Basics of Noise and Its Measurements

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Lecture 10
1-D Wave Equation

Welcome again. Welcome to Basics of Noise and its Measurements. In this continuing series at least for these week we are exploding, how the1-D Wave Equation is derived its formulation and what we are going to actually do today is combine all the three equations, which we have developed our last three sessions the Wave Equations and continuity equation, the momentum equation, the continuity equation and the gas law and synchronize them into one single Wave Equation. This is the equation which governs the propagation of wave in a fluid media, which is elastic in nature and also which inviscid nature.

It is important to understand, that is in this particular equation in the Wave Equation we assume. That the fluid is not viscus and as a consequence, because of viscus affects no heat is generated and lost in the process. So, that is what we are going to do.

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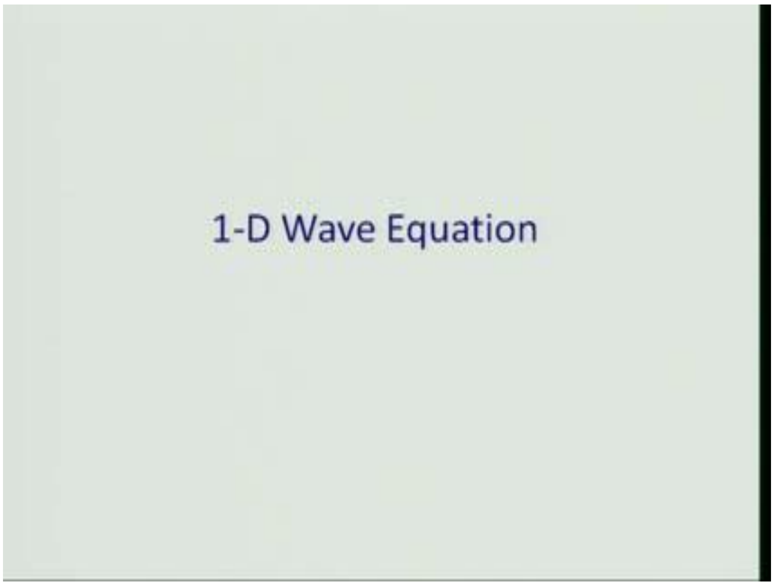

We are going to develop the 1-D Wave Equation for pressure we also going to develop 1-D Wave Equation for velocity. And once we have done that then, we will also look at we are not going to exactly develop other equation, but we will definitely look at two other forms of Wave Equations; one is the Wave Equation in three d condition frame. So, the1-D Wave Equation only gives this propagation of sound in one particular direction.

Suppose, you have long tube with a uniform cross section and sound is traveling along the length of the tube. So, this type of sound propagation is governed or detected by1-D Wave Equation for cartesian frame. So, will look at that will develop that then will also expand it into three d Wave Equation for cartesian different frame. Then we will also look at a special form of Wave Equation for this variable system, where you have a point source its emitting sound. That sound is spreading out readily in all the directions uniformly. So, that is other form of Wave Equation which we will look at and comment upon.

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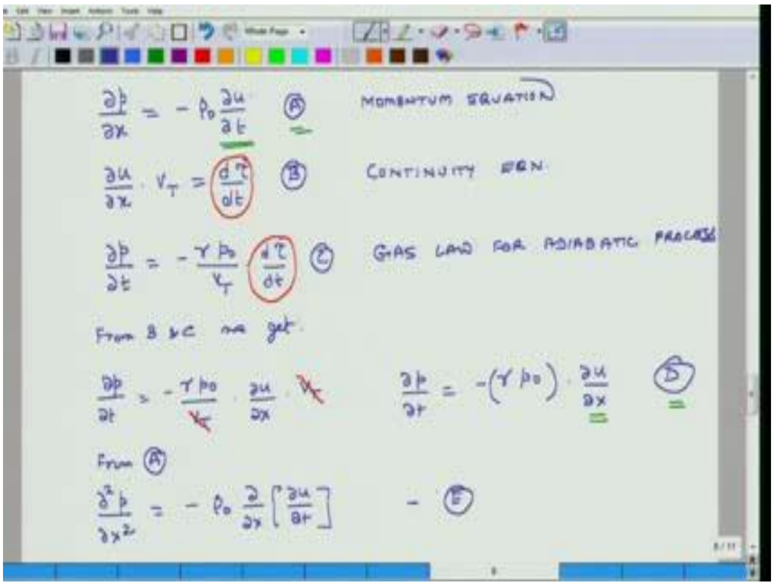

So, very quickly I will recap for the three different equations which we developed earlier. So, the first one was the momentum equation, and that is partial of pressure with respect to x equals minus rho naught partial of velocity with respect to time. So, let us number this as equation A, and this is my Momentum Equation. The second equation which we had developed was the1-D continuity equation, and that equation is partial of u with respect to x times VT equals d tau over d t and we are going to number it as equation B, this the equation for continuity. The final equation which we have developed was, for gas law adiabatic gas law, and this is partial of p with respect to t equals minus gamma p naught over VT times d tau over d t this is equation C. This is my gas law for adiabatic process that is equation number C.

So, what we are going to do is we are going to eliminate. Our aim is eliminate to u our aim is to eliminate t. From these three equations an ultimately come with one single relationships for pressure. So, when we inspect these two equations, what we see is that we have this term, d tau over d t and d tau over d t in equations B and C. So, I can eliminate d tau over d t from these two and what I can get is. So, from B and C we get del p over del t equals minus gamma p naught VT and instead d tau over d t, I can get del u over del x times VT and this thing cancels out. So, what I end up of getting is del p over del t, equals minus gamma p naught. I can put this in parenthesis times del u over del x. So, let us call this equation D.

So, the next thing we do is we compare equation D with equation A. What we see here and if our aim is to eliminate u, what we see here is, that u has been differentiated with respect to time and here it has been differentiated with respect to x partial differentiate with respect to x. So, if I have to eliminate u, then what can I do is I can, differentiate the whole of the equation A with respect to x. So, then I will get a term del u, excuse me partial of u second derivative of u with respect to x and t and I will also get a very similar term here in equation D. So, that is what I will do. So, if I differentiate A with respect to x. So, from A what I get is, I am differentiating the whole equation A with respect to x. So, I get partial of p, second derivative with respect to x is equal to minus rho naught del over del x, this is my equation E. Similarly, I am going to differentiate equation D with respect to time.

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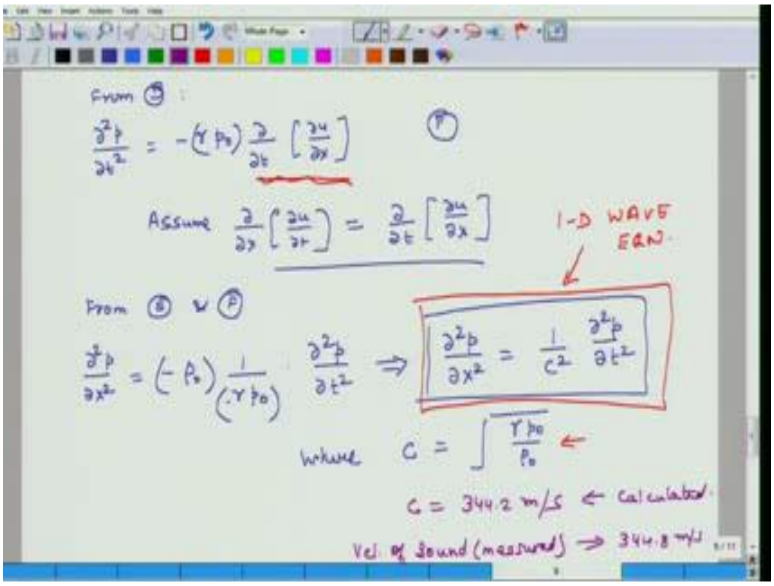

So, from D we get partial of pressure; second derivative of pressure with respect to time, equals minus gamma p naught, del of with respect to t times del u over del t. This is equation F. Now we will look at this term, and we look at this term. And we here we assume oh I think, I made an error here it should be del x.

Here, we assume that these cross derivative of u, where I am first differentiating time and then with respect to x is same as this cross derivative. And this assumption is would be valid if my u is continuous in the whole field, and also its derivatives both with respect to time and then space. They are not only the first, but also subsequent derivatives they are continuous in time and space across the whole field then this assumption will be valid.

If they are not continuous then we cannot necessary say, that this assumption is valid for instance if you have a branching a floor. So, in that kind of situation this assumption cannot be held true. Anyway, because those kinds of situation are not here, so I take these things to be the same, and then what I can do is, I can now eliminate these two terms, from the equation. So, what we get is from E and F essentially, what I get is second derivative p with respect to x, is equal to minus rho naught and instead of this cross derivative, I get 1 over gamma p naught times del 2 p over del t square.

So, what this gives me is this final equation, where c equals, so this is the Wave Equation, this is the1-D Wave Equation. The value of c is this and it just happens, if you plug in the values for air for ambient pressure density of air and the value of gamma for air, you find at the value of c, it comes out of the 344.2 meter per second. So, this is the calculated value of c.

At this stage I am not saying that the c is same as velocity of sound, but if I actually measure the velocity of sound. Then, velocity of sound measured, that turns out to be for the same ambient conditions, 344.8 meter per second. So, its look like that the value of c is pretty close to the velocity of sound. Later we will see as to why the velocity of c the value of c or the c is nothing, but actually indeed it is the velocity of sound. So, what you have seen is how the 1-D Wave Equation is derived I will right that again.

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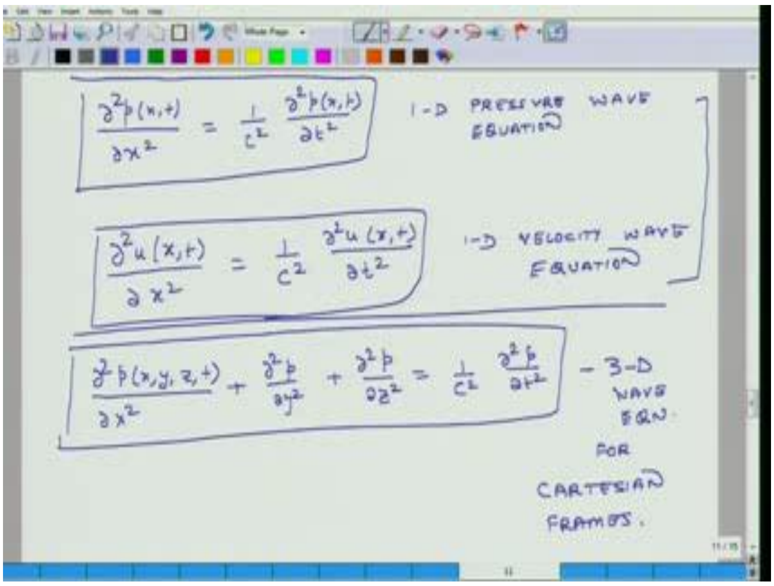

So, its second derivative of partial derivative pressure with respect to x that equals one over c square second partial derivative of pressure, with respect to time and that is my 1-D Pressure Wave Equation.

What this does is that it connects the second derivative of pressure with respect to 2, it is second time derivative. So, it connects how pressure is changing in space to how pressure is changing in time. That it is what it connects and the thing which connects these two is this constant called c. Similarly, if we do the same process, if we follow a very similar process what we did earlier. Something very similar to that then we can also develop a 1-D velocity Wave Equation. I will write that equation directly and that is. So, actually this is not v, but u.

So, this is 1-D velocity Wave Equation. So, this very much similar to pressure it connects rate of change or changes in velocity with respect to x, positional changes to their time related changes. In both these equations we have assumed, that the floors one dimensional that is the changes in x and y as 0. Second assumption is the things that perturbations are extremely small in nature. The third assumption is that the material is we have constant material particles, flowing the system. So, these are some of the important assumptions.

The fourth assumption is that the things are, there is no heat generated because of viscosity. There are no shear effects because of this. So, this is for one dimensional flow and specifically one dimensional for a Cartesian Frame reference. Now, I just wanted you to, at least have a look at three dimensional wave equations for pressure, so for three dimensions what happens is that it is pretty much very similar.

So, it is second derivative of pressure and here pressure is not just a function of x and t, but it is a function of x, y, and z and time. Then second derivative of pressure with respect to y square, and I am not going to write in parenthesis x, y, z for sake of gravity, but if you have to be mathematically correct, then it has to be there and this equals one over c square t square. So, this is my 3-D Wave Equation for Cartesian Frames and lastly I will write another relation and that is valid for spherical frame of references. So, I first write the equation.

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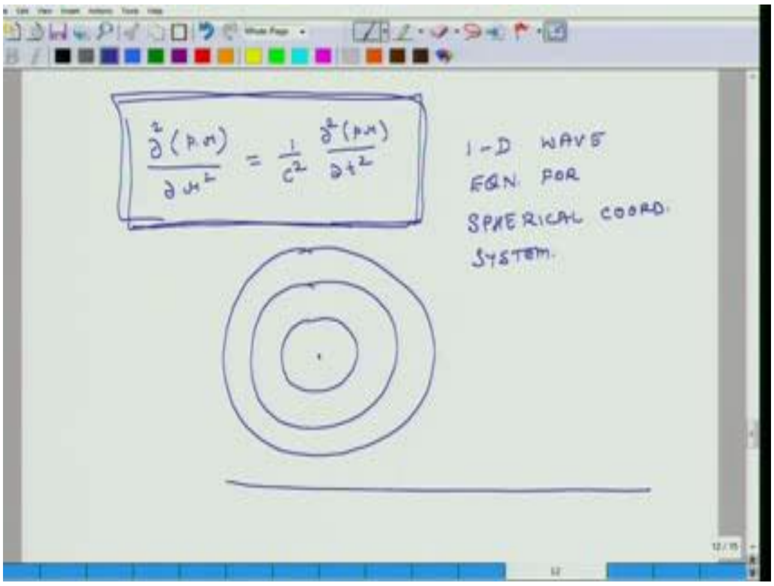

So, here instead of p, it is actually p times r over del r square, is one over c square. So, this is1-D Wave Equation for spherically and this particular equation is valid. If you have a point source or sound, and it spreads in a spherical symmetric way. So, its spreads out, excuse me I have to look at it at the center as its spreads out the progressive spreading out of these is dictated by this particular equation. So, that is all, what I intended to cover for today's lecture and will continue this discussion tomorrow look forward to see you tomorrow.

Thank you very much.