Viscous Fluid Flow

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Module - 10
Stability Theory
Lecture - 02
Viscous Stability

Hello everyone, so in last class we derived the Orr-Sommerfeld equation using linear stability analysis. In today’s class first we will write the non dimensional form of this Orr-Sommerfeld equation, then we will discuss about the viscous stability and then we will discuss about the stability map.

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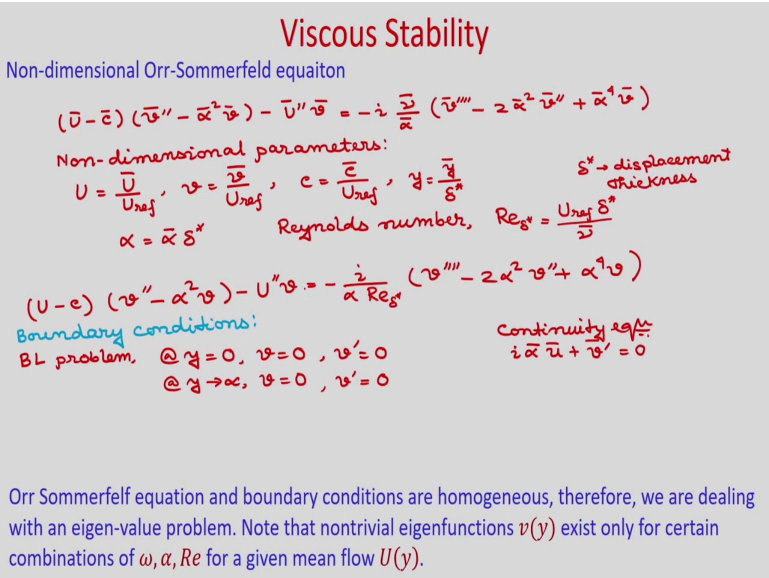

So first, let us write down the Orr-Sommerfeld equation which we derived in last class. So, it was u bar minus c bar v double prime bar minus alpha bar square v bar minus u double prime bar v bar is equal to minus i nu bar by alpha bar v bar four prime minus 2 alpha bar square v double prime bar plus alpha to the power 4 v bar, ok.

So you know that here, c bar is the wave propagation speed and alpha bar is the wave number and this double prime is the 2nd derivative and u double prime is d 2 u bar by d y square. So now, let us use these non dimensional parameters to non-dimensionalize the Orr-Sommerfeld equation. So, we will use this non dimensional parameters.

So, u we will write as u bar by some u reference ok. So, if it is flow over a flat plate, we will use u infinity, v is equal to v bar by u reference, c is equal to c bar by u reference and, y we will just denote as non dimensional y and if y bar is the dimensional y divided by we will use the displacement thickness delta star ok. So, delta star is displacement thickness ok.

So, here we are using y as non dimensional parameter that is why we have written y bar by delta star. So, length scale or reference length is delta star we are using ok. And, alpha we will write as, alpha bar delta star. So, this alpha is non dimensional wave number and this c is non dimensional wave propagation speed.

So, using this reference velocity and these reference length scale, we will define Reynolds number as Re based on delta star is equal to u reference delta star divided by nu bar ok. So, using this non dimensional parameter, you can convert this Orr-Sommerfeld equation in non dimensional form as u minus c v double prime minus alpha square v minus u double prime v is equal to minus i alpha Re based on delta star v four prime minus 2 alpha square v double prime plus alpha to the power 4 v.

So now, to solve this non dimensional Orr-Sommerfeld equation we need 4 boundary conditions right. Because, it is linear fourth order ordinary differential equation. So we need 4 boundary conditions.

So, you can see that the all the disturbances should go to 0 at the wall if you are considering boundary layer flow and y tends to infinity; also these all disturbances should go to 0. So, from here using the continuity equation we can write the 4 boundary conditions. So we will have this boundary conditions.

So, if we consider boundary layer problem then the disturbances you can write at y is equal to 0, v is equal to 0 and at y tends to infinity, these disturbances also will go to 0.

Now, use the continuity equation ok continuity equation. So, if you remember, we have derived i alpha bar u bar plus v prime bar is equal to 0 ok. So, if we use this continuity equation, from here you can see that this velocity disturbances u also will be 0 at the wall.

So from here you can see that v prime will be 0 at the wall as well as at y tends to infinity ok. From this equation you can see that the disturbances of u also must be 0 right, at the wall and at y tends to infinity. So, that means, here we can write v prime also 0 and v prime as 0 ok.

So, we need to solve this 4th order ordinary differential equation along with these 4 boundary conditions. For other problem you can write the boundary conditions suitably. So you can see that Orr-Sommerfeld equation and boundary conditions are homogeneous, because you can see from these two equations.

Therefore, we are dealing with an eigenvalue problem. Note that, non-trivial eigen functions v y exists only for certain combinations of omega, alpha, Re, for a given mean flow U y ok. So now, let us discuss about the temporal instability where we will see that omega is known ok.

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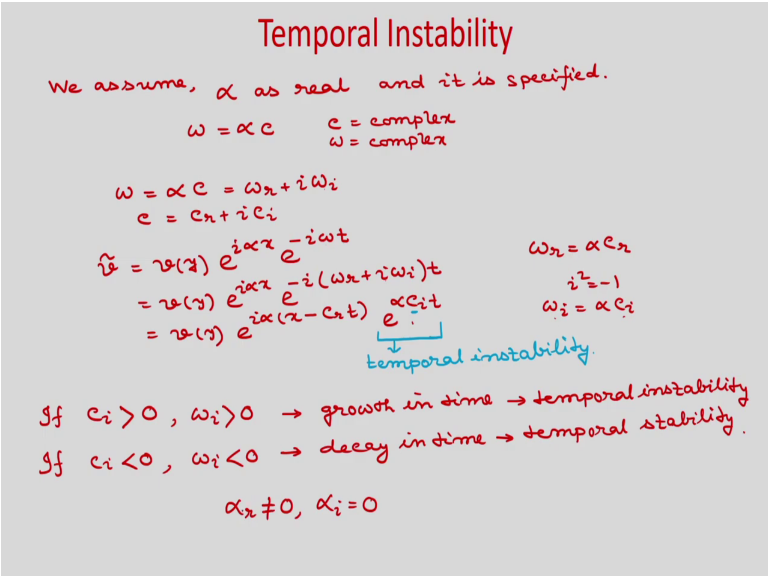

So, in temporal instability, now we assume alpha as real ok. And, if we write omega is equal to alpha c, then c is complex ok. So, you can see that wave propagation speed c is complex and we assume alpha as real and it is specified ok.

So, you can see that in order to find eigen functions v and the complex Orr speed c on the eigenvalue this alpha has to be specified. So, we are assuming alpha as real and omega is complex. So, as alpha is real, c has to be complex ok. So, as omega is complex and alpha is real then c has to be complex.

So, if you remember that we have written omega is equal to alpha in to c right, now we can write this as omega r plus i omega i. So this is the real part and this is the imaginary part. So, and c as complex, so we can write c r plus i c i ok. If you remember, that we have used v tilde as v, which is function of y e to the power i alpha x e to the power minus i omega t.

So, here omega is complex, so that you can substitute and we can write it as e to the power i alpha x e to the power minus i omega r plus i omega i ok. So, you can see that this will become minus i omega and that we can club with this. So it will be e to the power i alpha x minus c r into t.

So we have written, omega r is equal to alpha into c r ok. So, alpha is real right. So, this we can write and; obviously, you can see that we have this i and this i it will be i square and i square is minus 1, so this will become plus. So, you can write e to the power alpha c i t ok. Because, omega i is alpha c i right. So, we can write e to the power alpha c i t.

So, you can see that this term ok, it represents the temporal instability ok. So, it will determine whether it will be stable or unstable. So, which one we will determine here? c i will determine right. If, c i greater than 0 if c i greater than 0 from here you can see that it will be omega will be greater than omega will be greater than 0, that means, you will be growth in time right growth in time.

So, this term this so we will get temporal instability ok. And, if c i less than 0 ok, that means, omega i less than 0 from here you can see, that if c i is less than 0 omega i less than 0. So, you can see that it will be it will decay in time right. So, this term will decay in time, so that means it is a temporal stability ok.

So, you can see that temporal analysis predicts behaviours at t tends to infinity in given initial disturbances of alpha ok, where alpha r is not equal to 0 and alpha i is 0, because we have assumed that alpha as real. So, there is another type of instability that is known as a spatial instability, where omega is real ok.

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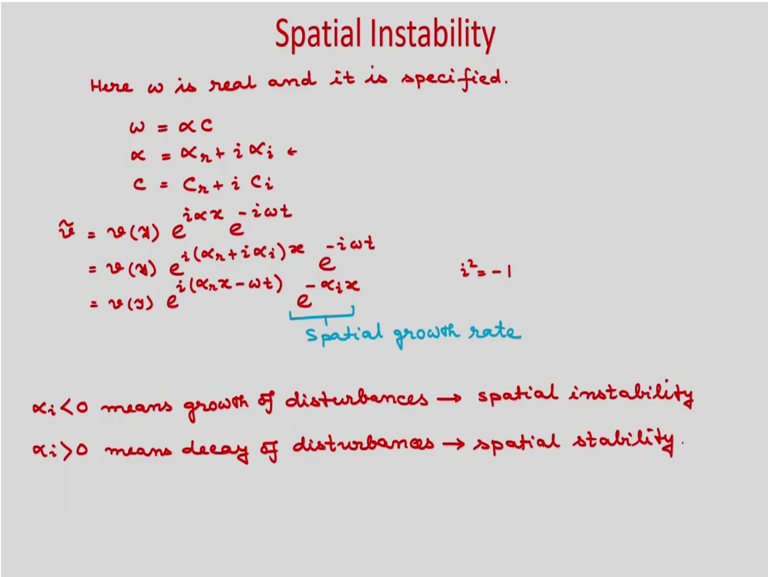

So now, we will consider omega as real and omega is specified. So here, omega is real and it is specified. So, we can write omega is equal to alpha in to c, where alpha is complex ok. So alpha is equal to alpha r plus i alpha i and c is equal to c r. So this is real part and this is the imaginary part.

So now, we can write v tilde is equal to v, which is function of y e to the power i alpha x e to the power minus i omega t. So, what we will do now? We will just substitute this alpha by this expression ok. So, we can write v e to the power i, so now alpha, we can write alpha r plus i alpha i x e to the power minus i omega t.

So now, we can rearrange and write as e to the power i alpha r x, and this we are taking here, so it will be minus omega t. And here, we have i square alpha i x and i square is equal to minus 1. So, we can write e to the power minus alpha i x. So, you can see that this term will determine, whether the disturbance will grow or decay right. So, this is your spatial growth rate.

So you can see, that if alpha i is less than 0, means growth of disturbances and if alpha i greater than 0, so it means that it is decay of disturbances ok. So obviously, you can see that this will lead to spatial instability and this will lead to spatial stability.

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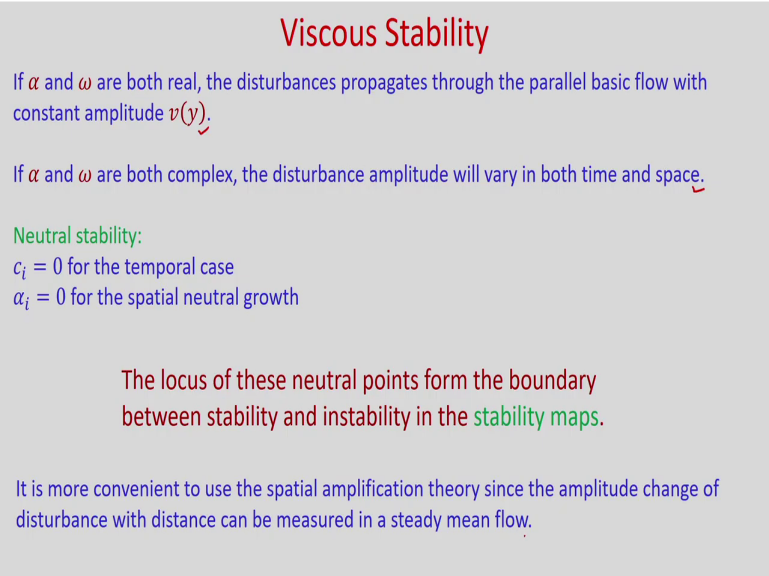

And if alpha and omega are both real, then the disturbances propagates through the parallel basic flow with constant amplitude v y and alpha and omega are both complex, the disturbance amplitude will vary in both time and space.

So, we are more interested in neutral stability. So for that if c i is equal to 0 for the temporal case and alpha is equal to 0 for the spatial neutral growth, and if we plot the c i is equal to 0, the locus of these neutral points from the boundary between stability and instability in the stability maps.

So, you can see that in both the cases we can draw the stability map, but it is more convenient to use the spatial amplification theory, since the amplitude change of disturbance with distance can be measured in a steady mean flow.

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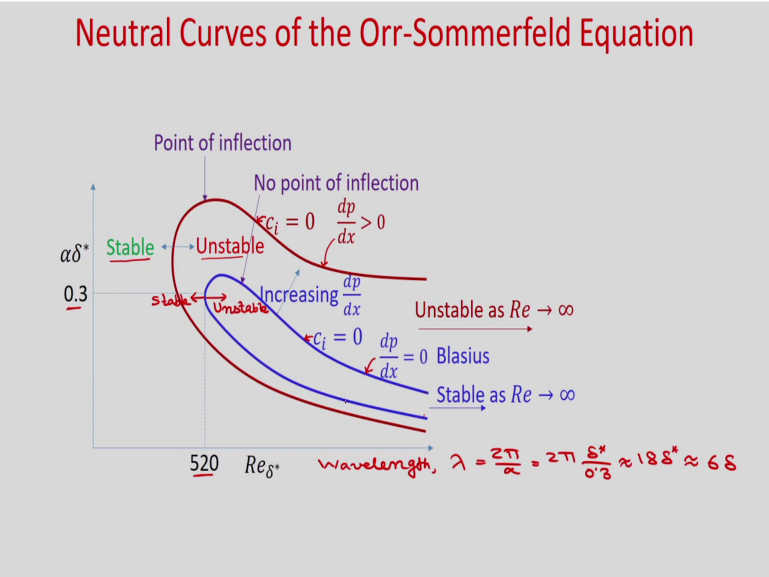

So now, we will discuss about the neutral curves of the Orr-Sommerfeld equation putting a c i is equal to 0. So, both the cases we will consider, one is viscous stability, where we will consider the boundary layer equation, where d p by dx is 0 and we will discuss about the inviscid instability, where d p by dx is greater than 0. And, more detail we will discuss in the next class about the inviscid stability.

So, you can see, we have plotted this in x axis Reynolds number based on delta star and in y axis alpha into delta star, where delta star is the displacement thickness. Now, for inviscid flow, if you plot this c i is equal to 0, so this is the curve where, obviously d p by dx is greater than 0.

And, if you plot this curve, so you can see that this is having the shape of thumb, so this is known as thumb curve as well. And, you can see that outside this curve ok, the flow is stable and inside this curve it is unstable ok.

So, you can see that there is a critical Reynolds number for this inviscid flow after which this become unstable. And, if you see this curve ok, so as Reynolds number tends to infinity this will not meet ok. So, for Reynolds number tends to infinity, the flow will be unstable.

Now, if you consider the special case, flow over flat plate where d p by dx is 0 ok. So, this is the Blasius solution. And, if you plot c i is equal to 0 curve ok, so this is the neutral curve. So, you can see that it is having a critical Reynolds number below which anyway it is stable ok. This is stable. And, after this critical Reynolds number, it is unstable ok.

If you carefully look here ok, so if you go Re tends to infinity; that means, if you go along the x axis, then somewhere these two curve will meet and after that it will become stable. Because obviously, inside this thumb curve we have a unstable region, but if you go very far away in the x direction; that means, Re tends to infinity, then this will become stable.

And, you can see that this is the viscous neutral curve and when you go increasing d p by dx, then obviously, you will get a inviscid neutral curve and you can see for Re tends to infinity this will remain unstable. And obviously, you can see that there will be no point of inflection in case of this Blasius neutral curve, but there will be point of inflection for this inviscid neutral curve.

So you can see that, this critical Reynolds number will be 520 for this Blasius flow and corresponding alpha delta star will be 0.3. So, if you calculate this wavelength of this unstable mode then you can see that wavelength will be lambda is equal to 2 pi by alpha. So you will get 2 pi delta star by 0.3, it will be around 18 delta star. And in terms of boundary layer thickness if you write, convert the delta star 2 boundary layer thickness for the Blasius flow then it will be 6 delta ok.

So, you can see, the first unstable wave at a very long wavelength and the boundary layer is always stable 2 short wavelength. So, in today's class first we wrote the non dimensional form of the Orr-Sommerfeld equation and we defined the Reynolds number based on the difference velocity u reference and the reference length as displacement thickness delta star.

Then we discussed about the temporal instability, where alpha is specified and alpha is real. Then we discussed about the spatial instability, where omega is real and omega is specified. After that, we discussed about the neutral map where we have plotted the c i is equal to 0 for spatial instability. And we have shown the thumb curve in the x y plot, where in x direction we have used Reynolds number based on the displacement thickness and in y direction we have alpha into delta star.

For Blasius flow, when d p by dx is 0, so we have the critical Reynolds number as 520 and corresponding alpha delta star is 0.3. And, you can see that for inviscid stability for c i is equal to 0 the critical Reynolds number is lesser than the critical Reynolds number for viscous stability.

For inviscid stability, if Re tends to infinity; obviously the flow will remain unstable. However, for boundary layer flow we have seen that as Re tends to infinity the flow will become stable.

Thank you.