Thermal Engineering: Basic and Applied
Prof. Dr. Pranab K Mondal
Department of Mechanical Engineering
Indian Institute of Technology - Guwahati

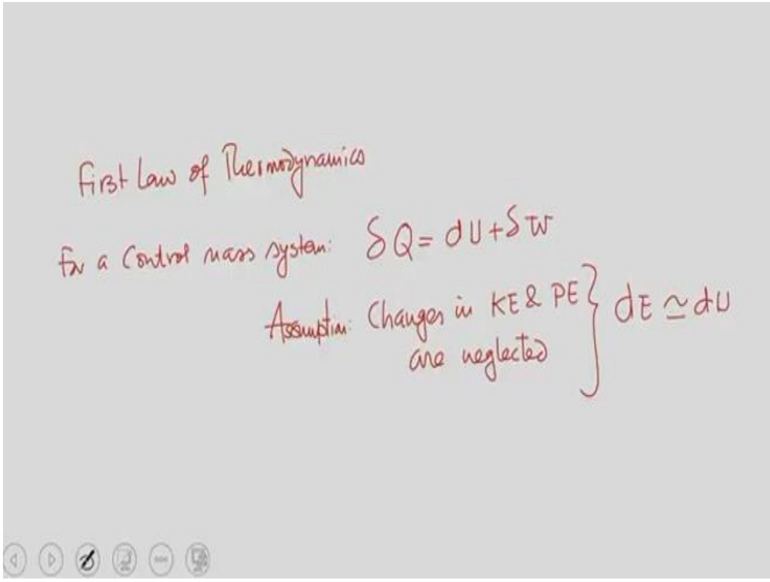
Lecture - 02
First Law of Thermodynamics for Control Volume System

I welcome you all to this session of thermal engineering. And in today's class, we shall discuss about the first law of thermodynamics applied to the control volume system. And if we try to recall that in the last class, we have established the mathematical expression of first law for a control mass system undergoing any general process. So, now, question is that having established the expression of first law for a control mass system, which is applicable to any processes, then why do we need to go for the expression of the first law which is valid for the control volume system?

So, you know that we have studied fluid mechanics and in this course, as I told you in the last class, we shall be discussing a few systems to be precise thermal systems like power system, refrigeration system, systems which are used to operate the internal combustion engines also known as mechanical systems. So, in all the systems we shall learn slowly that there is a working substance, not only the working substance is fluid; rather there will be a continuous flow of the working substance. So, in all these systems net transport of energy, either in terms of heat or work is done through the transport of a working substance which is fluid. And in fluid mechanics you know that we have learned that the transport equations are generally expressed from a control volume approach rather than a control mass approach.

Since, all the processes involves flow of a fluid transport, equations are generally expressed from a control volume perspective rather than the control mass system. Here also the underlying transport of heat and work, I would say the underlying transport of energy is or can be studied conveniently for a system in which fluid flow is involved from a control volume perspective. And that is why we should see today how you can express the first law of thermodynamics from a control volume perspective.

(Refer Slide Time: 04:13)



So, just for the recapitulation whatever we have discussed in the last class, the first law of thermodynamics for a control mass system we could write the expression

$$\delta Q = dU + \delta W$$

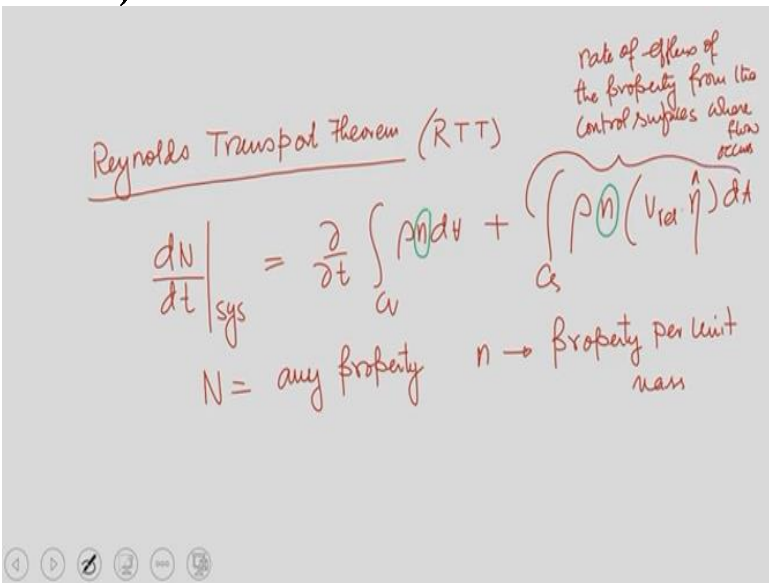
Here one important assumption is that the changes in kinetic energy and potential energy are neglected. So, because of this assumption we could write

$$dE \approx dU$$

So, this is the first law of thermodynamics for the control mass system and this is valid for any general process. It is not necessary that the process has to be a cyclic process. So, now, today as I was talking about the flow system and that is what we should discuss today. By flow system, I mean thermal systems in which flow of a fluid is involved. In fact, we shall be discussing all those systems in our subsequent classes. In these flow systems, if you would like to study the transport of energy by heat and work, then it is better rather it would be convenient to study that particular aspect if we can express the first law of thermodynamics from a control volume perspective. So, this is the first law of thermodynamics for the control mass system. If you would like to express this particular system from the control volume perspective, then a mathematical expression will be required.

So, we need to have mathematical transformation of this particular equation so that we can conveniently study the transport of energy from a control volume perspective. I am not going to discuss in details though you have studied that particular aspect in fluid mechanics. But if you would like to express the transport equations from the control volume perspective, one important theorem is used and that is known as Reynolds transport theorem. So, Reynolds transport theorem is used to express the transport equation from a control volume perspective.

(Refer Slide Time: 08:20)



So, what is Reynolds transport theory or RTT in short? I am writing the expression assuming that you have studied this part in fluid mechanics course.

Reynolds transport theory (RTT)

$$\left(\frac{dN}{dt}\right)_{Sys} = \frac{\partial}{\partial t}\int \rho n dV + \int \rho n (V_{rel}.\hat{\eta}) dA$$

But this is the Reynolds's transport equation which allows us to write the transport equation from the control volume perspective. So, you know that here

$$N = \text{any property}; n = \text{property per unit mass}$$

N is essentially an abstract it can be anything. So, it is any property.

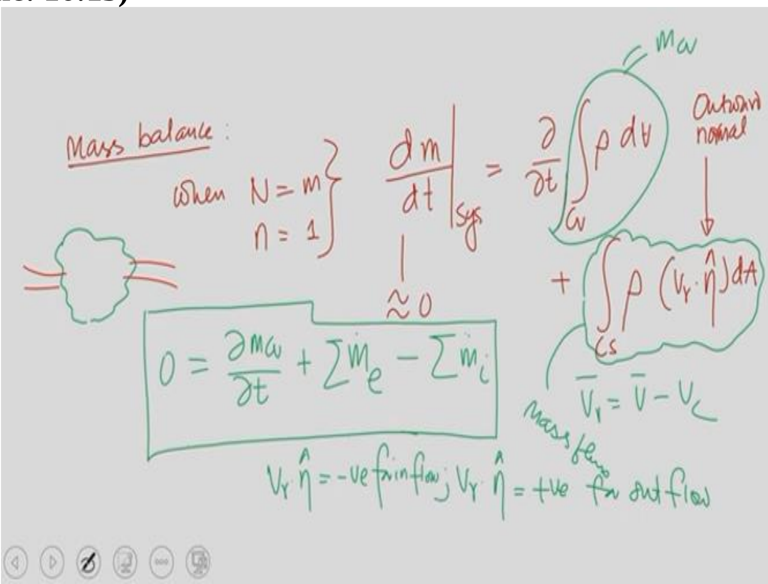
So, if we look at this particular equation, you can see that left hand side of this equation is time rate of change of the property and as I told you it can be any property of the system. So, the rate of change of any property within the system can be expressed in terms of the time rate of change of the property within the control volume plus this extra term is there. So, this extra term is the rate of efflux from the control surface through which flow will take place.

So, as I told you, our objective is to express first law of thermodynamics from a control volume perspective, why? Because all the systems that we shall study in this course is involved with the transport of any particular fluid as a working substance. So, when there is a flow of fluid, so, the net transport of energy because of the transport of fluid is essentially this term. So, basically this is time rate of change of the property within the system which can be written in terms of the time rate of change of the property within the control volume plus the rate of the efflux of the property from the control surface or rather from the control surfaces in which flow is there.

Now, as I told you, this transformation equation will be used to write the first law of thermodynamics for the control volume system. So, essentially the first law is a statement of the conservation of energy. So, for the control volume system, there is efflux through the control surfaces where flow occurs. So, now, when there is a flow, mass flux is there. So, this term essentially indicates transport of energy because of the transport of mass. In a control volume system we may have multiple inlets and multiple exits. So, through multiple inlets, there will be a flow and again through multiple outlets, there will be a flow. So, the energy transport due to the mass flow is taken care by this term.

So, essentially, if you would like to write first law of thermodynamics, we also need to have the mass balance equations. Because if you would like to write first law of thermodynamics for the control volume, the energy balance that first law essentially talks about is not balanced in an isolated manner, rather, the energy balance has to be coupled with the mass balance, why? Because mass flux is there, because of the flow and the energy transport associated with this mass transport should be taken into account.

(Refer Slide Time: 16:15)



So, let's try to write the mass balance using this equation. So, why we need to start with this mass balance; because in a control volume system, when you are trying to establish the energy balance equation, this mass balance is an integral part.

Mass Balance

$$\text{When } N = m, n = 1;\ \frac{dm}{dt}_{sys} = \frac{\partial}{\partial t}\int \rho dV + \int \rho(V_r.\hat{\eta})dA$$

Here this $\hat{\eta}$ this is a normal pointing outwards. Basically, if we try to complete this equation by doing 1 or 2 steps you know more what we can write that $dm/_{dt}$ for a system. So, essentially left hand side talks about the control mass system. So, in this case, so, a time rate of change of any property within the system that is control mass system can be expressed in terms of control volume plus the efflux term. So, by definition mass is fixed for the control mass system.

$$\text{So,} \frac{dm}{dt}_{sys} = 0; \int \rho dV = m_{c_v}$$

So, you know control volume is fixed in a reference in which velocities are described. So, this is relative velocity which is nothing but the fluid velocity minus the velocity of the control volume. So, I mean it is the definition.

$$\bar{V_r} = \bar{V} - V_c$$

Now, this term if you try to recall that this is efflux from the control surface $V_r.\hat{\eta}$ if we integrate it over the control surface and assuming that the properties are remaining uniform that means ρ is remaining uniform over that particular cross section. So, here important concept is that if we integrate this term and if we assume ρ is remaining constant over that particular cross section, I mean as we have multiple inlets, multiple outlets through which you know fluid flow is taking place. The term $\int \rho(V_r.\hat{\eta})dA$ is essentially mass flux from the control surface. So, you will know that we will be having mass flux to the inlet, we will be having mass flux to the outlet. So, if we try to write the equation, so, this would be

$$\Rightarrow 0 = \frac{\partial m_{c_v}}{\partial t} + \sum m_i + \sum m_e$$

Here *i* stands for inlet *e* stands for exit.

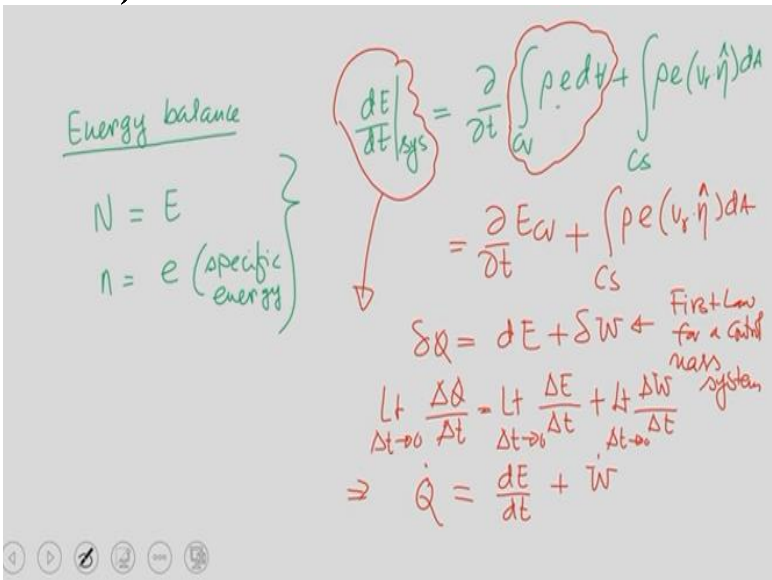
I am using the same notations which are followed in the textbook. So, they say is mass flux that is rate of mass leaving from the inlet. As we have multiple inlets and multiple outlets that is why I had used the summation sign.

Now, I would like to discuss this $V_r.\hat{\eta}$, which is outward normal. So, if we take a control volume as shown in slide and so, we have this outlet and inlet. So, you know that $V_r.\hat{\eta}$ is positive for the outlet or exit because normal is always pointing outward from the control surface. So, this unit vector is outward normal I mean this is pointing outward from the control surface. So, the dot product will be you know negative for the inlet and it would be positive for the outlet. So, if we try to change the; I mean if we try to write it correctly, it would be

$$\Rightarrow 0 = \frac{\partial m_{c_v}}{\partial t} + \sum m_e - \sum m_i$$

So, this is basically mass balance equations where e stands for exit i stands for inlet. I have discussed about the sign convention because this $V_r.\hat{\eta}$ is positive for outflow and negative for inflow. So, this is the mass balance equation and summation sign is taken if we have multiple inlets and multiple exits. So, by using this equation that is Reynolds's transport theorem we could write the mass balance from a control mass system to the control volume system.

(Refer Slide Time: 24:34)



Next we are going to discuss about the energy balance. We will be writing energy balance equation from a control volume perspective. Now, we are using this Reynolds's transport theorem. Now, N can be used to describe any property.

So, $N = E$

$n = e$ (Specific energy)

If we try to write the Reynolds transport theorem,

$$\frac{dE}{dt}_{sys} = \frac{\partial}{\partial t}\int \rho e dV + \int \rho e(V_r.\hat{\eta})dA$$

So, now it is very important that what we can do further. So, essentially this left hand side gives us a clue about the change in energy within the system. So, it is essentially the first law of thermodynamics. So, one important assumption is that properties are uniform over the respective cross sections through which flow occurs. So, this is an important assumption which is used to describe this equation.

So, you know that ρdV that is mass times e, small e I mean that may vary within the control volume, but, you know that if we multiply that mass will be getting total energy.

$$\frac{dE}{dt}_{sys} = \frac{\partial}{\partial x} E_{c_v} + \int \rho e(V_r.\hat{\eta}) dA$$

We shall come to this particular term later. Now, this gives us an information about the time rate of change of energy within the system. I would like to tell you one important thing in this context that you have studied about thermodynamics.

So, why we are calling it thermodynamics, why not thermo statics? So, in thermodynamics, the word dynamics is coming from the fact that whenever we are trying to study something or whenever we are trying to describe something, we use time rate of change of any property. So, basically if you would like to study time rate of change of anything, then only the word dynamics will come into the picture. In thermodynamics sometimes we would also prefer to write the equations in the rate form and that is why we are writing ${dE}/{dt}$ with the system that is the time rate of change of energy within the system. Since, we are trying to write the time rate of change, that is why the word dynamics is coming into the picture & so, it is not thermo statics.

So, this gives us impression about the first law of thermodynamics of a control mass system. So, basically if you would like to know that time rate of change of energy within the control mass system, then that is essentially, giving us a clue about the knowledge about the first law.

So, as I told you, first law essentially talks about the energy conservation, so, it is a statement of energy conservation. So, the left hand side term in equation, which is clearly the time rate of change of energy in the system and that is giving us knowledge about the first law of thermodynamics. So, this can we retain from the first law, which is applied to a control mass system. So, if we try to write first law applied to the control mass system,

$$\delta Q = dE + \delta W$$

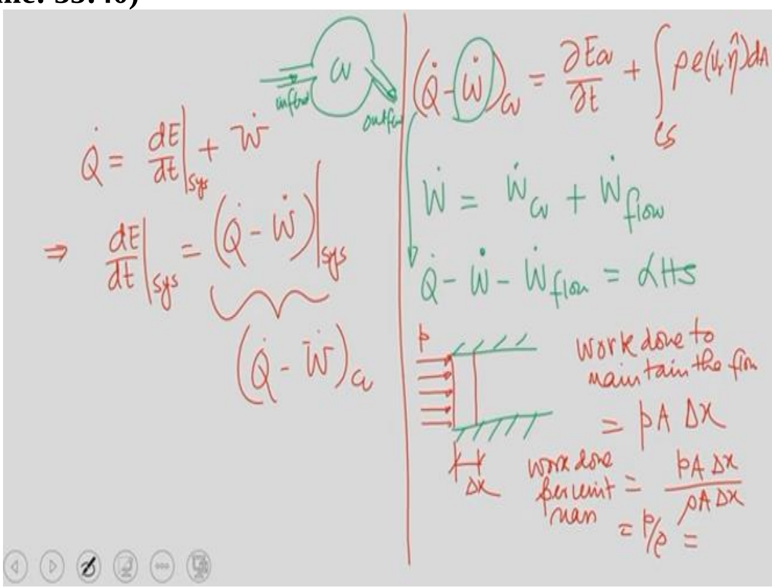
If we again ignore the changes in kinetic and potential energy, then we can write it in internal energy form. Since we are trying to express the general form of the first law of thermodynamics, so it is better to write *E*. So, this is first law for a control mass system essentially that is the change of energy within the system. So, here all these quantities are written in the differential form, but we have to keep in mind that *Q* and *W* are the path functions, so, we have written in the form of in exact differential, whereas *E* is the point function, because this is the property of the system and that is why we could write in the form of an exact differential. So, now, if you would like to write these quantities in the rate form, we can write like this,

$$\lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta E}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Again let me tell you one important thing why we considering $\Delta t \to 0$, because the derivation of the Reynolds transport theorem is based on important assumption is that $\Delta t \to 0$, that means, the system and control volume are coincident. If you cannot recall Reynolds transport theorem, I would suggest for you to consult with any standard fluid mechanics books, you will find that the Reynolds transport theorem is derived based on an important assumption that is $\Delta t \to 0$ and this is considered keeping in mind that the overlap area that is the system and control volume will be coincident. So, that is why we have written $\Delta t \to 0$ as our objective is to express the rate of change of energy from a system to the rate of change of energy to the control volume. So, basically we are trying to express the rate of change of energy from the control volume perspective and that is why you are trying to do it. So, we can write

$$\dot{Q} = \frac{dE}{dt} + \dot{W}$$

(Refer Slide Time: 35:40)



As you know this is for the control mass system, so, we can write this as

$$\dot{Q} = \left(\frac{dE}{dt}\right)_{Sys} + \dot{W}$$

$$\Rightarrow \left(\frac{dE}{dt}\right)_{Sys} = (\dot{Q} - \dot{W})$$

So this is for a system or for control volume? It is essentially for a system. So, the quantity written here in the right hand side of this equation is also applicable for the system. I mean,

$$\Rightarrow \left(\frac{dE}{dt}\right)_{Sys} = \left(\dot{Q} - \dot{W}\right)_{Sys}$$

But, we can write this quantity applicable for control volume as well. So, this quantity can be retained, I mean this is as good as

$$\Rightarrow \left(\frac{dE}{dt}\right)_{Sys} = \left(\dot{Q} - \dot{W}\right)_{CV}$$

Because, we have taken an important assumption that $\Delta t \to 0,$ so, system and control volume are coincident. So, if we try to look at the expression which we have written earlier,

$$\left(\dot{Q} - \dot{W}\right)_{CV} = \frac{\partial E_{CV}}{\partial t} + \int \rho e(V_r.\hat{\eta})dA$$

So, basically you can see that starting from the first law of thermodynamics for a control mass system, we could write this expression. So, here $\dot{Q}$ is the time rate of change of heat within the control volume, $\dot{W}$ is time rate of change of total work within the control volume and the difference of these two quantity is nothing but the time rate of change of energy within the control volume plus the rate of energy flux. So, basically this is the rate of energy in into the system and rate of energy out from the system because of the mass flux.

Now, $\dot{W}$ is having 2 different components, what are the components? If you talk about a control mass system, still there will be $\dot{W}$ because it is energy transport either by heat or by work. In this particular course, in fact, you have studied in thermodynamics that energy transport is either by heat transfer or by work transfer. So, components of $\dot{W}$

$$\dot{W} = \dot{W}_{CV} + \dot{W}_{Flow}$$

So, $\dot{W}_{CV}$ is basically that is work done within the control volume. So, either work is added to the control volume or work is taken away from the control volume. On the top of that, one additional component will be there. Because we are trying to study the first law of thermodynamics for the control volume system, why because most of the thermal system that will be discussed in the subsequent classes involves transport of a fluid. So, when there is a flow of fluid within this control volume, then a part of work will be associated with that to maintain the flow in the presence of a pressure.

So, without pressure difference flow will not take place as you have studied in fluid mechanics. So, if you need to maintain continuous flow in the presence of a pressure then there will be work done. So, this work done is $\dot{W}_{Flow}$, flow work. So, basically the total work comprises of 2 parts. One part is the work done by the control volume or work is done on the control volume;

either work is added to the control volume or work is taken away from the control volume. On the top of that the 2nd component takes care of the intrinsic work done associated to maintain the flow in the presence of pressure.

If this part ($\dot{W}_{Flow}$) of energy is not there, I mean if you do not have energy associated with the transport or flow, then flow will not survive. So, basically to maintain the flow, some part of energy will be associated to that, than that is $\dot{W}_{Flow}$. Now the equation becomes

$$\dot{Q} - \dot{W} - \dot{W}_{Flow} = \frac{\partial E_{CV}}{\partial t} + \int \rho e(V_r . \hat{\eta}) dA$$

So, what is flow work? You have studied in fluid mechanics, say we have a parallel flow channel, a very simple example I am taking and if we need to have continuous flow. Say if we have a control volume and there is one inlet and one outlet, so there is a continuous flow to the control volume and there is again continuous outflow from the control volume. If you need to have continuous inflow and continuous outflow, some part of energy should be associated with that otherwise, it would be difficult to maintain the continuous flow.

So, if we take out this small part, say this is the inflow part and I have tried to draw the zoomed in view. So, it is a continuous flow at this inlet and if we are considering pressure here as p and because of this pressure, the flow has advanced a distance say Δx through the channel. So, this is inlet channel.

Now, to maintain the flow in a continuous manner, we need to have flow work or work done. So, work done to maintain the flow in the presence of pressure because and if we assume that the cross section length of this particular portion is so small that the pressure is remaining almost constant. So, you can write

$$\text{Work done to maintain the flow} = pA\Delta x$$

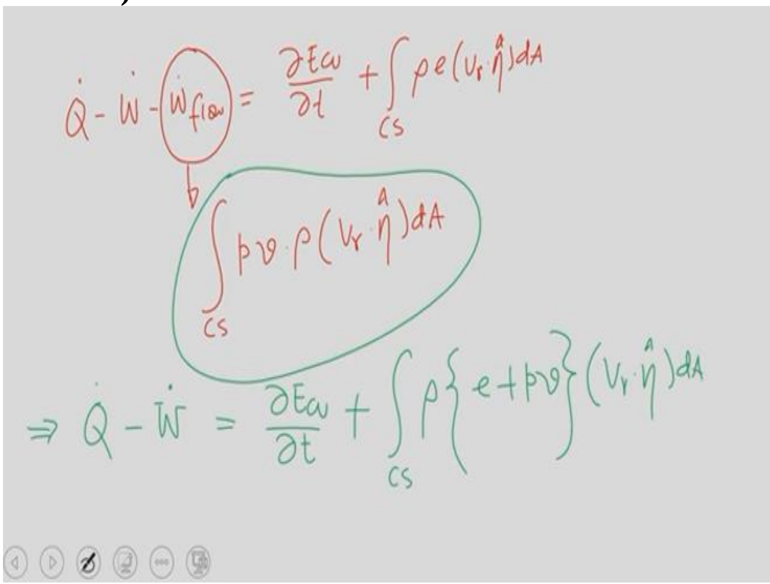
So, that is the work done. If you would like to write per unit mass, let us consider that density of the fluid is ρ and that is also remaining constant in this particular cross section, as I told you that properties are not varying over the cross section through which flow occurs. So, basically mass would be $\rho A \Delta x$.

$$\text{Work done per unit mass} = \frac{pA\Delta x}{\rho A \Delta x} = \frac{p}{\rho} = pv$$

So, that is what you have studied in fluid mechanics course as flow work.

So, the energy required to maintain flow in the presence of pressure is p/ρ. In thermodynamics, it is convenient to write in terms of the specific volume, v because the specific volume can be easily calculated from the linear interpolation from the property chart so, this is the specific volume.

(Refer Slide Time: 45:44)



So, if we try to write the equation again

$$\dot{Q} - \dot{W} - \dot{W}_{Flow} = \frac{\partial E_{CV}}{\partial t} + \int \rho e (V_r . \hat{\eta}) dA$$

So, basically this $\dot{W}$ is work done by the system or on the system, that is whether work is added to the system or work is taken away from the system. You know that flow is there. On the other hand that same amount of work done will be there within the control volume. See we have written work done per unit mass, so, work done due to the mass flux, which is mass flux into the system and work done due to mass flux, which is mass flux out from the system. So, this is work done per unit mass, if we can calculate the mass flux into the system and mass flux out from the system then we can calculate total.

So, you know that this quantity $\rho(V_r . \hat{\eta}) dA$ is mass flux on the control surface, which is the difference of mass flux out and mass flux in, if we have control surface at the inlet and control surface at the outlet.

$$\dot{Q} - \dot{W} - \dot{W}_{Flow} = \frac{\partial E_{CV}}{\partial t} + \int \rho e (V_r . \hat{\eta}) dA$$

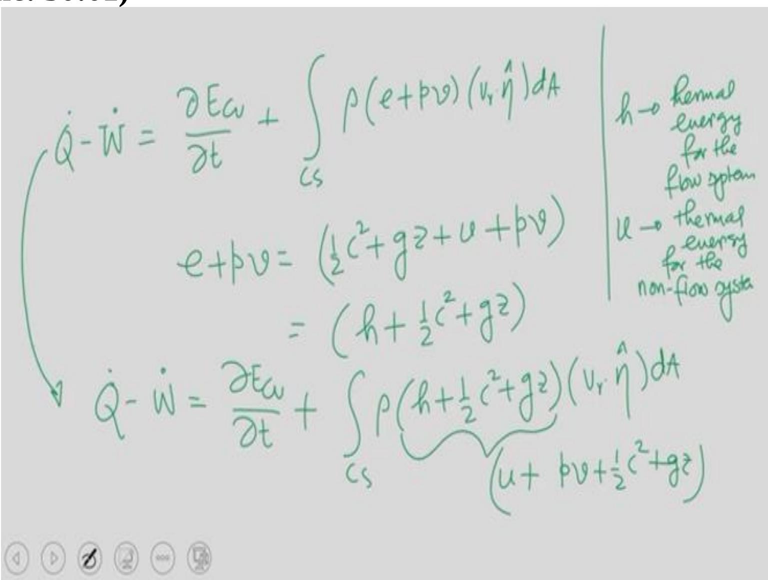
$$\Rightarrow \dot{Q} - \dot{W} - \int pv\rho(V_r.\hat{\eta})dA = \frac{\partial E_{CV}}{\partial t} + \int \rho e(V_r.\hat{\eta})dA$$

$$\Rightarrow \dot{Q} - \dot{W} = \frac{\partial E_{CV}}{\partial t} + \int \rho\{e + pv\}(V_r.\hat{\eta})dA$$

So, I have taken this term in the right hand side and just tried to club these 2 terms together. So, what is this $e + pv$? So, this e is specific energy

$$e + pv = \left(\frac{1}{2}c^2 + gz + u + pv\right)$$

(Refer Slide Time: 50:01)



You have studied in thermodynamics that if we write $u + pv$, we are getting another important property that is enthalpy. So

$$e + pv = \left(h + \frac{1}{2}c^2 + gz + pv\right)$$

So, that means, we can write on step further that

$$\dot{Q} - \dot{W} = \frac{\partial E_{CV}}{\partial t} + \int \rho \left(h + \frac{1}{2}c^2 + gz + pv\right)(V_r.\hat{\eta})dA$$

So, this is first law of thermodynamics of a flow process across the control volume. So, this is very important. Now if the system is non-flow system that is control mass system, we could write this total energy in terms of u. So, if it is a non-flow system instead of h we can write u, that is $\left(u + \frac{1}{2}c^2 + gz\right)$, but for the flow system that is control volume system where there is a flow across the control surface the total energy is written $\left(h + \frac{1}{2}c^2 + gz\right)$.

So, this is nothing but the internal energy & this total part is nothing but $u + \frac{1}{2}c^2 + gz + pv$. So, even if you do not consider the changes in kinetic and potential energies, this quantity for the flow system is nothing but the internal energy of the system plus energy required to maintain the flow in the presence of a pressure. Again I am telling, even if we do not consider the changes in kinetic and potential energy, this quantity indicates the total energy which is nothing but the internal energy plus the energy required to maintain the flow in the presence of a pressure. So, this is the difference between the control mass and control volume system. So, for the control volume system we are getting 1 extra term that is enthalpy and this is basically thermal energy for the flow system. So, this h is the thermal energy for the flow system and u is the thermal energy got the non-flow system.

So, this is the important difference. So, we will discuss again, we will go on a few steps further to describe first law of thermodynamics for the flow processes, across the control volume. So, taking a few assumptions, we can write this expression in different forms. That part we shall discuss in the next class. So with this I stop here today. Thank you.