

**Viscous Fluid Flow**  
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**Module - 08**  
**Laminar Boundary Layers - II**  
**Lecture - 03**  
**Separation of Boundary Layer**

Hello, everyone. So, we have already discussed about the Boundary Layers, but boundary layer has tendency to separate. If boundary layer is separated then the wake will be formed and due to this wake formation there will be much pressure difference which leads to increase the form drag, but in most of the application it is not desirable to increase the form drag.

So, we will discuss in today's class the Separation of Boundary Layers and how to control the boundary layer separation.

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### Separation of Boundary Layer

Condition for separation

Pressure gradient  $\frac{\partial P}{\partial x}$

$\frac{\partial P}{\partial x} > 0$  adverse pressure gradient  
- pressure will increase in flow direction.

$\frac{\partial P}{\partial x} < 0$  favourable pressure gradient  
- pressure will decrease in flow direction

$\frac{\partial P}{\partial x} = 0$  zero pressure gradient.

Bernoulli equation:

$$U \frac{dU}{dx} = -\frac{1}{\rho} \frac{\partial P}{\partial x} \quad U = U(x)$$

So, first let us discuss about the condition for boundary layer separation. So, that will decide based on the pressure gradient. So, we have the pressure gradient let us say the axial pressure gradient is  $\frac{\partial p}{\partial x}$ . So, we can have either favourable pressure gradient or adverse pressure gradient.

So, obviously, if  $\frac{\partial p}{\partial x}$  greater than 0, then we know that we have adverse pressure gradient. So, what does it mean? That in the adverse pressure gradient the pressure will increase in the flow direction. And in most of the cases whatever we have discussed in this course we have considered favourable pressure gradient when  $\frac{\partial p}{\partial x}$  is less than 0.

So, this is your favourable pressure gradient and you know that from high pressure region to low pressure region these flow occurs. So, in this case pressure will decrease in flow direction and in some cases we have considered  $\frac{\partial p}{\partial x}$  is equal to 0; that means, it is zero

pressure gradient. So, we have the this example that flow over flat plate in this particular case  $\frac{dp}{dx}$  is equal to 0 which is your zero pressure gradient.

So, obviously, you can see that in the boundary layer this pressure gradient is determined by the outer flow and from Bernoulli's equation we know; Bernoulli's equation we know that outer flow velocity if is  $U$  then  $U \frac{dU}{dx}$  is equal to  $-\frac{1}{\rho} \frac{dp}{dx}$ . So, that we have already discussed where  $U$  is function of  $x$  ok. So, for flow over flat plate; obviously,  $U$  is constant. So,  $\frac{dp}{dx}$  becomes 0.

Now, let us discuss that when this flow separation occurs and what is the onset of flow separation? So, obviously, we have already discussed that when the velocity gradient at the wall becomes 0, then you can see that fluid particle will not fill any stresses. To discuss about the onset of vortex setting let us talk about this velocity gradient at the wall. So, you know that at the wall we have the velocity gradient should be 0 and the shear stress will become 0.

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### Separation of Boundary Layer

The point of separation is defined as the threshold of forward and reverse flow in the immediate neighbourhood of the wall.

At the point of separation,

$$\left. \frac{\partial u}{\partial y} \right|_{y=0} = 0$$

Shear stress at the wall is zero.

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = 0$$

So, we can see that the point of separation is defined as the threshold of forward and reverse flow in the immediate neighbourhood of the wall ok. So, you can see that at the point of separation  $\frac{\partial u}{\partial y}$  at  $y$  is equal to 0 should be 0; that means, shear stress at the wall is zero, right because we know  $\tau_w$  is equal to  $\mu \frac{\partial u}{\partial y}$  so, at  $y$  is equal to 0.

So, if at the point of separation  $\frac{\partial u}{\partial y}$  at  $y$  is equal to 0 is 0. So,  $\tau_w$  will become 0 this velocity gradient at the wall becomes 0 when we have adverse pressure gradient; that means,  $\frac{\partial p}{\partial x}$  greater than 0. So, if further this adverse pressure gradient continues then there will be reverse flow in the flow field.

So, now we will discuss about this flow separation in terms of the velocity gradient. So, we will consider first favourable pressure gradient, then we will discuss about the velocity

gradient and then we will consider the adverse pressure gradient and we will show how the flow separates.

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### Separation of Boundary Layer

Favourable pressure gradient

Boundary Layer equation,

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$$

At wall,  $u=0, v=0$ ,  $\frac{\partial^2 u}{\partial y^2} \Big|_{y=0} = \frac{1}{\mu} \frac{\partial p}{\partial x} \leftarrow$

For favourable pressure gradient,

$$\frac{\partial p}{\partial x} < 0 \quad \frac{\partial^2 u}{\partial y^2} \Big|_{y=0} < 0$$

At the free stream (edge of BL)  $\frac{\partial^2 u}{\partial y^2} < 0$

The curvature of a velocity profile  $\frac{\partial^2 u}{\partial y^2}$  is always negative.

So, you can see this is for favourable pressure gradient, this is the boundary layer flow. So,  $u$  is function of  $y$ ; obviously, at wall you have  $u$  is equal to 0 and at  $y$  tends to infinity  $u$  is infinity. So, we have boundary layer equation in general we are considering. So, we have  $u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$ . So, we are considering in general case so,  $\frac{\partial p}{\partial x}$  is present, but for flow over flat plate this will be 0.

So, now if you satisfy this boundary layer equation at the wall so, if we consider the wall then, at wall, we know  $u$  is equal to 0,  $v$  is equal to 0. So, what will be this equation at the

wall? So, obviously, you can see that  $u$  is 0,  $v$  is 0. So, this will become  $\frac{\partial^2 u}{\partial y^2}$  is equal to; so, it is  $\mu$  by  $\rho$  so, it will be  $\frac{1}{\rho} \frac{\partial p}{\partial x}$ , ok.

So, now we will explain the flow separation in terms of the second derivative of this velocity ok. So, this is  $\frac{\partial^2 u}{\partial y^2}$  at  $y$  is equal to 0. So, now, you consider a favourable pressure gradient. So, for favourable pressure gradient we know that  $\frac{\partial p}{\partial x}$  is less than 0, right for favourable pressure gradient ok.

If  $\frac{\partial p}{\partial x}$  is less than 0, from this equation what you will get? We will get  $\frac{\partial^2 u}{\partial y^2}$  at  $y$  is equal to 0 should be less than 0. So, that means, you can see that the curvature of a velocity profile  $\frac{\partial^2 u}{\partial y^2}$  at  $y$  is equal to 0 it should be negative.

So, now from this profile if you plot the  $\frac{\partial u}{\partial y}$ , so, you can see that it is velocity gradient will be highest at the wall and it will decrease as  $y$  increases and at  $y$  tends to infinity obviously, it will become 0 because you have free stream velocity is function of  $x$  only, it is not function of  $y$ . So, this velocity gradient will tend to 0.

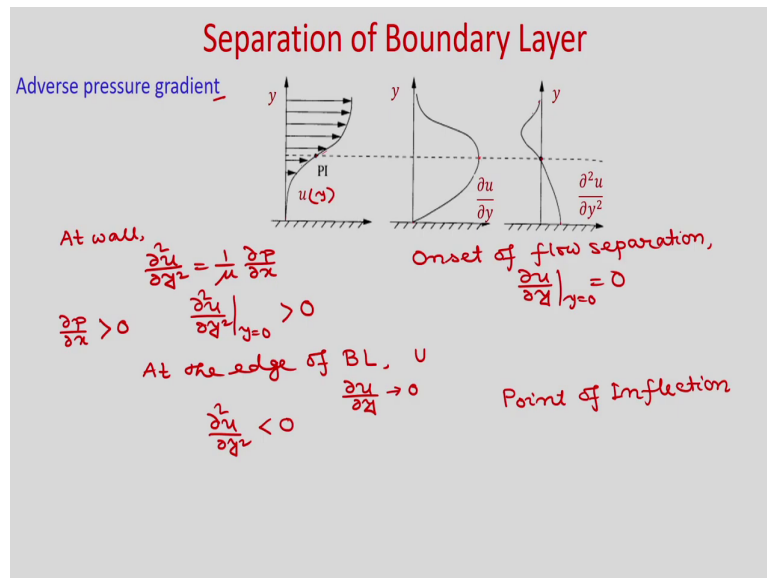
Now, if you plot  $\frac{\partial^2 u}{\partial y^2}$  ok. So, from here you can see that at the wall it should be negative. So, you can see that it will be a negative value and now, if you go further towards the free stream then obviously, as it is you can see that it is continuously decreasing. So, this  $\frac{\partial^2 u}{\partial y^2}$  also will decrease and it will be tending to 0 near to the free stream ok.

So, that means, at the free stream or edge of the boundary layer this  $\frac{\partial^2 u}{\partial y^2}$  also will be less than 0. It will be tending to 0, so obviously,  $\frac{\partial^2 u}{\partial y^2}$  will be less than 0. So, that means, that the curvature of a velocity profile  $\frac{\partial^2 u}{\partial y^2}$  is always negative in this case ok. So, from here we can see that for favourable pressure gradient this curvature of  $\frac{\partial^2 u}{\partial y^2}$  will be always negative. So, we have plotted here you can see.

Now, let us consider adverse pressure gradient; that means,  $\frac{\partial p}{\partial x}$  is greater than 0 and if  $\frac{\partial p}{\partial x}$  greater than 0, then at the wall you can see that  $\frac{\partial^2 u}{\partial y^2}$

will be greater than 0 because  $\frac{\partial p}{\partial x}$  is greater than 0. What about the free stream condition? So, in the free stream you can see that  $\frac{\partial^2 u}{\partial y^2}$  will be less than 0.

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So, let us discuss about this. So, you can see that for adverse pressure gradient we have this  $\frac{\partial^2 u}{\partial y^2}$  is equal to  $\frac{1}{\mu} \frac{\partial p}{\partial x}$  right at wall. So, obviously, you can see that for adverse pressure gradient; that means, when  $\frac{\partial p}{\partial x}$  is greater than 0 this  $\frac{\partial^2 u}{\partial y^2}$  at  $y=0$ ; that means, at the wall will be greater than 0.

Now, we have already discussed that the onset of flow separation will be when  $\frac{\partial u}{\partial y}$  will be 0 at the wall. So, we are considering this particular case when we have onset of flow separation ok; that means,  $\frac{\partial u}{\partial y}$  at  $y=0$  is 0. So, we are considering onset of flow separation and you know that it will occur when we have adverse pressure gradient; that

means,  $\frac{\partial p}{\partial x}$  is greater than 0 and at the point of onset of flow separation  $\frac{\partial u}{\partial y}$  at the wall will be 0.

So, this particular case we are considering. So, in this case you can see the velocity profile  $u$  which is function of  $y$  will look like this. So, obviously, you can see that  $\frac{\partial u}{\partial y}$  is 0 at the wall, ok. So, if  $\frac{\partial u}{\partial y}$  is 0 at the wall then shear stress is also 0, right and as  $\frac{\partial u}{\partial y}$  is 0 here. So, this velocity profile will cut the boundary normally ok.

So, this is your  $u$  profile and now, if you plot the velocity profile  $\frac{\partial u}{\partial y}$ , then you can see that as  $y$  increases obviously, first it will you can see that at the edge of the boundary layer obviously, you we have  $u$  is equal to  $u_{\infty}$ , right. So, we know that at the edge of the boundary layer at the edge of boundary layer, we have this free stream velocity  $u$ , right.

So, obviously,  $\frac{\partial u}{\partial y}$  will be tending to 0 you can see and obviously, you can see that when this velocity gradient  $\frac{\partial u}{\partial y}$  decreases near to the free stream then  $\frac{\partial^2 u}{\partial y^2}$  also will be negative and will be tending to 0 near to the edge of the boundary layer.

So, that means, you have  $\frac{\partial^2 u}{\partial y^2}$  at the edge of boundary layer will be less than 0, ok. So, you can see for this particular case when you are considering adverse pressure gradient  $\frac{\partial^2 u}{\partial y^2}$  at the wall it is positive and  $\frac{\partial^2 u}{\partial y^2}$  near to the edge of the boundary layer it is tend into 0; that means, it will be negative. And, obviously, so, there will be some point where  $\frac{\partial^2 u}{\partial y^2}$  will become 0.

So, you can see when this curve from positive to negative it goes so, it becomes 0 here and this point is known as point of inflection. So, you can see when  $\frac{\partial^2 u}{\partial y^2}$  is 0 then  $\frac{\partial u}{\partial y}$  becomes maximum in the flow domain at this point and this point you can see in the velocity profile  $u$ , this point is known as point of inflection. So, this point is known as point of inflection ok.

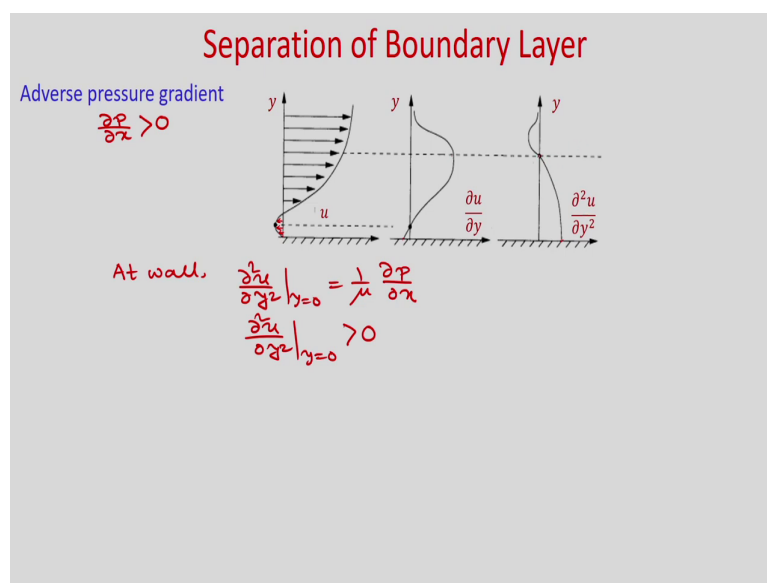
So, that means, when we have this flow separation there will be also point of inflection in the velocity profile. So, you can see that it changes its gradient ok. So, at this point this velocity



profile  $u$  vs  $y$  it changes its gradient ok. So,  $\frac{\partial u}{\partial y}$  becomes maximum at the point of inflection.

So, if we increase further this adverse pressure gradient then there will be flow separation and there will be a reverse flow. So, let us consider the case when we have adverse pressure gradient and  $\frac{\partial p}{\partial x}$  is higher.

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So, you can see in this case when we have adverse pressure gradient  $\frac{\partial p}{\partial x}$  when greater than 0. So, in this case you can see that these fluid particles actually which we are having 0 shear stress, so obviously, if you increase further this pressure gradient there will be a reverse flow ok.

So, and if reverse flow occurs if you see that obviously, then  $\frac{\partial u}{\partial y}$  will have some value at  $y$  is equal to 0 and  $\frac{\partial^2 u}{\partial y^2}$  will be positive because from this equation at wall obviously, will have  $\frac{\partial^2 u}{\partial y^2}$  at  $y$  is equal to 0 is equal to  $\frac{1}{\mu} \frac{\partial p}{\partial x}$ .

So, if we have adverse pressure gradient  $\frac{\partial p}{\partial x}$  is greater than 0. So,  $\frac{\partial^2 u}{\partial y^2}$  at  $y$  is equal to 0 will be greater than 0 and this will be 0 and near to the free stream obviously, it will be less than 0. So, there will be point of inflection and you can see this point of inflection here and in this case you can see that  $\frac{\partial^2 u}{\partial y^2}$  from positive value to negative value it goes and at some point  $\frac{\partial^2 u}{\partial y^2}$  becomes 0 in the flow domain.

So, you can see that for the flow separation we need that adverse pressure gradient and for the reverse flow we need higher adverse pressure gradient. So, you can see that always if  $\frac{\partial p}{\partial x}$  is greater than 0; that means, we have adverse pressure gradient there may not be any flow separation. But, to have the flow separation we should have point of inflection in the velocity profile.

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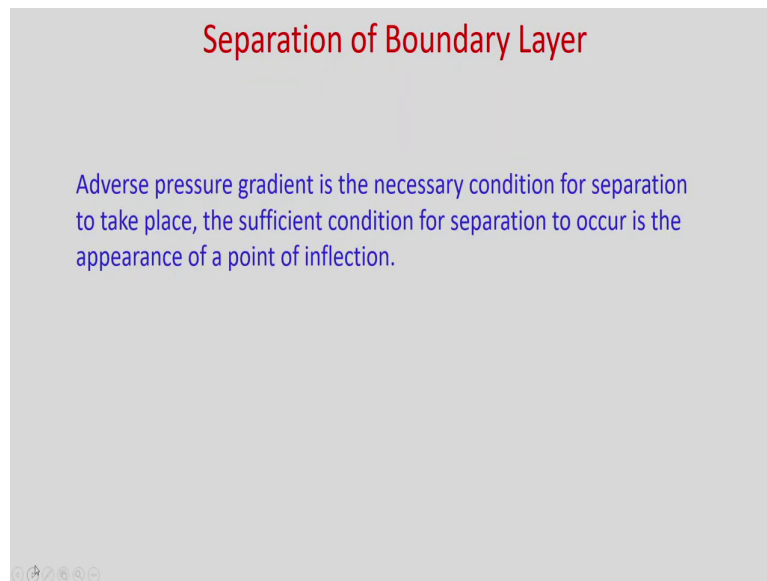
**Separation of Boundary Layer**

For the onset of flow separation,  
At wall,  $\frac{\partial u}{\partial y} = 0$ ,  $\frac{\partial^2 u}{\partial y^2} > 0$  [adverse pressure gradient]  
At the edge of the boundary layer,  $\frac{\partial^2 u}{\partial y^2} < 0$

So, if you see that for the onset of flow separation,  $\frac{\partial u}{\partial y}$  should be 0 at the wall and  $\frac{\partial^2 u}{\partial y^2}$  should be greater than 0; that means, we have adverse pressure gradient right adverse pressure gradient. So, that means, this should be at wall and at the edge of the boundary layer  $\frac{\partial^2 u}{\partial y^2}$  is always negative, right  $\frac{\partial^2 u}{\partial y^2}$  should be less than 0.

So, if there is a point of separation then there must exist a point of inflection in the velocity profile, right. We have seen that if we go higher this adverse pressure gradient then  $\frac{\partial^2 u}{\partial y^2}$  will change the sign from the positive at the wall to negative at the edge of boundary layer. So, you can see that point of inflection can be definitely associated with an adverse pressure gradient.

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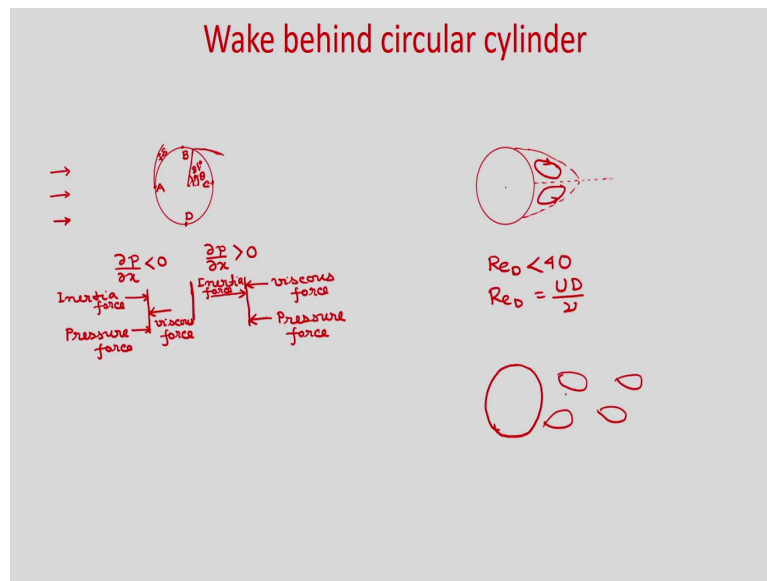
**Separation of Boundary Layer**

Adverse pressure gradient is the necessary condition for separation to take place, the sufficient condition for separation to occur is the appearance of a point of inflection.

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So, from here let us summarize that adverse pressure gradient is the necessary condition for separation to take place, but the sufficient condition for separation to occur is the appearance of a point of inflection in the flow domain.

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So, now let us discuss about the separation when it happens for flow over a circular cylinder. So, we are considering laminar flow over this circular cylinder of diameter  $d$ . So, some uniform flow is coming. So, this is the circular cylinder we are considering 2-dimensional situation. So, you can see when the flow comes in the front there will be one stagnation point ok. So, that is A and let us say this is B, this is C and this is D.

So, you can see A and C will be the stagnation point. When flow will come obviously, it will heat at this point A and the velocity will become the pressure heat in this stagnation point. This flow when pass over this cylinder obviously, you can see that when it will go from A to B the flow area decreases.

So, obviously, you can see there will be a favourable pressure gradient. So,  $\frac{\partial p}{\partial x}$  will be less than 0 when it passes through this section A to B ok. So, B is the position if you

measure theta from here. So, theta around say if you measure theta from this point then at theta is equal to 90 degree. So, area decreases so, velocity increases. So,  $\frac{dp}{dx}$  will be less than 0. Then when this flow passes through this section B to C, so obviously, you can see that flow area increases.

So, there will be some adverse pressure gradient. So,  $\frac{dp}{dx}$  will be greater than 0 in this area B to C. So, now, if you see the forces what are the forces acting? Say, if you this is the theta is equal to let us say 90 degree. So, obviously, in the A to B position you can see there will be inertia forces acting in this direction inertia force and pressure force also will act in this direction.

So, there will be pressure force, but there will be viscous force acting negative direction to this inertia force and pressure force. So, this is your viscous force, but when you will come to this section B to C you will see that obviously, inertia force acts in this direction. However, you can see that viscous force will act negative to this inertia force. As you have adverse pressure gradient in this region so, pressure force also will act in the same direction of viscous force.

So, you can see that when these flow passes through this section B to C so, inertia force is balanced by viscous force and pressure force ok. So, depending on this magnitude of this pressure gradient, somewhere near to the 90 degree the fluid particles in the boundary layer are separated from the wall due to the imbalance force.

So, you can see around theta is equal to 81 degree if this is 81 degree in the around this position flow will separate, ok. So, there will be a flow separation ok. So, there will be boundary layer formation from section A this is delta and flow separation takes place around theta is equal 81 degree. So, it varies as Reynolds number varies so, due to this imbalance force. So, there will be flow separation.

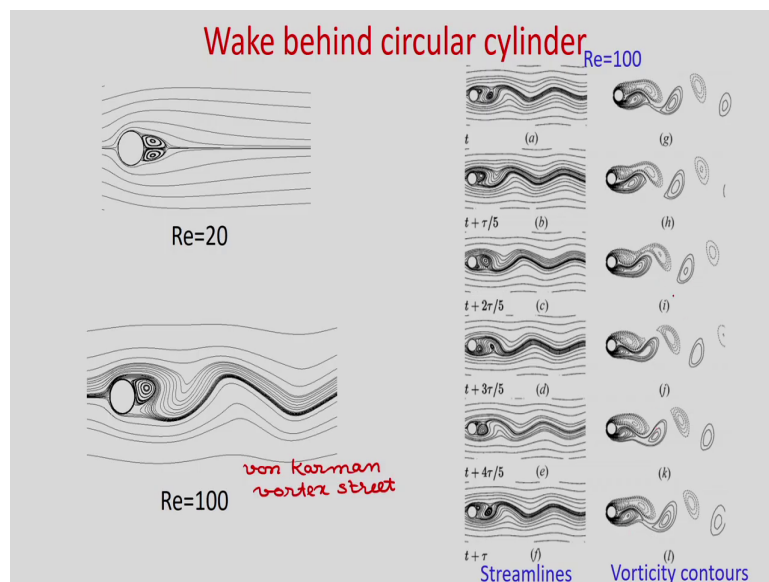
As flow separates from here so, there will be a it is from behind this cylinder and it depends on the Reynolds numbers. So, if Reynolds number less than 40 degree based on diameter ok. So, Reynolds number based on diameter; that means you have free stream velocity  $U$  and

diameter  $D$  divided by kinematic viscosity  $\nu$  then if Reynolds number is less than 40 then you can see that there will be a symmetric vortices behind this cylinder ok.

So, you can see there will be symmetric vortices behind the cylinder and it will rotate like this. As Reynolds number increases, if it is greater than 40, then what will happen? There will be oscillation of these vortices and these will periodically shaded behind the cylinder ok.

So, it will detach from the surface of the cylinder and this wake region you can see that this vortices will be shaded behind this cylinder. So, this is known as vortex shedding.

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So, you can see these are some numerical results of wake behind circular cylinder. So, if Reynolds number 20 you can see that it is less than the critical Reynolds number where the flow becomes unsteady after Reynolds number greater than 40. So, obviously, you can see

that two symmetric vortices are formed behind this cylinder ok. So, these are stream lines plot.

And, if Reynolds number greater than 40, so, for Reynolds number 100 you can see that there will be vortices shaded behind the cylinder periodically and these vortices are known as von Karman vortex street von Karman vortex street. So, here you can see in the right hand side these are some streamlines plot and vorticity contours at Reynolds number 100 for flow over circular cylinder.

So, in one time period in equi-space six figures are there. So, you can see how these vortices are formed in the upper side it increases in size, then it detaches from the surface, then another vortices starts increasing from the lower side, then it increases and then it is shaded. So, you can see these are periodically shaded behind the cylinder.

And, these are some vorticity contours and similar observation you can see that vortices are shaded behind the cylinder and this is known as von Karman vortex street. So, we have seen that if flow separation takes place then obviously, there will be high pressure difference and it will attribute to the higher form drag and in many application it is not desirable. So, we need to control the flow separation.

So, how to control the flow separation, so, that we will discuss now. So, we know that total drag is a summation of form drag and the skin friction drag. What is form drag? So, form drag is coming due to the pressure difference and skin friction drag is coming due to the shear stress on the wall. So, most of the time or in most of the application we see that form drag is dominant. So, we need to decrease the form drag to avoid the flow separation.



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### Control of Boundary Layer Separation


In order to reduce the form drag, the boundary layer separation should be prevented or delayed so that somewhat better pressure recovery takes place and the form drag is reduced considerably.


So, you can see that in order to reduce the form drag the boundary layer separation should be prevented or delayed, so that somewhat better pressure recovery takes place and the form drag is reduced considerably.


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### Control of Boundary Layer Separation

Boundary layer tripping:  
*Increase in surface roughness effects in enhancing turbulence which in turn would delay separation.*

Streamlining:  


Wall suction:  


Wall blowing:  


So, how we can do that? So, one of the way is boundary layer tripping. So, what is boundary layer tripping? So, if we increase the surface roughness then it attributes to the turbulence and if turbulence occurs, then there will be delay in flow separation and we will have less form drag.

So, in boundary layer tripping, so, you can see that increase in surface roughness increase in surface roughness effects in enhancing turbulence which in turn would delay separation. Flow separation can be avoided in streamlining the body. So, that means, you can see that if you have a streamline body then there will be less form drag.

So, you can see that if you have a shape like this which is known as black body, it is a circular cylinder of diameter  $d$  ok. So, if you measure that drag coefficient, then you will see that it will be around 1.2, but now if you use some streamline body let us say aerofoil like this ok.

So, in this case the drag coefficient will be 0.07 ok. So, these are the value in the range of Reynolds number 10 to the power 3 to 10 to the power 7.

So, you can see that if you streamline the body then obviously, you can reduce the form drag and that way you can avoid the flow separation because in this case you can see that due to this body shape there will be flow separation and there will be wake formation behind the cylinder. So, obviously, the wake zone is formed, then we will have higher form drag.

Another way you can control the flow separation using wall suction; that means, you can see that when the flow separates the fluid particle fills no shear stress right. So, and it is it has tendency to move from that place and from outer surface there will be other fluid flow other fluid particle will come. So, and flow separation takes place.

So, to avoid that what you can do you can actually suck the less velocity particle inside, so that from the higher layer higher velocity particle or with higher kinetic energy particle will come to that place. So, that you can do using a wall suction and you can see that if you have a shape like this so, this is some aerofoil shape and now fluid flow is taking place.

Now, if flow separation is taking place on this boundary so, these low kinetic energy fluid particles are sucked through this. So, then obviously, higher kinetic energy particle will come here and flow separation will be delayed. Another way to control the boundary layer separation is the wall blowing.

So, obviously, you can see that if you have this aerofoil and the lower kinetic energy fluid particle will be residing near to the wall and if you can actually inject some high velocity from this porous wall, then those particle will get more kinetic energy ok.

So, there was say due to the onset of flow separation maybe these fluid particles are having less kinetic energy now you are actually blowing high kinetic energy fluid particle and obviously, this will replace this low kinetic energy fluid particle and flow separation maybe delete.

So, in today's class we discuss about the onset of flow separation. So, we have seen that if we have a adverse pressure gradient then the onset of flow separation will take place when the velocity gradient at the wall is 0; that means,  $\frac{\partial u}{\partial y}$  at wall is equal to 0. If you further increase the adverse pressure gradient then obviously, there will be a point of inflection in the velocity profile.

So, how it occurs? So, for that we have actually seen the boundary layer equation and we have seen the value of  $\frac{\partial^2 u}{\partial y^2}$  in the flow domain we have seen that when we have adverse pressure gradient, then  $\frac{\partial^2 u}{\partial y^2}$  is always positive at the wall. However, this value is always negative at the edge of boundary layer.

So, obviously, you can see that  $\frac{\partial^2 u}{\partial y^2}$  will change the sign from positive to negative in the domain. So, obviously, you can see that to have the flow separation we should have this point of inflection in the flow domain when we have adverse pressure gradient, then we discussed different ways to control the boundary layer separation to reduce the form drag.

Thank you.