Basics of Noise and Its Measurements
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 12
Complex Time Signals and Transfer Functions

Hello, welcome to Basics of Noise and its Measurements. Today is the last day of the second week of these lectures. In the earlier part of this week, what we have essentially done is develop the 1-D wave equation, also develop its generic solution and using that generic solution, we have been able to interpret the physical meaning of this constant C which appears in the wave equation. And we explained that this C actually, indeed represents the velocity of sound as it propagates through an elastic inviscid medium. What we are going to do today is cover two important concepts, and these concepts are also important, because where and when you will be learning more about sound related measurements, these concepts will come in handy and they will be very frequently used.

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These two concepts are complex time signal and transfer function. So both of these are important concepts; they are not very difficult to grasp. And I will try to explain them, and also help you understand their physical significance, and that is what we are going to do today. So, the first concept which I have mentioned is known as complex time signal and essentially it is a mathematical concept. So mathematical concept and then we actually interpret it in the real world, and then relate physical quantities to it. So we will discuss this.

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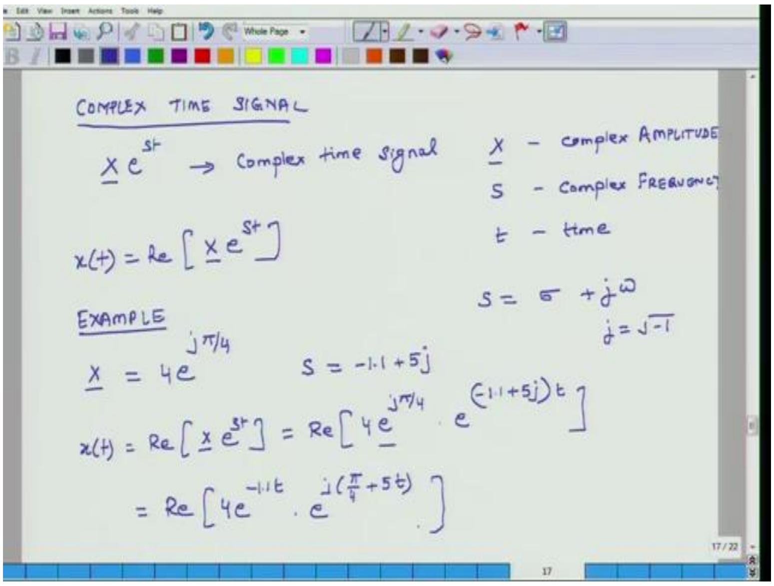

So this is complex time signal. So, what does it mean first thing is that it represents mathematical number which changes with time, so it is a series of numbers in digital domain; or if it is an analog, it is doing changes continuously. So it is a time signal, it is a signal which changes with respect to time. And the other feature of it is that, it is complex. So, if we have suppose a number 2, it is a real number; you have a number 3, it is also a real number; minus 5 - real number; 5 - real number, but these numbers this time signal they it can be complex in nature, so it can have an imaginary components as well as a real component and because of this, as it is known as a complex time signal and at various as a function of time.

So, a very generic representation of this complex time signal is like this. So here X e to the power of s t so this is a complex time signal. Here, X can be complex number, so it can have a real as well as an imaginary component, and then S can also be real complex. This X bar is actually a complex number, and it is actually referred as complex amplitude. So, both X bar and S, both are complex numbers. X bar is known as complex amplitude and S is known as complex frequency.

So, a complex time signal has two complex numbers embedded into it, complex amplitude and complex frequency, and t is time is time. So you can have complex pressure, you can have complex voltage, you can have complex current, you can have complex velocity and it all these time signals could be complex entities. But, if you want that entity to be mapped into the actual pressure or actual velocity or actual then you take its real component. So real, excuse me, so suppose this X e s t represents complex pressure then lower case x t will represent real actual physical pressure and that will be real of X bar e s t. The other thing I wanted to mention is this s is complex frequency, so it can have a real component sigma and an imaginary component omega. And j is equal to square root of minus 1, so it is a square root of negative 1. So the real component is sigma and the imaginary component is omega.

So let us look at an example. So consider that X bar as I mentioned it is a complex number, so I can assign it any value. And in this case, I am going to call it as 4 e to the power of j pi over 4. And then my complex frequency, I want to choose, and in this case its value is 1.1 the nothing significant at least currently plus 5 times j. So then the complex time signal will be X times e to the power of s t. And the real time signal will be x t will be real value of X bar e to the power of s t, which is equal to real of 4 e j pi over 4 times e to the power of s t, so minus 1.1 plus 5 j times t. So, this is equal to real of 4 e to the power of minus 1.1 t, what I am doing is I am separating in you know complex and real parts separating them out. So this is one component times e to the power of j and then I take pi over 4 from this one and then plus 5 t from here.

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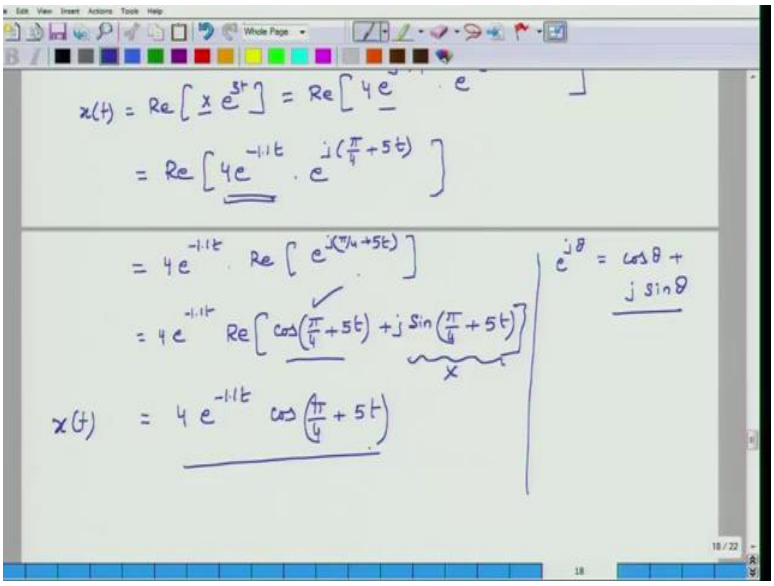

And this is entirely real, so I can take it out, so it is 4 e to the power of minus 1.1 t times real component of e to the power of j pi over four plus 5 t. Now we know that e to the power of j theta equals cosine of theta plus j sine of theta. So I use this into this, so what I get is 4 e to the power of minus 1.1 t times real of cosine pi over 4 plus 5 t. So this just to make it clear, this is 5 t plus j sine of pi over 4 plus 5 t. And this is nothing but 4 e to the power of minus 1.1 t times cosine of pi over 4 plus 5 t, because this is the real component, this is the imaginary component; so, I take this, I do not take this one. So, my x t is the real component of that complex time signal and its value is this. So, now, what I am going to do is give you a physical interpretation of sum of these numbers and, but before I do, what I will do is I will actually plot.

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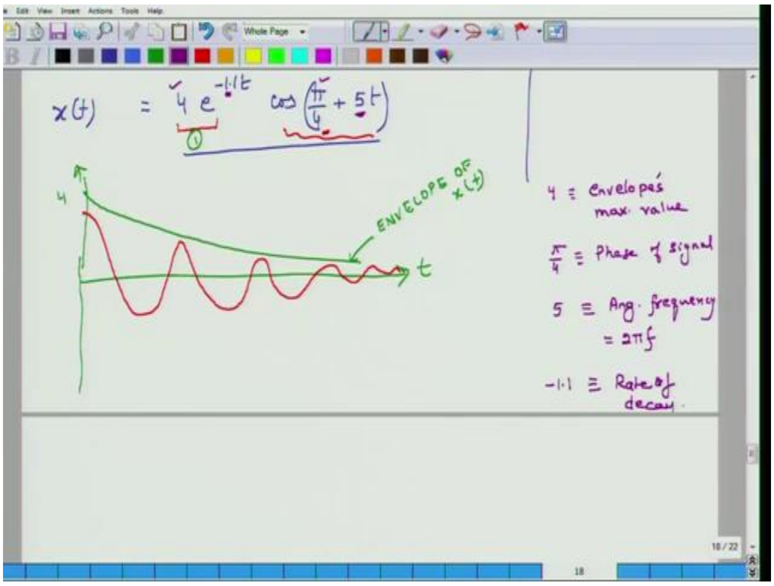

So, what you see is that you have this term here, and then you have this term here. So what I am going to do is I am going to plot, in my x-axis, it will be time; and on my y-axis, I will plot some magnitude. So, first I am going to plot this thing. So time t is equal to 0, this value is 4, and it goes like this. So this thing is called envelope of x t. Why is it called envelope, because the whole function x cannot go beyond this green curve, and the actual curve will actually do this, if we do something like this. Because there is this pi over 4 difference here, the actual curve - this is a red curve does not touch the green curve at x is equal to 0. If there was no pi over 4 in this case, then it would have touched it, but because of the presence of pi over 4, it does not touch it.

And then it will be important to make some observations. So, what does this tell? That 4 it corresponds to envelopes maximum value. Where did this 4 come from, it was a part of the complex amplitude portion. Then what else, pi over 4, it corresponds to phase of signal. Where was phase of signal embedded, it was again a part of complex amplitude. Complex amplitude was 4 times e j pi over 4, so the complex amplitude defines the maximum value of the envelope, and it also defines the phase of the signal. Then you have this term this number 5, so we have discussed this we have discussed this.

A number 5, what does it correspond to, so you see this 5 is here. And this is nothing but angular frequency, and this is same as 2 times pi times f. And where did this pi come from it came from complex frequency complex frequency was defined as minus 1.1 plus 5 j. So the frequency - angular frequency is nothing but that pi, it is coming from there. And finally, you have this minus 1.1, which is this thing. What does this imply? It tells you, how fast this green curve is decaying, so this is rate of decay.

Where does this comes from, it also comes from complex frequency. So complex frequency influences two things; one is the angular frequency of the system and also the rate of the decay. And the complex amplitude, it influences two other things; and those two things are envelopes maximum value and the phase of the segment. So this is important to understand because when we do measurements, you will come across some of these terminologies and parameters, and you should be able to relate to these parameters.

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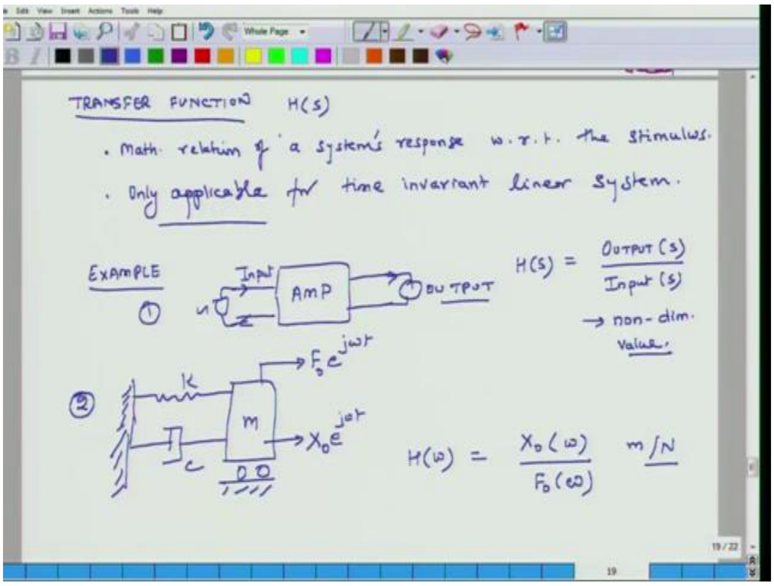

The second term I had mentioned is known as transfer function. And lot of times, we write it as H of s, s is your frequency. So what is transfer function, it is basically first thing is it is a mathematical representation, I am sorry mathematical relation of a systems response with respect to the stimulus, this is the first feature. What does it do, so suppose

I have a motor, and I throwing some current, so what is the stimulus, the current is the stimulus. What is the output or the system response, the motor is rotating at some frequency, so it is a relation, which connects the stimulus or the input to the output which are the parameters in which I am interested in, so this is one. But then this transfer function is only applicable for time invariant system - time invariant linear system. So it is not invalid for time changing system.

So, what is time invariant system? So, once I again to take the example of a motor, if I excited so with some voltage then initially the speed of the motor is 0, and then it is slowly or within a small finite amount of time, it increases and becomes stationary, their speed becomes constant, if constant current is flowing through. So, then the systems performance is not changing with time, it is either harmonic or it is a steady state system so that is what is time varying. If it is fluctuating up and down, up and down, or arbitrarily fluctuating the or if it is a transient, if it is not in a steady state, if it is in a transient state then this transfer function is not applicable.

The other qualification is that is only valid for linear system. So what is a linear system in general, what it means is that if I double my input then my output also will proportionally go up. So whatever is the proportional increase, there is also some correlation between input and output; and that relationship is that of a straight line. It may not be double, but it may not be, but it is the factor between change in input and change in output is constant. So, if you plot the input and output on x, y graph, you will get a straight line. And so this is the case.

So, we will take two or three examples, things will become clear. So, first we will take the example of an amplifier. So amplifier, what I am giving it is some input, and I am measuring some output. So, let us I am providing exciting it with some voltage and providing an input to it, and it is running on some power source also, and I am measuring the output voltage to it. So, then my transfer function, and it will be a function of frequency, it will be output. And if I am interested in how much voltage it is generating then it will be in volts, and it will be a function of frequency divided by excitation voltage. So, here the ratios the input and output quantities which have been measured are same voltage, so this is a non-dimensional value, which changes with frequency. So this is one example.

The other example could be I could have a spring mass dashpot system. So I have a spring mass dashpot system; mass is m, damping is c, stiffness is K. I am exciting it with force e j omega t. And I am interested in finding out how much it moves, so let us say it moves by some amplitude X naught e j omega t. Then my transfer function the excitation or the stimulus is been provided by F naught, so here and it can change with omega, so this could be a function of omega, and that is basically so stimulus is from force and what I am measuring is the displacement, so X naught which is the function of s or actually omega here divided by F naught omega. So, in this case, the units will be meters per Newton, so transfer function does not have to be necessarily unit less.

A third example could be I am having a loud speaker, and it is getting excited by a voltage source, and the voltage can change for different frequencies. And what I am interested in measuring is how much sound pressure level, it is generating the loudspeaker and I can measure it in decibels. So, in this case, the transfer function will be decibels per voltage. And again this could be a function of frequency; it could be complex frequency or actual omega. But, the most important the most important thing to remember about such functions is that, they are valid only if the system is in steady state; and second if the system is linear; both of them has have to be true. So that covers my coverage of these two concepts, transfer function as well as this complex time signal and I hope you had a wonderful day today.

Thank you very much, and we will meet once again next week.

Thank you. Bye.