Basics of Noise and Its Measurements
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Lecture-16
A Semi-Infinite Tube and Overall Summary

Hello again, welcome to Basics of Noise and its Measurement.

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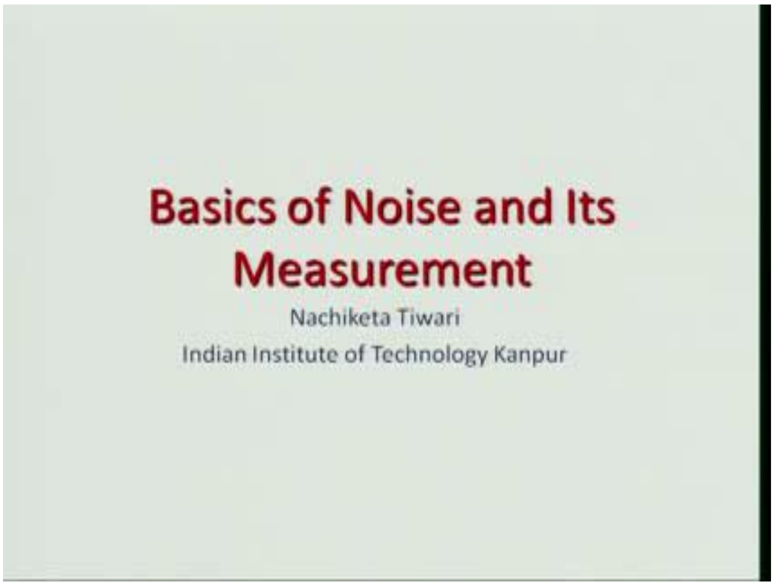

This week we have been discussing the transmission line equation and using it we have been solving for propagation of sound waves in 1-D tubes and till so for, what we have accomplished is that we have been able to solve for some propagation in a close tube as well as in an open tube. Today what we will do is essentially we will cover two things. The first thing is we will see as to how sound propagates in an extremely long tube and mathematically we can call it as semi-infinite tube. So, this tube goes on forever, and it does not have a terminating point. So, how does sound travel in such tube? So, that is one, and the second thing is that we will introduce today this concept of specific acoustic impedance. So, earlier we had talked about another impedance you known as characteristic impedance and it was defined as z naught and we found that its value was rho naught times c, and did not change with respect to position or time and it was property of the medium.

And today we are going to define type impedance, specific acoustic impedance and see what we make of it. So, once again, welcome to today's lecture and we are going to discuss the first start is the semi-infinite tube.

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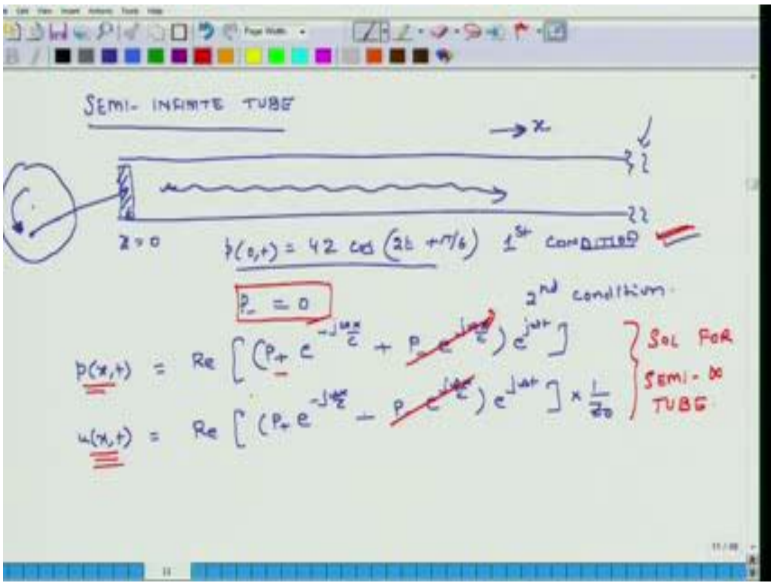

So, semi-infinite tube, so the tube would be something like this, and what these terms these symbols designate is that the tube is keeping on going in the positive x direction forever. So, it does not terminate at any point at the other end. So, it has a fixed end at the other point. So, its length is infinite in positive direction, but it has a beginning. It has a beginning, but it has no ends. So, that is why we call it as a semi-infinite tube and at this end what we are doing. So, this is again exiting it, using piston. So, what we know is that. So, this is, in this case let us say x is equal to 0. So, in this case this is my positive direction, and the tube has infinite length in the positive direction and then x is equal to 0. We know that the pressure is given by p at 0 t is given by some number 42 for instance cosine of 2 t plus 5 over 6.

So, when we write transmission line equations for this, the equations will be same and as discussed earlier is when as explained earlier in the case of this close tube. We would have four unknowns and two equations, and to solve for 4 unknowns, we need 2 extra conditions. Now, one condition we know is, that at x is equal to 0 the pressure is this. So, this is my first condition, and this is basically boundary conditions at x is equal to 0.

Now, the other in the case of close tube, and an open tube we had also a known boundary at the other end, but in this case; because, the tube is extremely long, infinitely long. It has no other end at has only one end. So, there is no other boundary condition. So, then what would be the second condition, because without prescription of the second condition, we cannot find all four unknowns. Because, now we have only two transmission line equations this is the third equation, but then what is the fourth condition?

The fourth condition we can infer based on the fact, that the length of this tube is infinite. So, and then again you have to look at the physics of this problem. What is happening is that you have a piston at x equal to 0 and it producing sound. And the sound is traveling, and we have seen that the solution for pressure wave could be that there is a wave propagating in the positive x direction, and there could be wave propagating in the negative x direction.

Now in this case physically, because the tube is extremely long. There is no wave propagating in the negative x direction, because the sound it gets generated here, and it just keeps on traveling. And it does not hit a boundary to reflect back in case of close tube, or an open tube there was boundary. And at that boundary, the sound was getting reflected back. Here that reflection is not happening. So, that reality has to be now expressed as a mathematical relation and that will be that my P minus which represents the complex amplitude of the backward traveling wave, that is equal to 0. So, this is my second condition.

Please note that this second condition is not a boundary condition, but it is the condition which reflects the reality of the fact, that the length of the tube is infinite in positive x direction. So, these are the two conditions and with these two conditions, we can now solve our p n u. So, once again I will write down the transmission line equation. So, p (x, t) is equal to real of p plus e minus j omega x over c plus p minus e j omega x over c e j omega t and u of x t is equal to real of p plus e minus j omega x over c minus p minus e j omega x over c e j omega t times 1 over z naught and because; p minus is 0. So, these are general equations, but because p minus is 0. This thing goes away, and this thing also goes away, now with this p minus terms gone. These two equations represent solution for semi-infinite tube.

Now, we have two equations, and we still have 3 unknowns P plus this function is unknown and u of x t is unknown. So, we have two equations, 3 unknowns. So, the third condition is this one, and we will use this condition to eliminate and solve for all of these unknowns.

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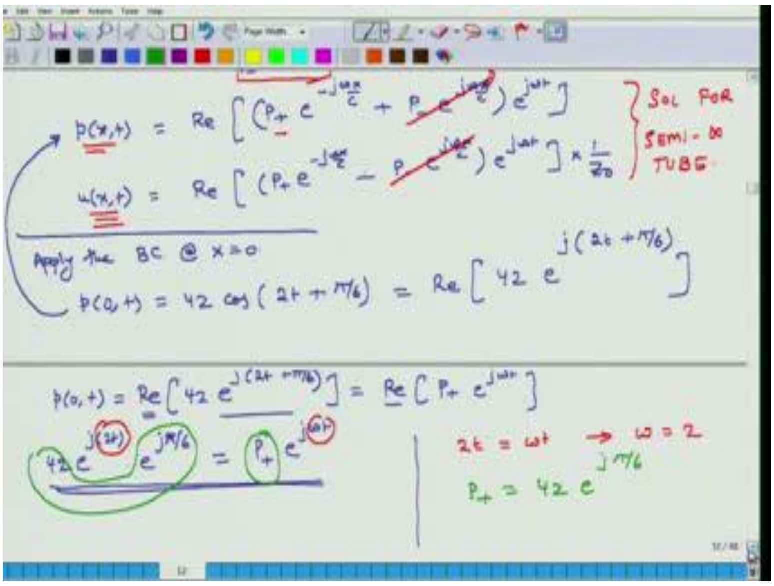

So, that is what we have going to do. Now, what we do is we apply the boundary condition, at x is equal to 0 and that is p of 0, t equals 42 cosine of 2 t plus pi over 6 and this, I can write it as real of 42 e j 2 t plus pi over 6. So, now, I applied this thing in the equation for pressure and what I get is p of 0, t is equal to real of 42 e j 2 t plus pi over 6 and that equals real of p plus e minus j omega and the value of x is 0 in this case. So, this term goes away. So, it is p plus e j omega t.

I am going to expand this side further, and rewrite this equation. So, and I will omit this thing called real. So, what this means is that 42 e j times 2 t times e j pi over 6 equals p plus e j omega t. What this means? So, this equation has to be valid for all values of time. It has to be valued for all values of time. That will happen only if this 2 t term is same as omega t which means 2 t should be omega t implying that omega equals 2. The second condition is that this term in green should be equal to p plus, which means p plus equals 42 e to the power of j pi over 6. Now, we have a specific solution for semi-infinite tube which is a specific because it corresponds to this boundary condition.

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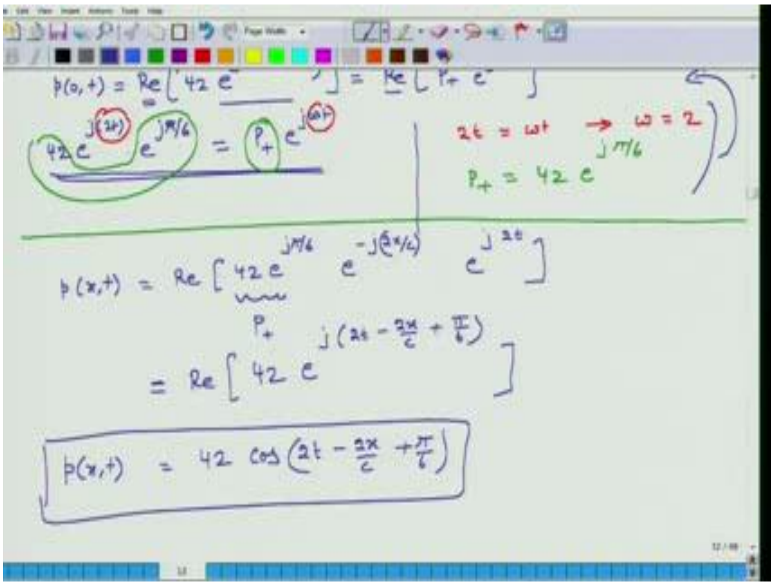

So, with this simplification my solution is p of x and t. So, what I do is, I put these things back in the expression for pressure. So, p of x and t is equal to real of 42 e to the power of j pi over 6. Because this thing is p plus right times e to the power of j omega x over c and omega will be 2. So, I am going to erase it. j time two x over c I will make it clearer minus j times 2 x over c where two is the value of omega times e j times 2 t real of 42 e to the power of j 2 t minus 2 x over c plus pi over 6. So, the value of pressure is p of x and t equals 42 cosine of 2 t minus 2 x over c plus pi over 6. So, that is my pressure function.

Similarly, I can compute the value of u as a function of t, because here the expression for u is given, and all I have to do is substitute p plus by 42 e j pi over 6 and omega by 2 and

I will get an expression for u also. So, that is the solution for a semi-infinite tube. So, what are the important characteristic for these tubes that for semi-infinite tube the value of p negative that is the complex amplitude of backward traveling wave is 0. For a close tube the value of u at the closed end is 0 and for an open tube the value of p that is pressure at the open end is 0. So, if we are apply these three conditions then we should be able to solve for the pressure velocity for all these three tubes. So, at this stage I would like to introduce a new term.

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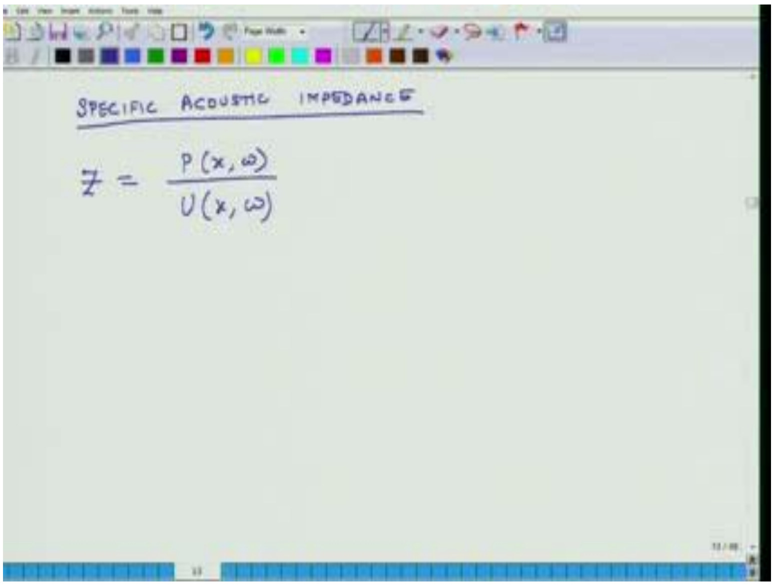

And it is called specific acoustic impedance and this it is designated by term z, and this is nothing, but the ratio of complex pressure at any point in a tube. So, this definition of acoustic impedance right now is for one dimensional wave only. So, it is the amplitude of complex pressure at location x and at corresponding to a frequency and also, this ration of pressure and velocity amplitudes. Complex pressure amplitude, and complex velocity amplitude and these amplitudes can vary with respect to frequency and also they can were vary with respect to position.

So, now what I am going to do is I am going to, now you have seen how to calculate the value of u and p for open, closed and semi-infinite tube. So, what we will do is we will develop a table which lists the characteristic impedance as well as the specific acoustic impedance for all these three tubes.

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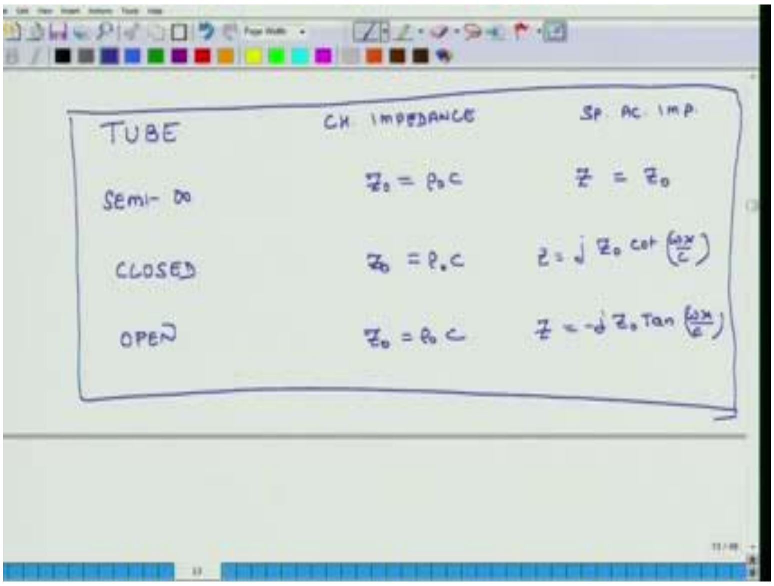

So, you have tube and the first one is semi-infinite the second one is closed and the third one is open. The characteristic impedance is z naught for all the three because it does not change it does not depend on the system it is a function the medium. Depends on the medium only and its value is rho naught c. Where rho naught is the density and c is the velocity of sound in the medium, but specific acoustic impedance that is defined as z and if you do the math for semi-infinite tube you will find that, the specific impedance specific acoustic impedance for semi-infinite tube comes out to be z naught.

For closed loop, for a close tube it works out to be j times z naught cotangent of omega x over c. For an open tube it corresponds to j times minus j times z naught tangent of omega x over c. So, this is an important table. What this says is that the specific acoustic impedance actually depends on the boundary conditions of the system. If it is a semi-infinite tube it same as z naught. If it is a close tube then it depends on the cotangent of omega x over c. If it is an open tube then it depends the tangent of omega x over c. So, this is the overall summery and discloses this particular lecture and what we do will in the next lecture is a continuation of this and now we will start talking about imperfect terminations so.

Thank you very and I look forward to seeing you tomorrow bye.