Viscous Fluid Flow
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Module - 12
Turbulent Flows - II
Lecture - 03
Turbulence Modelling

Hello everyone. So, you know that we have already derived the Reynolds averaged Navier-Stokes equations and there we have unknown Reynolds stress and later we used Boussinesq eddy viscosity approximation and we have written the apparent viscosity and summation of molecular viscosity plus turbulent viscosity.

Now, the question is how to model this turbulent viscosity mu t? For that today we will discuss few turbulence model and we will see how to find this mu t.

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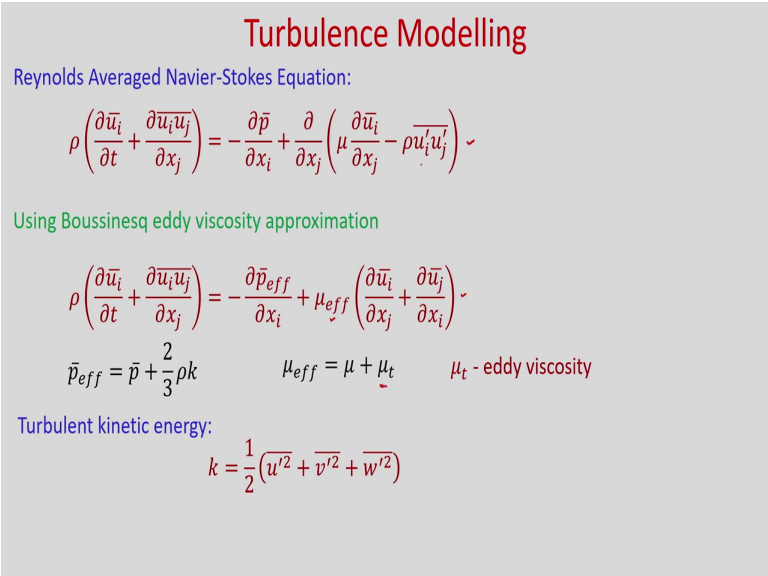

So, you can see this is the Reynolds averaged Navier-Stokes equations we derived and this is the additional term Reynolds stress.

And then we used Boussinesq eddy viscosity approximation and we have written this equation, where this mu effective is mu plus mu t where mu t is eddy viscosity and it is still unknown. And this is the effective pressure, this is the mean pressure plus 2 3rd rho k, where k is the turbulent kinetic energy from this fluctuating velocity.

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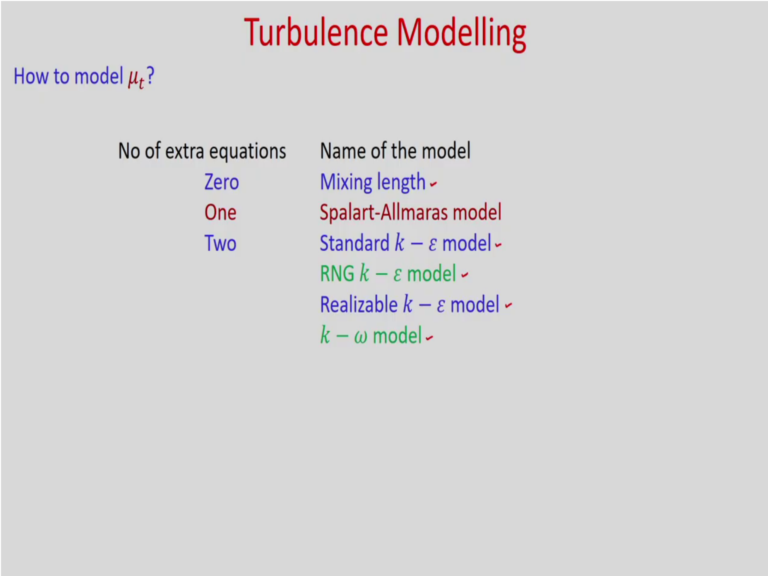

So, now how to determine this eddy viscosity? So, you can see that there are different models available. Already, we have discussed this Prandtl's mixing length hypothesis, where we do not need to solve any extra equation. So, that is a zero equation model. We have Spalart-Allmaras model, where we need to solve one extra equation and several two equation models are available like standard k epsilon, RNG k epsilon, Realizable k epsilon and k omega model.

So, if we want to model this turbulent flow numerically, then we need to solve these Reynolds averaged Navier-Stokes equations along with we need to solve some other equation to find the unknown eddy viscosity mu t.

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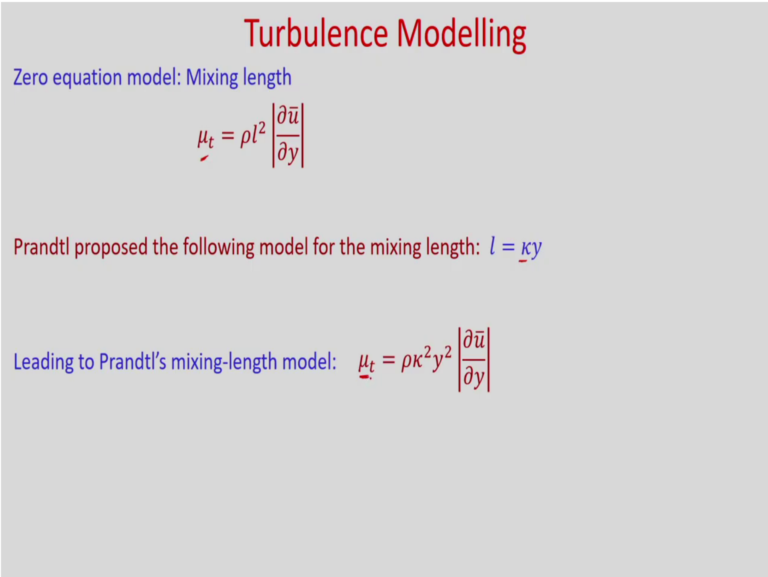

You have seen that in zero equation model we just model this turbulent viscosity as rho l square, the absolute values of del u bar by del y, where l is the mixing length. So, you model this l using this Prandtl mixing length hypothesis as l is equal to kappa y.

So, you can see it is the simplest way to model it and if you use it then this turbulent viscosity you can write rho kappa square y square this absolute value of del u bar by del y, and you know that this is the absolute value we are taking such that the value of this turbulent viscosity will be positive.

So, you can see that it has advantages as it is easy to implement and it is computationally cheaper as you do not need to solve any additional equations to calculate these turbulent viscosity. But, it has disadvantages like it is completely incapable of describing the flows with separation or circulation.

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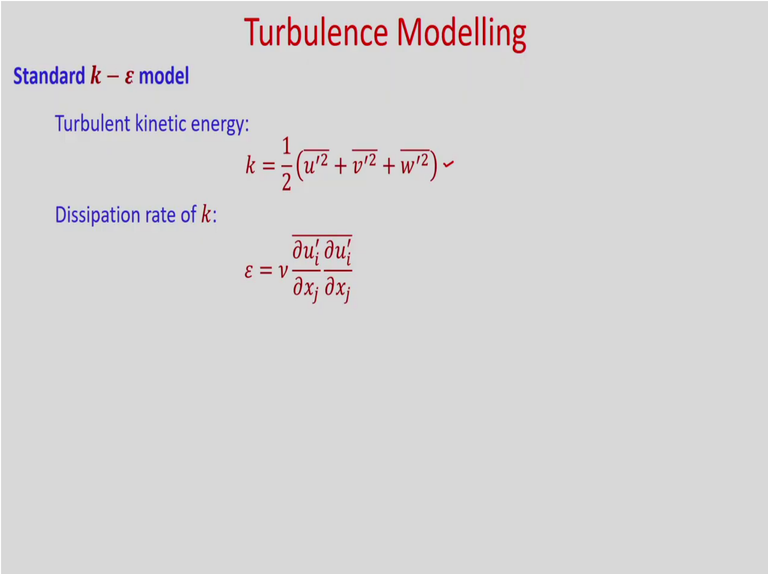

Now, let us discuss one of the two equation turbulence model. So, we will first discuss about the standard k epsilon model. So, you can see k stands for turbulent kinetic energy and this epsilon is the dissipation rate of this turbulent kinetic energy. So, epsilon generally is represented as nu del u prime by del x j del u prime by del x j and time average.

So, now, let us derive the equation for this turbulent kinetic energy and then we will write down the equation for this dissipation rate epsilon.

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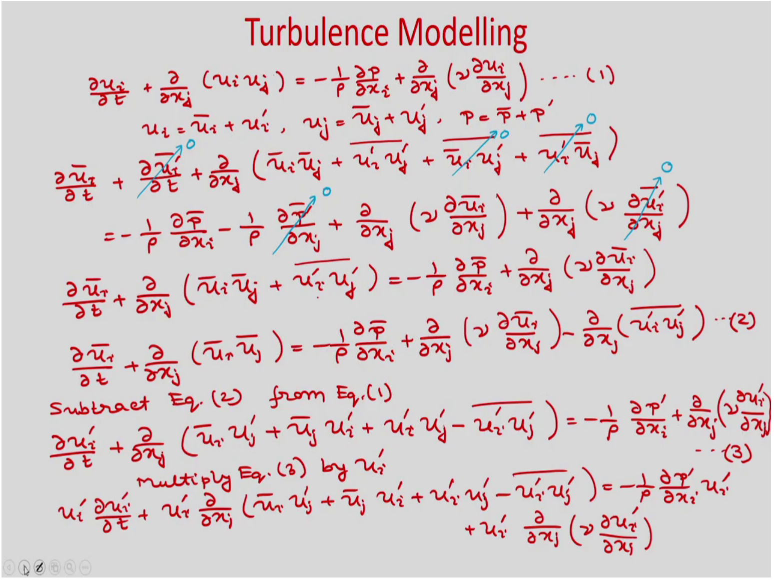

So, we will start from the Navier-Stoke equations an intentional from and we will derive the equation for turbulent kinetic energy. So, you know that the in general, we have the Navier-Stoke equation, del u i by del t plus del of del x j u i u j is equal to minus 1 by rho del p by del x i plus del of del x j nu del u i by del x j ok. Let us say this is equation number 1.

Now, if you substitute u i is equal to u i bar plus u i prime and u j is equal to u j bar plus u j prime and p is equal to p bar plus p prime ok. So, these are Reynolds decomposition.

So, if you substitute it here ok and if you take the average of the entire equation, then we will get del u i bar by del t plus del u i prime bar by del t plus del of del x j u i u j bar plus u i prime u j prime bar plus u i bar u j prime bar plus u i prime u j bar is equal to minus 1 by rho del p bar by del x i minus 1 by rho del p prime bar by del x i plus del of del x j nu del u i bar by del x j plus del of del x j nu del u i prime by del x j bar.

So, you can see that some of the terms will be 0. So, because you know that the time average of this fluctuating component u i prime bar will be 0. So, this will be 0 ok, you can see that this term ok. So, this will be 0, this term will be 0, this term will be 0 and you can see this is also the time average of this fluctuating component. So, this will be also 0 ok.

So, you can see that after substituting this here and taking the average, we just wrote this and after simplification we can write this equation as del u i bar by del t plus del of del x j u i u j bar plus u i prime u j prime bar is equal to minus 1 by rho del p bar by del x i plus del of del x j nu del u i bar by del x j ok.

And; obviously, this is the Reynolds state. So, that we can take in the left hand side and we can write del u i bar by del t plus del of del x j u i bar u j bar is equal to minus 1 by rho del p bar by del x i plus del of del x j nu del u i bar by del x j minus del of del x j. So, we are taking this term in the right hand side which is your Reynolds stress bar, let us say that this is equation number 2.

So, now you subtract equation number 2 from equation number 1, then let us write the equation in terms of the velocity fluctuations. So, you can see that; obviously, if you subtract this equation 2 from equation 1, then you will get this u i minus u i bar which we can represent in terms of this fluctuating velocity u i prime. So, we can write subtracting, subtract equation 2 from equation 1 ok, then what we will get?

We can write del u i prime by del t because u i bar by minus u i sorry, u i minus u i bar will be u i prime plus del of del x j. It will be u i bar by u j prime plus u j bar u i prime plus u i prime u j prime minus u i prime u j prime bar. So, you make some additional simplification then you will get this.

Then right hand side we have minus 1 by rho del p prime by del x i plus del of del x j nu del u prime by del x j ok. So, this is equation number 3. Now, multiply equation 3 by u i prime ok.

So, that we can write in terms of the turbulent kinetic energy, you can see that if you multiply u i prime here. So, it will be u i prime square and that is the representation of this turbulent kinetic energy.

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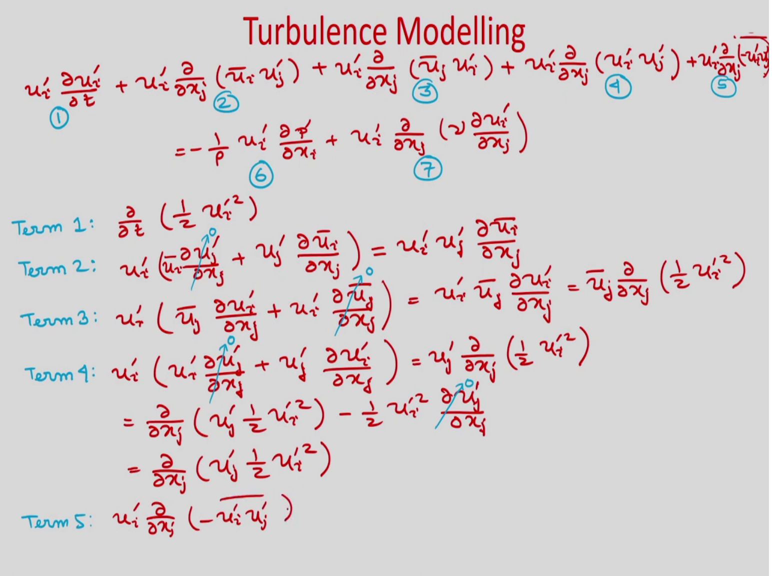

So, if you write that then you can see that we can write u i prime del u i prime by del t plus u i prime del of del x j u i bar u j prime plus u j bar u i prime plus u i prime u j prime minus u i prime u j prime bar is equal to minus 1 by rho del p prime by del x i plus del of del x j.

So, here you need to multiply with u i prime and if you multiply here by u i prime then we can write this term. So, now, each term let us individually see ok. So, if you write this equation as u i prime del u i prime by del t plus u i prime del of del x j u i bar u j prime plus u i prime del of del x j u bar j u i prime plus u i prime del of del x j u i prime u j prime plus u i prime del of del x j minus u i prime u j prime is equal to minus 1 by rho u i prime del p prime by del x i plus u i prime del of del x j nu del u i prime by del x j.

So, now let us look each an individual these terms ok. So, let us say this is. So, this is your 1, this is term 2, this is 3, this is 4, this is 5, this is 6 and this is 7 ok. So, let us say term 1 ok. So, what we can write? So, if you take u i prime inside. So, we can write this term as del of del t half u i prime square ok. So, u i prime if you take inside then; obviously, it will become half del of del t u i prime square.

Now, term 2. So, this term you can see that this we can write as u i prime ok and this we can write del u i bar del u j prime by del x j plus u j prime del u i bar by del x j ok and then we can write you can see that this is nothing but the continuity equation satisfying the fluctuating velocity component. So, this will be 0, right.

So, you can see that this term is 0 because this is actually the continuity equation satisfying by the velocity fluctuating components ok. So, that we have already derived. So, this will get as u j prime del u i bar by del x j and we have another u i prime. So, we can write this term as u i prime u j prime del u i bar by del x j.

Now, term 3 ok. Similarly, we can write u i prime u j bar del u i prime by del x j plus u i prime del u j bar by del x j. So, here you can see that this is the continuity equation satisfied by the mean velocity. So, this will become 0 ok. So, this is 0. So, we will get u i prime u j bar del u i prime by del x j. Now, you take this u i prime again inside this derivative. So, what we will write u j bar del of del x j half u i prime square. Now, let us write this term 4. So, you can see this is the term 4.

So, similarly you can write u i prime; u i prime del u j prime by del x j plus u j prime del u i prime by del x j. So, this is also 0, right. So, this is 0. So, we can write. So, this now we can write. So, u i prime u j prime this. So, you can write u j prime and u i prime if you take inside. So, we can write del of del x j half u i prime square ok and now, if you take this u j prime inside this derivative.

So, what we can write? We can write del of del x j u j prime half u i prime square as we have taken inside. So, there will be minus half u i prime square del u j prime by del x j ok. So, you can see again this is 0 right ok. So, we can write this term as del of del x j u j prime half u i prime square. Now, let us consider this term 5 ok. So, term 5 if you write ok. So, we can see that we can, we will keep as it is ok u i prime del of del x j minus u i u j prime bar ok.

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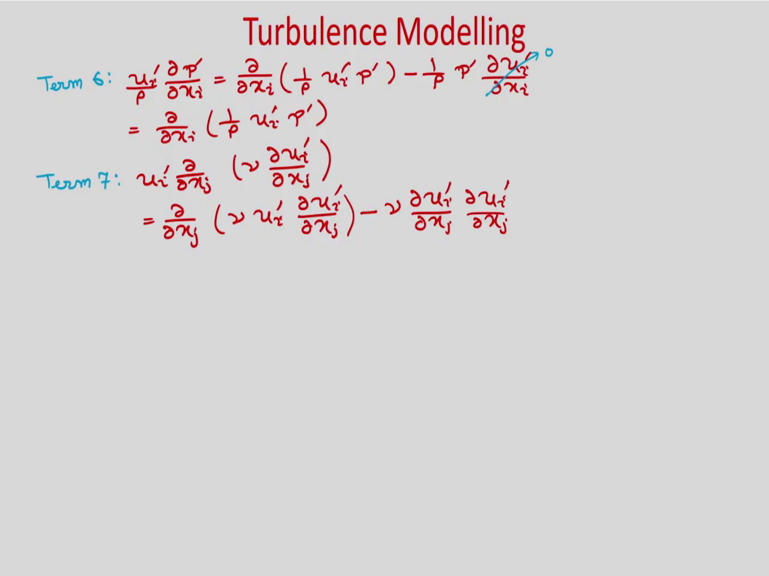

Now, part 6 and part 7. Now, let us simplify part 6 ok. So, we have this term u i prime del p prime by del x i. So, we will take u i prime inside. So, you can write this as del of del x i, let us keep the viscosity also. So, we can write 1 by rho u i prime p prime minus 1 by rho p prime del u i prime by del x i ok.

So, now this is again continuity equation. So, this will be 0. We can write this term as del of del x i 1 by rho u i prime p prime. And the last term part 7, sorry you were writing term. So, this is term 6 and term 7. So, you have u i prime del of del x j nu del u i prime by del x j. So, if you take this inside. So, you can write del of del x j nu u i prime del u i prime by del x j minus nu del u i prime by del x j and del u i prime by del x j.

So, you can see that all the terms we have written in such a way that we can define the turbulent kinetic energy or you can represent the turbulent kinetic energy.

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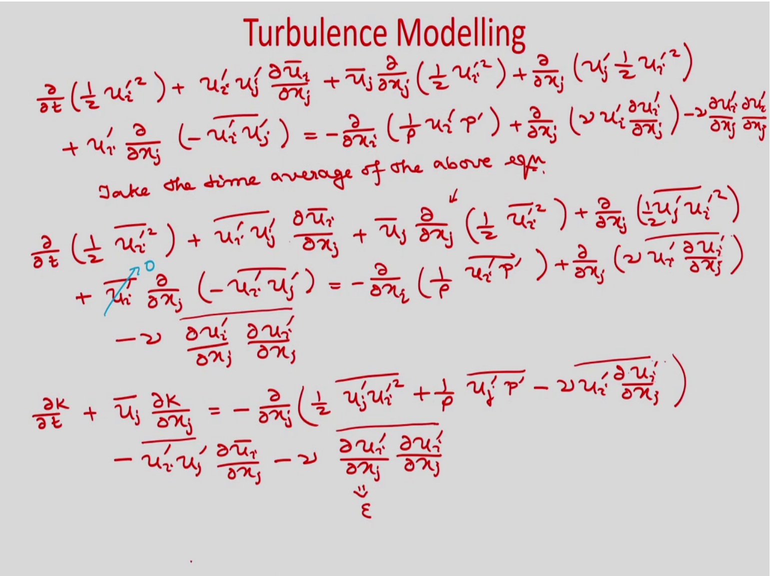

So, now all these terms, from term 1 to term 7, let us put in the original equation. So, if you put we will be get del of del t half u i prime square plus u i prime u j prime del u i bar by del x j plus u j bar del of del x j half u i prime square plus del of del x j u j prime half u i prime square plus u i prime del of del x j minus u i prime u j prime bar.

So, in the right hand side we have minus del of del x i 1 by rho u i prime p prime plus del of del x j nu u prime u i prime del u i prime by del x j and minus nu del u i prime by del x j del u i prime by del x j. So, next what we will do? We will take the time average of this whole equation ok. So, take the time average of the above equation ok. So, then we can represent as del of del t half u i prime square bar plus u i prime u j prime bar del u i bar by del x j plus u j bar del of del x j half u i prime square bar plus del of del x j.

So, we have this u j prime u i prime square bar plus this will be u i prime bar del of del x j minus u i prime u j prime bar and right hand side we have del of del x i 1 by rho u i prime p prime bar plus del of del x j nu u i prime del u i prime by del x j bar minus nu del u i prime by del x j del u i prime by del x j bar. So, you can see that this we have already defined as dissipation rate per unit kinetic energy right.

So, this is the dissipation rate. So, now, we can see this is the representation of this turbulent kinetic energy and you can see that this is u i prime bar. So, u i prime bar will be 0. So, this term is 0. So, now, we can write the equation ok. So, this we will write as del of del t. So, this we will write k because this is the turbulent kinetic energy half u i prime square bar plus now this term you see, this term ok.

So, this we write as u j bar and this will be del k by del x j ok, this is the turbulent kinetic energy. In the right hand side if you take so these terms. So, we can write minus del of del x j ok what are the terms we have. So, this is one term, half u j bar u i prime square bar plus 1 by rho. So, this will be x i ok.

So, now this x i were interchanging with j. So, you can write 1 by rho u. So, del of del x j we have written. So, u j you can write prime p prime bar ok. Then we have minus nu u i prime this term, del u i prime by del x j bar ok. What are the additional terms we have? So, this is one term. So, this will be minus u i prime u j prime bar del u i bar by del x j and another term we have this term.

So, it will be minus nu del u i prime by del x j del u i prime by del x j bar. So, this you can see this we have already defined as the dissipation rate epsilon ok.

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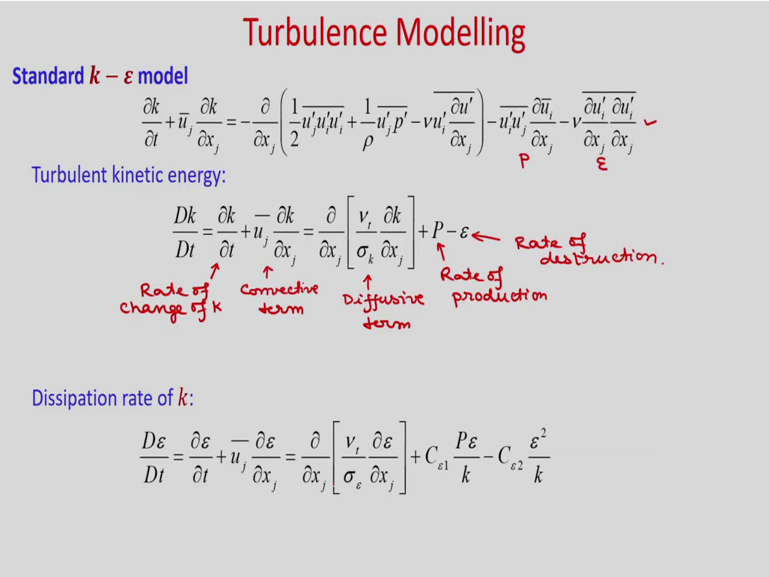

So, now, you can see that we have derived the equation for this turbulent kinetic energy k ok. So, let us see the significance of each terms. So, you can see this is the equation we have already derived ok and this term we can write as the production term ok. This term as we can write as production term P and this is the dissipation rate epsilon ok and you can see that these terms we can model because these fluctuating velocity components are unknown. So, that we can write del of del x j nu t by sigma k del k by del x j.

So, these are some model constant sigma k and; obviously, nu t is the turbulent viscosity. And this is the term P we are representing and this is the dissipation rate epsilon. So, you can see that this is a convective diffusive equation right. So, you can see this left hand side is the inertia term, this we can represent as a viscous term and some additional such terms are there ok.

So, you can see that it is a rate of change of k ok. So, this term is rate of change of k ok. So, k is the turbulent kinetic energy and this is the; obviously, you can see that inertia term ok. So, convective term and this you can see that this is diffusive term and this term ok we are representing as P which is your rate of production and this epsilon is your dissipation. So, it is rate of destruction.

So, you can see that to solve this equation, we need to know the value of nu t as well as the dissipation rate epsilon. So, we need another equation for dissipation rate epsilon. So, we are not going into detail of derivation of this equation, but we are writing the final equation for the dissipation rate. So, you can see dissipation rate of k is this after using some model constants and these model constants are found by picking the data from the experimental values ok.

So, you can see again this is the similar way, this is rate of change of epsilon, this is the convective term, this is the diffusive term and this is the rate of production and this is the rate of destruction ok. So, now, we need to know all these model constants, sigma k, sigma epsilon, C epsilon 1, C epsilon 2 and this P is the rate of production.

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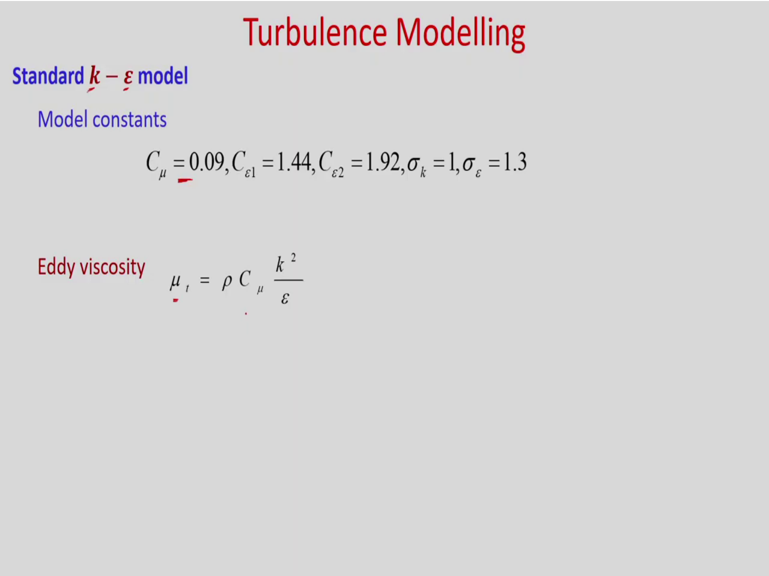

So, you can see that in standard k epsilon model, there are some model constants which are found from the experiments. So, these are empirical constants. So, you can see C mu is 0.09, C epsilon 1 is 1.44, C epsilon 2 1.92, sigma k is 1 and sigma epsilon is 1.3. Once you find the value of k and the epsilon, then you will be able to calculate the eddy viscosity mu t as rho C mu k square by epsilon, where C mu is this model constant.

So, you can see that this k epsilon model has some advantages as it is relatively simple to implement and it is widely validated turbulence model. However, it has some disadvantages like it gives poor prediction for swelling and rotating flows and flows with strong separation.

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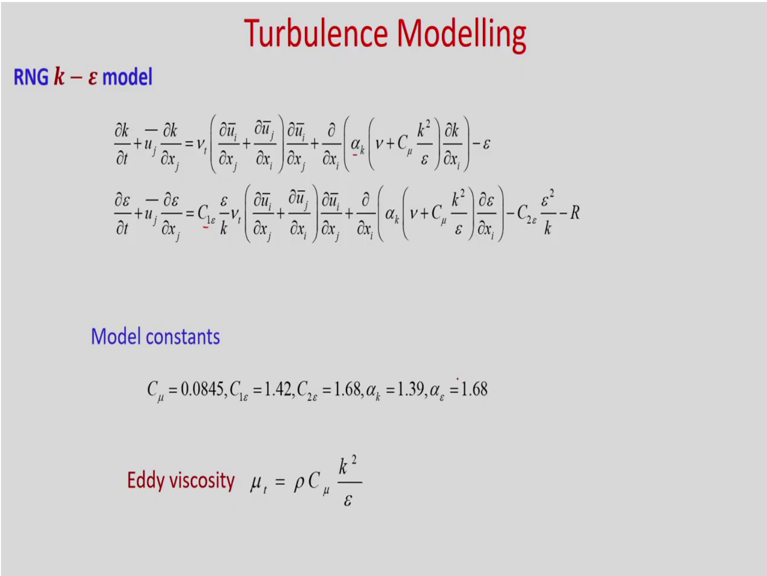

So, there are some improved models like RNG k epsilon model. RNG stands for renormalization group model and here we solve two equations, one for k and another equation is epsilon with some different empirical constants and you can see these are the model constants. So, C mu is 0.0845, C 1 epsilon is 1.42, C 2 epsilon is 1.68, alpha k is 1.39 and alpha epsilon is 1.68.

So; obviously, these equations you can see here these model constants are there and these you need to use these model constants and once you find this k and epsilon you will be able to calculate this eddy viscosity mu t as rho C mu k square by epsilon. So, obviously, you can see that it has impute prediction for flows with high steam line curvature and strain rate and transitional flows.

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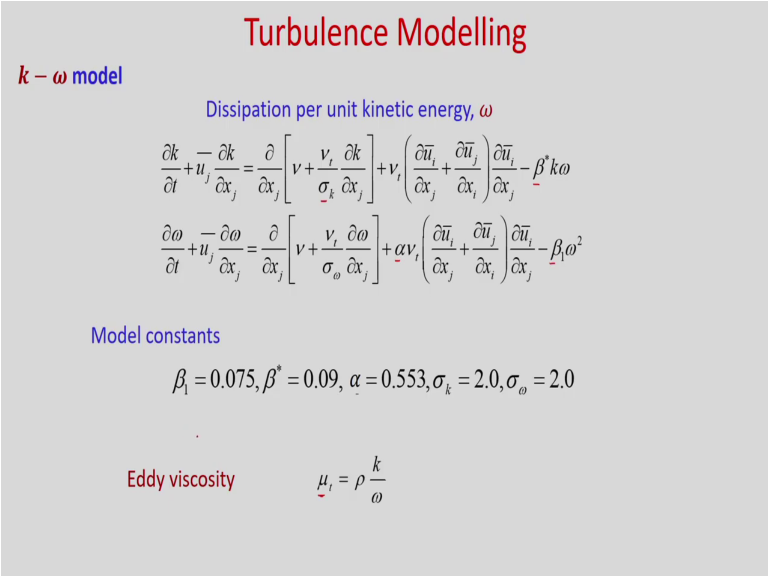

Another two equation model is there, that is known as k omega model, where omega is the dissipation per unit kinetic energy. So, we are not going into details of derivation of these equations, but we are writing this equation. So, this is the equation for k and this is the equation for this dissipation per unit kinetic energy omega.

So, here you can see that there are some model constants like here you have sigma k, beta star, alpha, beta 1. So, these are the empirical constants beta 1 is equal to 0.075, beta star is equal to 0.09, alpha is equal to 0.553, sigma k is equal to 2, sigma omega is equal to 2. So, once we solve these two equations, then we will be able to find k and omega and from there we can find the eddy viscosity mu t as rho k by omega.

So, this k omega model has much application in aerospace engineering. It gives improve performance for boundary layers under adverse pressure gradient. So, there are different turbulence model, we have discussed where standard k epsilon model is a versatile model where you have the flow separation and it is best for industrial internal flows and also this model is good for this aerospace application.

However, we have this RNG k epsilon model which is good for transitional flow and for this k omega where you have rotational flow we have much strain rate. So, in those cases we can actually use this k omega model.

Thank you.