regarded as perfect terminations, because in one case you have this perfect condition that velocity is 0 for a closed tube. In the other case, you have this perfect condition that pressure is 0 that is for an open tube, you can have a situation where the conditions are not very explicit and so clear and rather at one end of tube you know the impedance of the system; you know the impedance of the system.

So, if you know that there is a particular material which is being used to close the tube that will not necessary translate into perfectly closed end or a perfectly open end, because to have a perfectly closed end, you need a perfectly rigid closer which is not in the case in real systems and so it is imperfect termination.

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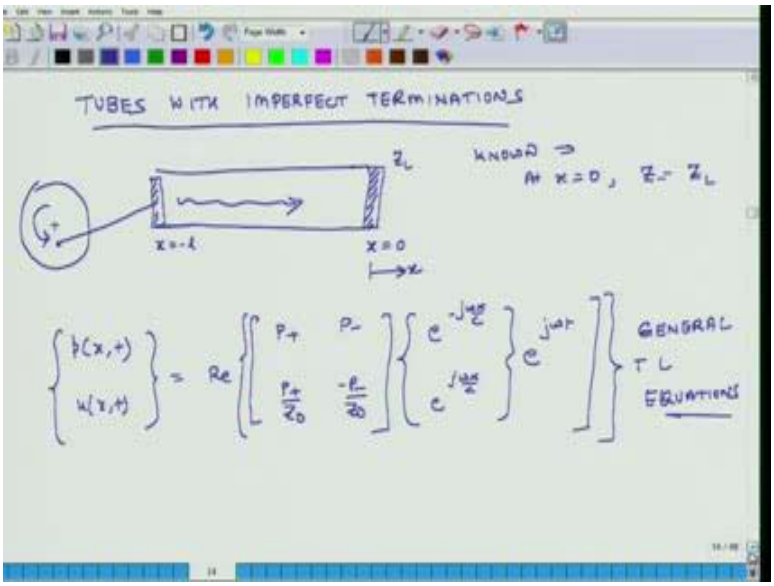

So, those are the tubes which we are going to discuss and we will see how we can address those tubes and what we can make out of such type of tubes. So, what we are going to discuss this tubes with imperfect terminations. So, what does this mean? Suppose, you have some tube of length L and at this end of the tube and using some material and this is not perfectly rigid material. So, it can absorb a part of the sound and it also bends because of the pressure of the sound, because of the sound pressure inside the tube. So, things are not as explicit as we saw in case of closed end, open tubes or even for the case of semi infinite tubes. So, all what we know is based on some measurement and we will discuss how we measure these things; that we have figured out that the impedance is specific acoustic impedance is Z L. So, what is known is that at x is equal to 0, Z that is specific acoustic impedance equals some values Z L.

How we will measure this Z L? We will discuss it in a next lecture, but at least today we will introduce this notion of imperfect termination and we want to see what kind of mathematical relations governed this tube. So, once again, this end is being exited by a piston and this is x is equal to 0, this is x is equal to minus L, this is my positive x. So, sound is travel traveling like this. So, what we do is again we start with our transmission line equations and this is equal to real of p plus p minus p plus over Z naught minus p minus over Z naught; e minus j omega x over c, e j omega x over c, e j omega t. So, once again these are the General TL equations, transmission line equations.

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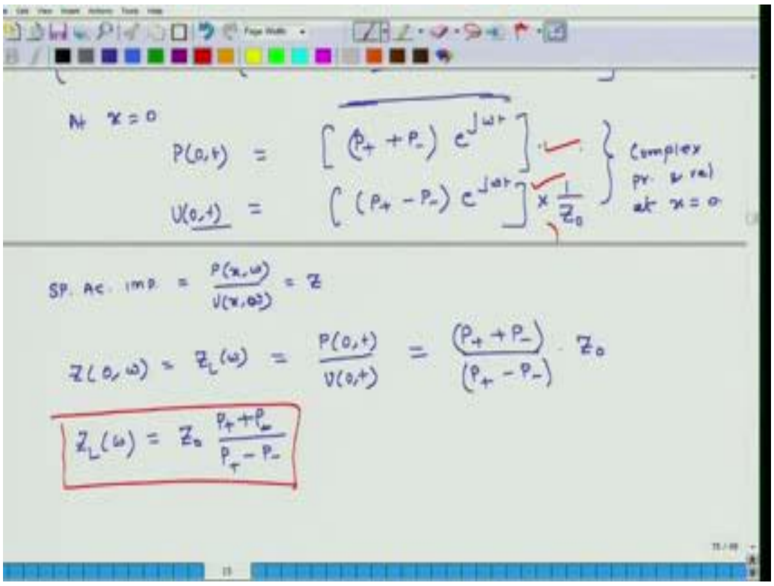

Now, let us see what we have at x is equal to 0. So, at x is equal to 0; we will write the same equations, at x is equal to 0 p of x and t is equal to p of 0 and t, and what I do is I put the value of x in this vector e minus j omega x over c and e j omega x over c. So, what I get is real of p plus plus p minus because e minus j omega x over c is 1, at x equals 0 and same things is true for e j omega x over c at x equals 0; e j omega t.

Similarly, velocity is equal to at origin u 0. So, I am going to erase these to eliminate confusion, at x is equal to 0, u of 0 of t is equal to real of p plus minus p minus e j omega t and of course, I have to multiply it by 1 over Z naught. So, this is the expression for the actual pressure e and this is the expression for actually velocity u, but the term in this parenthesize is complex pressure and complex velocity. If I remove this parameter real or function real, then what the left hand side is no longer real pressure or real velocity, but rather it is complex pressure. So, I will express it has capital P it is complex pressure and again this is complex velocity, so I will express it in capital letters. So, these are the expressions for complex pressure and velocity at x is equal to 0.

And again these are valid for all situations, because right now we have not introduced any boundary condition; we have not introduced any boundary condition. Now, we know that specific acoustic impedance is nothing but the ratio of complex pressure and complex velocity. So, if I have to compute the value of a specific acoustic impeded and this is designated as Z, right. So, if I have to compute the value of this specific acoustic impedance at the origin, then we have been already told that the value is given and it is Z at 0 omega is equal to Z L and this can change with frequency. So, it is Z omega and this is nothing but, p of 0 and t divided by u of 0 and t. And if I use these relations, if I use these two relations and put them here then what I get is p plus plus p minus divided by p plus minus p minus times Z naught. So, my Z L in this case it is known, if I know Z L then I can express Z L also in terms of p plus and p minus.

Now, at this stage I will introduce. So, this is my important relation that the terminal impedance which is known it influences the complex amplitude p minus and p plus based on this particular relation which we have written here. Now, we know that p minus is the complex amplitude of the wave which is called traveling in the negative direction, and p plus is the amplitude complex amplitude of the forward traveling wave. And then the forward traveling wave hits the end and hits this impedance, it gets reflected.

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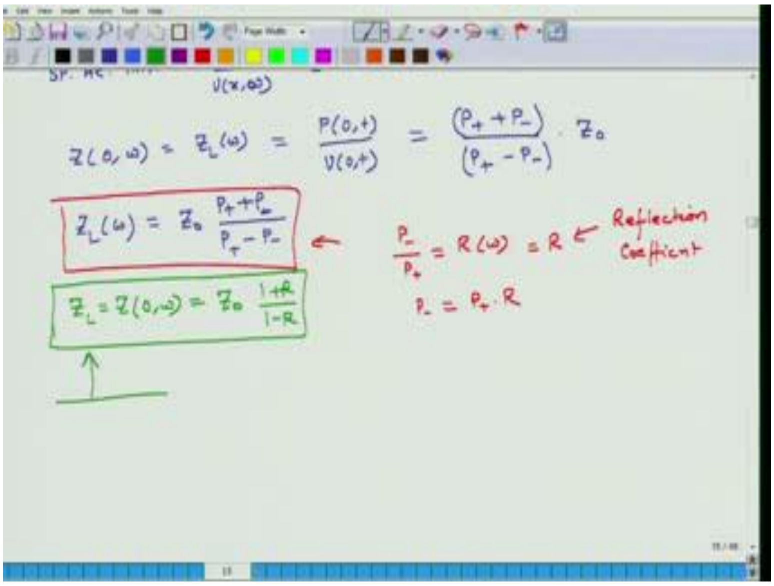

So, I can also express p minus divided by p plus as some number R, and the value of this R it can change with frequency because p plus depends on frequency we had seen originally that p plus and p minus are functions of s which is the complex frequency. But for purpose is the brevity, I will just write it as R, but R can always vary. It can always vary with respective to frequency.

And this R is known is as reflection coefficient. So, what I can do is, I can again use is the same expression that p minus is same as p plus times R and I can put this back here to get a new relation and that relation is Z L and again I am omitting omega just for purpose of a brevity Z L equals z naught p plus 1 plus R divided by 1 minus R and p plus is also in the denominator I can erase both of these things. And what I am going to do is I am going to erase the whole thing and write a simpler expression and that is Z L equals Z at the origin equals Z naught 1 plus R divided by 1 minus R .

So, this is the fundamental equation when we have an imperfect termination, and what this tells us is that I can calculate the value of R if I know Z L unless I know the conditions at the boundary, I can solve that thing. So, if I know the conditions at the boundary and if the impedance of the material which is used to plug in Z L, then I can use this particular relations Z L equals Z naught times 1 plus R over 1 minus R to compute the value of reflection coefficient. And once I know the reflection coefficient, I know the proportion of p minus and p plus once I know the proportion of p minus and p plus I can use that relationship, and use the other boundary condition at the end which is getting exited to compute p plus and p minus.

So, this is what I wanted to share in this particular module and what we have done today is introduced one more parameter known as reflection coefficient, and we have established the relationship between reflection coefficient and the terminating or terminal impedance in a tube, which is a Z L and the relationships says that Z L equals Z naught which is the characteristics impedance times 1 plus R over 1 minus R. Please bear in mind that R as well as Z L can vary with respect to function with respect to frequency. So, if you are handling these parameters you have to (Refer Time: 14:27) of this.

So, with this we close this particular lecture and tomorrow we will continue this discussion and we will actually learn as to how we can actually measure impedance, if we do not know using some actual instruments.

Thank you.