Basic of Noise and Its Measurements
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Lecture - 15
Planer Waves in 1-D Open Tubes

Welcome to Basics of Noise and Its Measurements. I am Nachiketa Tiwari and this week, we are having a discussion on transmission line equations, and how they can, these equations can be used to solve sound propagation problems in one-dimensional tubes. And today, what we are going to do is apply these equations to solve for the nature of sound propagation in an open tube. So yesterday, we have discussed a closed tube; today we will have another example, and we will discuss how sound propagates in an open tube. So, planer waves in 1-D open tubes, that is what we are going to discuss.

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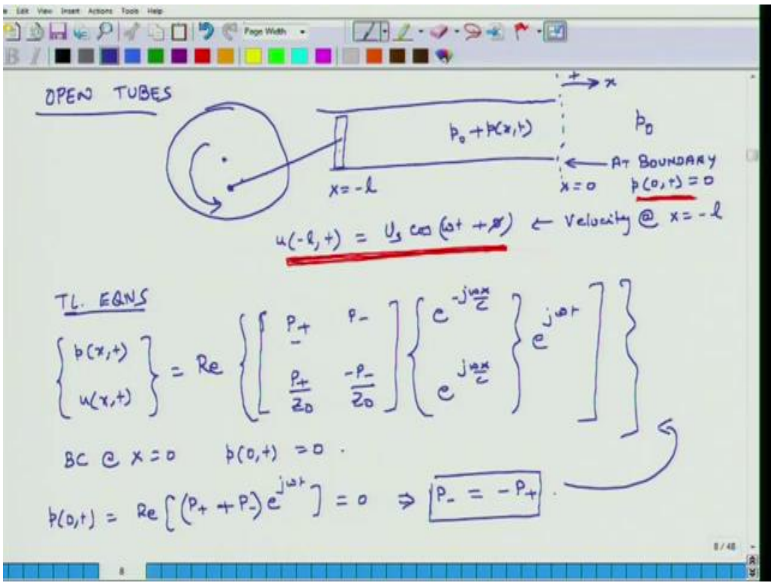

So, open tubes. So let us formulate the problem. Suppose, I have a tube and it is having a piston at one end and this piston is getting excited by this crank mechanism. Let us say this is my X equals 0; so it has one open end at X is equal to 0. And the positive x is in this direction and the length of the tube is l, so here at the other end were the piston is present X is equal to minus l. In the case of closed tube, we had said that we know the pressure at X equals minus l. Today, we will make a little different assumption that we will say that because I know how much piston is moving, so we know the value of U s or the source velocity. Because this is where the source of sound is, so we know the velocity at the source, so velocity at minus l at any time is given as some constant U s this is the real number cosine of omega t plus phi. So, this the velocity at X is equal to minus l.

Once again, we have two boundary conditions, there is an open tube at one end of the tube and then there is the other end X is equal to minus l where we know the source. Now, at the open end, we know because the end of the tube is open here, we have to now convert this physical situation into some mathematical relationship. So, we know that the ambient pressure here is p naught, because you have over all atmosphere is there, it is a big you know its p naught is there. And inside the tube, at all points, we have p naught plus p as a function of x and t.

So, the pressure inside the tube is p naught plus small perturbation in pressure that is lower case p, which is varies with x and t. And outside the tube the pressure is p naught and what that means, is that as I approach this boundary then the magnitude of p has to becomes less and less and add the boundary, p of 0 and t should be 0. So, this is one boundary condition from mathematical standpoint. That when you have an open tube, at its open end, in this case the open end is at X equals 0, the value of p which is the perturbation in pressure is 0.

Then the other boundary condition is this one; this is that I know the value of velocity - particle velocity at X equal to minus l and that is same as the velocity of the piston which is U s cosine omega t plus phi. So these are the two boundary conditions and just like we solved in case of four unknowns in case of closed tube. We again have four unknowns, two equations and then the two extra conditions are these two boundary conditions. So, once we apply these two boundary conditions, we get the solution for the system. So, we will do the same thing or the similar thing which we did in the case of closed tubes. So, my transmission line equations we begin by writing this transmission line equations and that equals, so these are general transmission line equation which are valid for all 1-D planer flow situations.

Now, we are going to apply these boundary conditions, and we will get transmission line equations which are specific for closed tubes. So, we apply the first boundary line conditions boundary condition at X equal 0; and at X equal 0, p of 0 t equals 0. So, when I put this equation in the first in the equation for pressure, then what I get is p of 0 t equals real of p plus and e minus j omega X over c at X equal 0 is 1, so it is not there. Then I have plus p minus times e j omega X over c, again since X is equal to 0, e j omega X over c is 1, so this there e j omega t, this is equal to 0 and this is possible only if p minus so these gives us the condition that p minus has to be equal to minus of p plus. So, I put this condition back in my transmission line equation.

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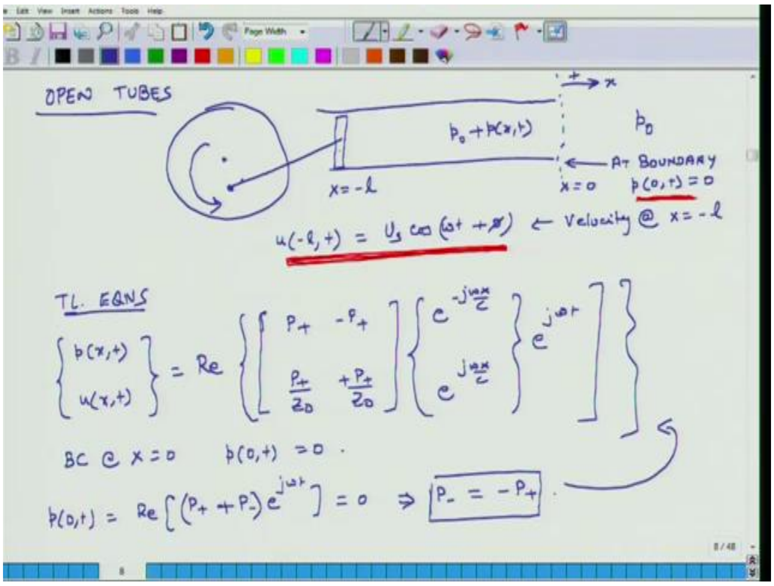

And I replace p minus with negative of p plus, so I am left with and here I get positive of p plus.

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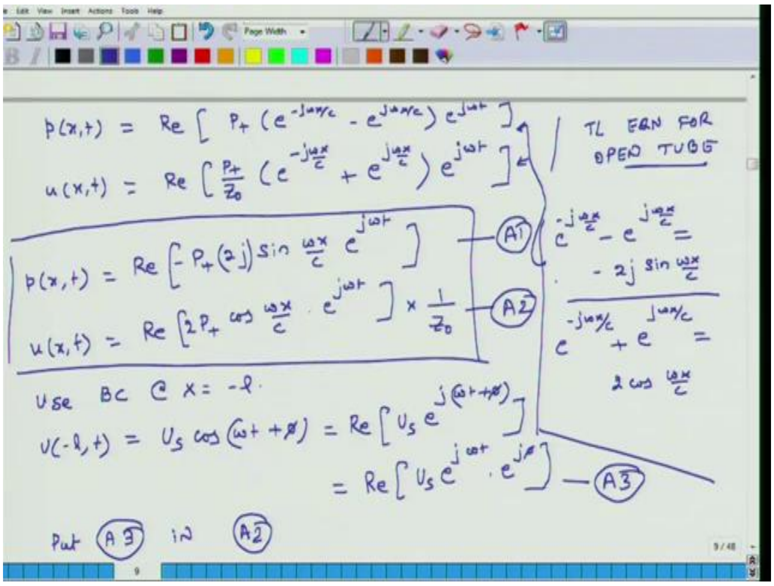

So, now, I have used one of my boundary conditions and what I will do is I will again rewrite these equations and this is equal to real of p plus e minus j omega X over c plus, excuse me, minus e j omega X over c e j omega t. So this is the equation for pressure; using this modified transmission line equation, and the equation for velocity is real of p plus over z naught e minus j omega X over c plus e j omega X over c e j omega t. These are the transmission line equations for open tube; does not matter what is the condition at the other closed other end where the piston is moving back and forth, but as long this end is open, these equations are good.

Now, again we use the second boundary condition, but before we do that we will simplify these equations further. So, we know that e j omega X over c negative minus e j omega X over c, and this equals minus 2 j sine omega X over c. Similarly, we also know that e minus j omega X over c plus e j omega X over c, actually I am going to make this a little clearer, so this equals 2 cosine omega X over c. So, I put these expressions back in these two relations; so, my expression for p x t becomes real of p plus 2 j sine omega X over c e j omega t and my expression for u simplifies to real of twice of p plus cosine omega X over c times e j omega t. Then of course, I have to multiply by this by 1 over Z naught. So these are my modified transmission line equations for an open tube. To calculate the value of p plus, I use the other boundary conditions which exits at the X equal to minus l and get the value of p plus and that is what we are going to do.

So, we know, so now, we use B C at X is equal to minus l, and which is that u minus l t equals some real number U s cosine of omega t plus phi. These I can write it in complex format nothing but has real of U s e j omega t plus phi, and this can be furthers modified as U s e j omega t times e j phi. So let us call this equation A 1; let us these call this equation A 2; and let us call this equation A 3. So what we do is we put A 3 in A 2, and we solve for p plus and that is what we are going to do now

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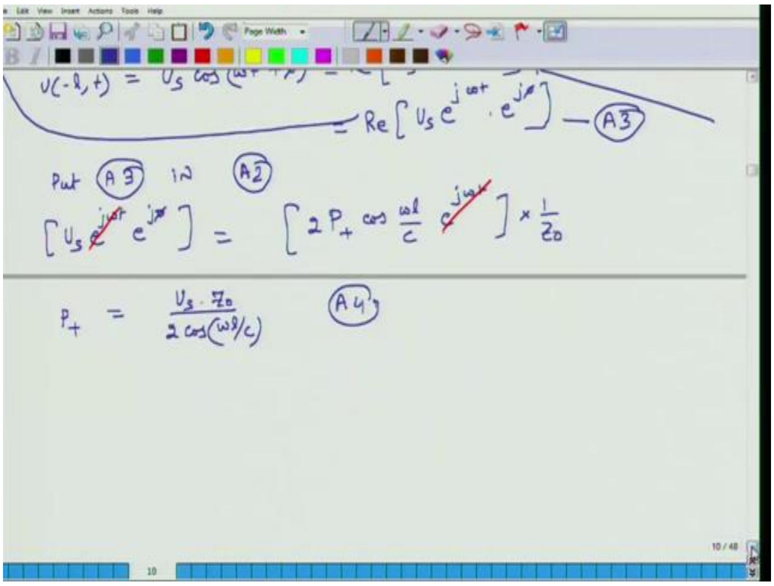

So this thing, it goes on the left hand side, so what I get is real of U s e j omega t e j phi equals real of 2 p plus cosine. Here, the value of X is minus l and cosine of minus omega a l over c is same as cosine of omega l over c. So this becomes cosine of omega l over c e j omega t times 1 over Z naught.

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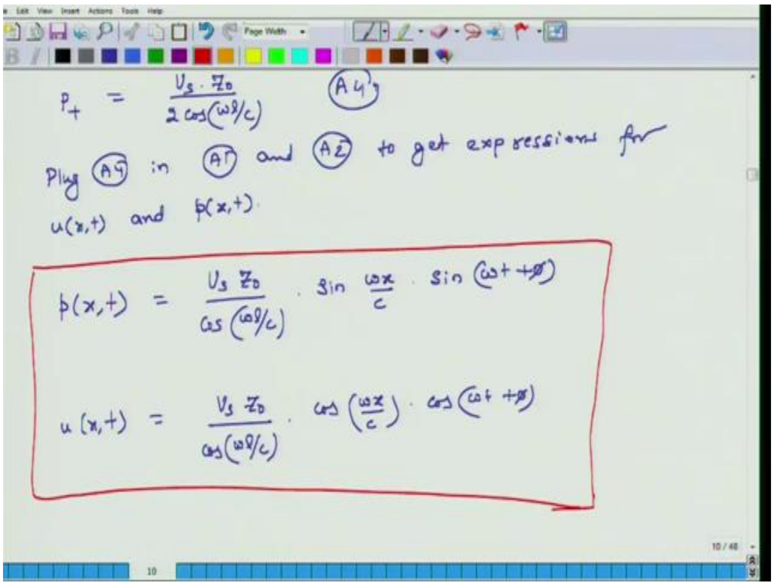

So, if these two sides have to be equal then the terms in the parenthesis have to be same, so I can drop this expression real and then e j omega t and e j omega t cancel out, and what I am left with this that p plus equals U s times Z naught divided by 2 cosine of omega l over c. So, this is my A 4. Now, what I do is I plug equation A 4 in A 1 and A 2 to get expression for pressure and velocity. So, what we do now is plug A 4 in A 1 and A 2 to get expressions for u of x and t, and p of x and t. If we do all that and if you do the math carefully then what I am going to do is I am going to write down the final relations, then the final relations look like this. So pressure is nothing but U s Z naught over cosine omega l over c times sine omega x over c times sin omega t plus phi; and u of x and t is equal to U s Z naught over cosine omega l over c times cosine omega x over c cosine omega t plus phi.

So, once again, so these two equations represent propagation of sound in an open tube; and these are the functions for pressure and velocity. As we saw, in the case of closed tube, here also we have standing waves, but the difference is that in the case of closed tube, the standing wave for velocity had a value of 0 at the closed end; and the pressure was maximum. In this case, the pressure at open end is 0; so it is minimum it is amplitude is minimum, but the velocity is maximum at X equals 0, so this is the fundamental difference between these two standing waves. So this closes the treatment of an open tube. Thanks for listening to this lecture, and I look forward to seeing you tomorrow.

Thank you very much.