Viscous Fluid Flow

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Module - 06
Lubrication Theory
Lecture - 03
One Dimensional Slider Bearing

Hello everyone. So, in last class, we derived the general Reynolds equation for lubrication to estimate the pressure distribution inside the slider bearing. Today, we will carry out the analysis of One Dimensional Slider Bearing. This slider bearing is also known as slipper bearing or thrust bearing.

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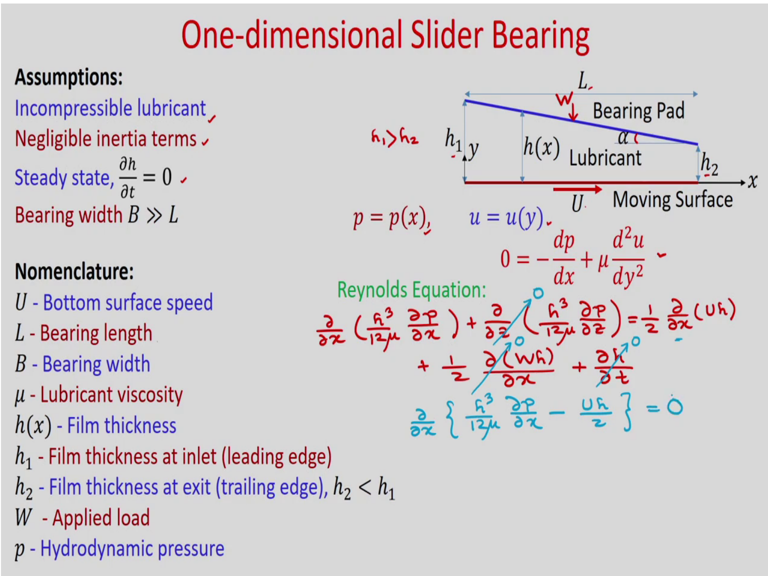

So, this is the schematic of this one-dimensional slider bearing. So, bottom surface is the moving surface with a constant speed U and the upper surface is the bearing pad of length L. You can see that this bearing pad makes an small angle alpha with the moving surface. So, inside we have lubricant, and we will make these assumptions, while doing this analysis.

We will use incompressible lubricant; we will neglect the inertia terms and we will consider steady state. That means, del h by del t is equal to 0. What does it mean? That there is no normal velocity or vertical velocity in y direction. That means, B is equal to 0. And the bearing width, this perpendicular to the surface in the z direction is B and this B is much much greater than L. So, the any gradient in this direction, you can neglect and the velocity in perpendicular to the surface is also 0.

So, obviously, from this figure you can see that p is function of x only and u is function of y only and we have this nomenclature U is the bottom surface speed, L is the bearing length, B is the bearing width, mu is the lubricant viscosity, h x. So, at any location x, this is the film thickness, h 1 is the film thickness at inlet; that means, that the leading edge and h 2 is the film thickness at exit or trailing edge and h 1 is greater than h 2. And W is the applied load on the bearing pad and p is the hydrodynamic pressure.

So, you can see that for this one-dimensional case, we have only this governing equation to find the velocity distribution and in general, we have derived this Reynolds equation in last class del of del x h cube by 12 mu del p by del x plus del of del z h cube by 12 mu del p by del z is equal to half del of del x of U h plus half del W h by del x plus del h by del t. So, this is the general Reynolds equation we derived.

Now, invoking this assumptions, we can simplify this equation. Obviously, we have seen that width is very very large compared to the length of the bearing pad. So, del of del z of any quantity will be 0. This will also be 0 and as it is steady state that is no vertical velocity v. So, this will also be 0. So, you can see after simplification, we can write the Reynolds equation for this one-dimensional slider bearing as del of del x h cube by 12 mu del p by del x minus U h by 2. So, this term, we have taken in the left hand side is equal to 0.

So, while carrying out this analysis, first let us find the velocity distribution, then we will calculate the volumetric flow rate and then, we will use the lubrication, this Reynolds equation to find the pressure distribution inside this slider bearing.

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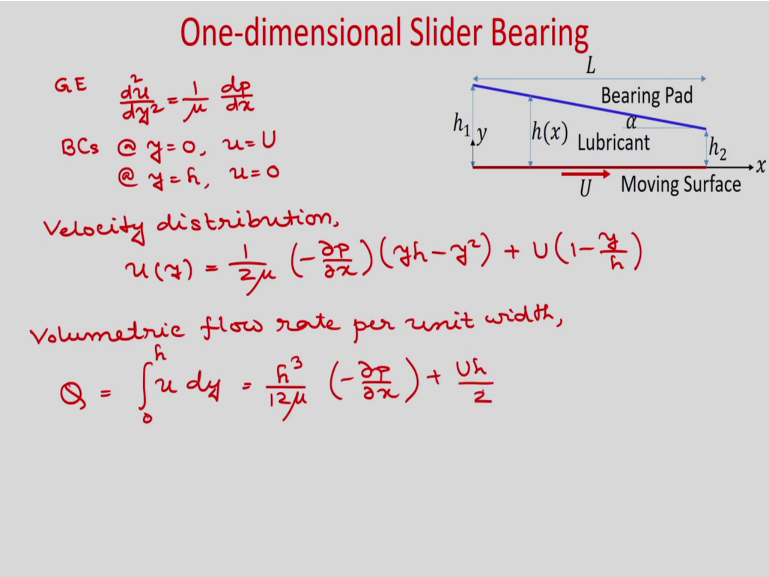

So, we have the governing equation to find the velocity distribution as d 2 u by d y square is equal to 1 by mu d p by d x; p is function of x only because del p by del y is equal to 0 and in z direction del p by del z is equal to 0. So, p is function of x only; similarly, u is function of y only. So, we have the boundary conditions at y is equal to 0; u is equal to this moving surface velocity U and at y is equal to h, h is function of x, u is equal to 0. So, this bearing pad velocity obviously, it is stationary. So, it is 0.

So, if you invoke these boundary conditions and if you integrate this governing equation, find the constants then finally, you can write the velocity profile u which is function of y as 1 by twice mu minus del p by del x y h minus y square plus U 1 minus y by h ok. So, now, let us find the volumetric flow rate and this let us calculate per unit width; that means, per unit B ok. So, B is the width. So, per unit width, we will calculate ok.

So, this Q, now we can write as integral 0 to h u dy ok. So, per unit width we are calculating. So, we are not writing here B. So, if you perform this integration, invoking this velocity distribution, you will find this as h cube by 12 mu minus del p by del x plus U h by 2. Now, let us consider the Reynolds equation and let us find the pressure distribution.

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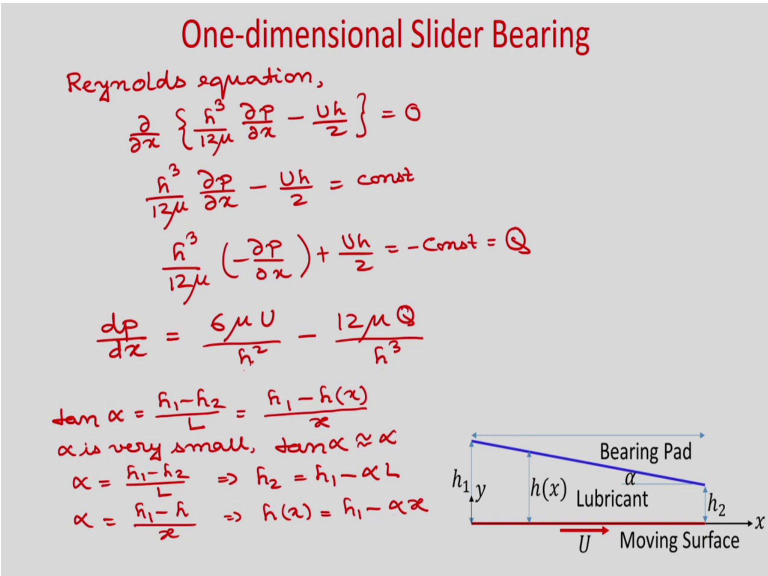

So, Reynolds equation for this particular case, we have simplified as del of del x h cube by 12 mu del p by del x minus U h by 2 is equal to 0. So, if you integrate this, then you will get h cube by 12 mu del p by del x minus U h by 2 is equal to some constant ok or if you modify it, so if you multiply both side with a negative sign, then you can write h cube by 12 mu minus del p by del x plus U h by 2. So, this will be also this minus constant and you can see this is nothing but the volumetric flow rate which we have derived. So, it is Q ok.

So, from here, now you can find. So, as p is function of x, we can write d p by d x instead of del p by del x is equal to 6 mu u by h square minus 12 mu Q divided by h cube. So, now we have found the pressure gradient ok. So, now, let us find this h as a function of alpha and the height h 1 and h 2.

So, you can see that we can write tan alpha. So, from this figure you can see you can write tan alpha is equal to h 1 minus h 2 divided by L right and this also, you can write as h 1 minus h which is function of x divided by any length x. So, any length x if you see it is h x, then you can write h 1 minus h x divided by x.

So, as we know that in this particular case alpha is very small. So, you can write tan alpha is equal to alpha. So, from here, you can see that we can write alpha is equal to h 1 minus h 2 divided by L; that means, h 2 is h 1 minus alpha L or alpha is equal to h 1 minus h divided by x. So, we can write h which is function of x as h 1 minus alpha x. So, now, let us substitute this h in terms of this x and let us integrate.

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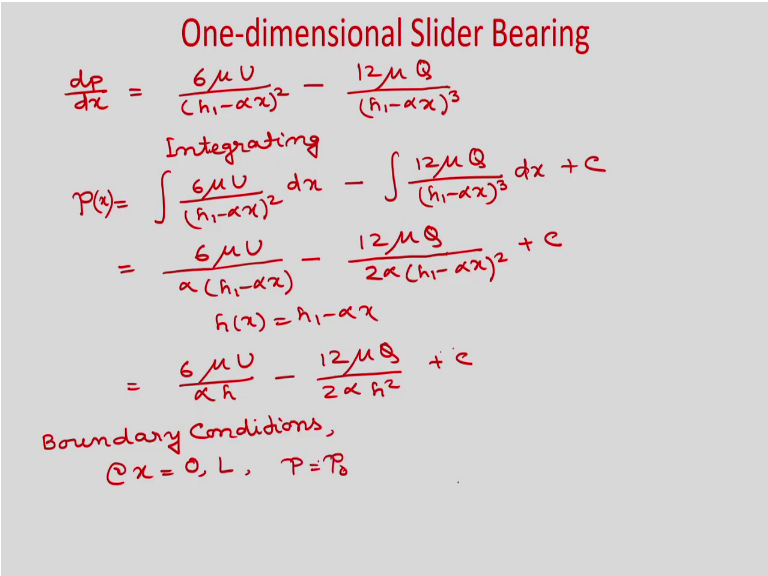

So, if you substitute it, you will get d p by dx is equal to 6 mu u divided by h 1 minus alpha x square minus 12 mu Q divided by h 1 minus alpha x whole cube ok. So, after integrating, you will get p is equal to integral 6 mu u by h 1 minus alpha x square d x minus integral 12 mu Q divided by h 1 minus alpha x whole cube d x plus some constant c. So, we perform this integration. So, you will get 6 mu U divided by alpha h 1 minus alpha x minus 12 mu Q divided by 2 alpha h 1 minus alpha x square plus constant C.

Now, this h 1 minus alpha x, we will write in terms of h. So, you can write it as 6 mu u by alpha h minus 12 mu Q divided by 2 alpha h square plus c. So, now, this integration constant, we will find invoking the boundary conditions. So, you can see that as two sides are open to atmosphere. So, we have the same pressure at exit and at inlet.

So, we can write at x equal to 0 and at x equal to L, p is equal to p 0. So now putting these boundary conditions at x equal to 0 and x equal to L, p is equal to p 0 let us first find the volumetric flow rate Q and then, we will find the integration constant C.

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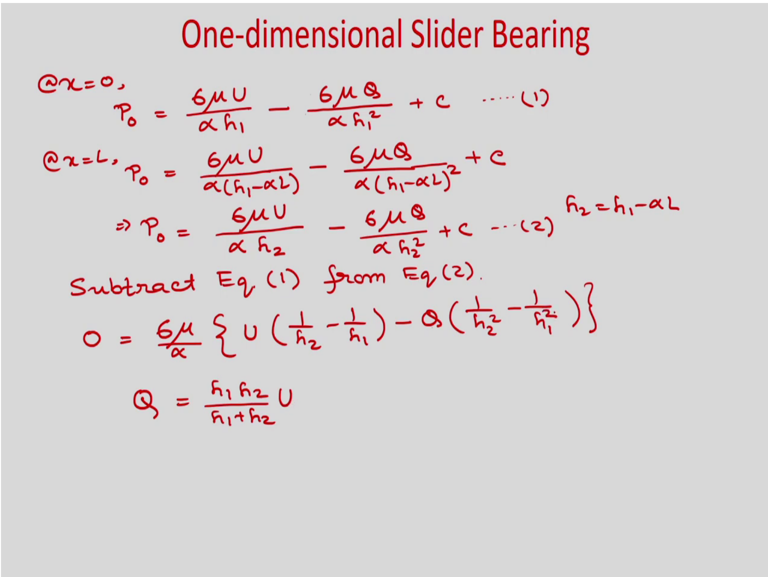

So, if you put at x equal to 0, then you will get p 0 is equal to 6 mu U divided by. So, at x equal to 0 if you put, you will get alpha into h 1 minus 6 mu Q divided by alpha h 1 square plus c.

And if you put at x equal to L, then you will get p is equal to p naught and you will get 6 mu U divided by alpha h 1 minus alpha L because x is equal to L minus 6 mu Q divided by alpha h 1 minus alpha l square plus constant c or this you can write as p naught is equal to 6 mu u divided by alpha and h 2 is h 1 minus alpha L ok. So, we will write alpha h 2 minus 6 mu Q divided by alpha h 2 square plus c.

So, let us say that this is the equation 1 and this is the equation 2. So, now, if we subtract equation 1 from equation 2, then what you will get? Subtract equation 1 from equation 2 ok. So, you can see left hand side, it will be 0 and here you will get 6 mu by alpha, you can take it outside, then we can write U 1 by h 2 minus 1 by h 1 and this you will get minus Q 1 by h 2 square minus 1 by h 1 square ok.

So, if you rearrange and if you find the Q from here, you will find that Q is h 1 h 2 divided by h 1 plus h 2 into U ok. So, from here, we have found the Q in terms of the geometrical parameters and the velocity U. So, now, if you put this Q in this equation of pressure distribution and at x equal to 0, we note p is equal to p 0 and we can find the integration constant c.

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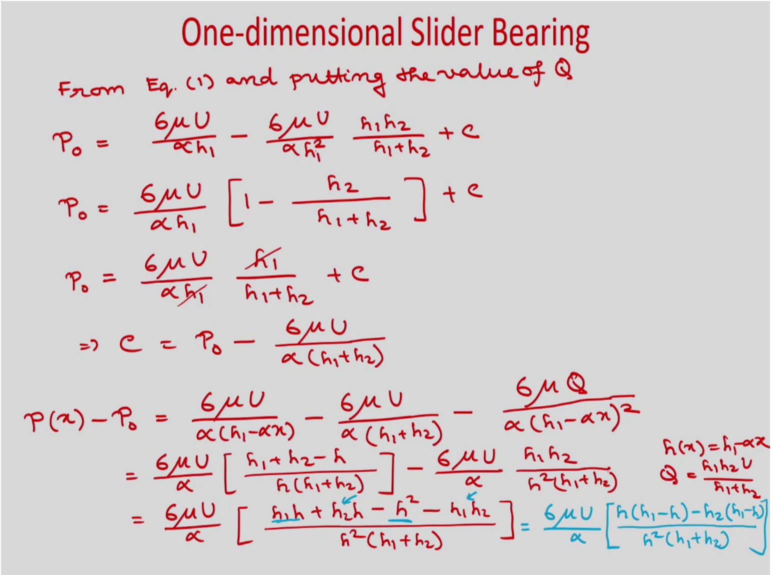

So, from equation 1 and putting the value of Q, we can now write p naught is equal to 6 mu U by alpha h 1 minus 6 mu U by alpha h 1 square and put the value of Q. So, now put the value of Q; h 1 h 2 divided by h 1 plus h 2 plus c ok. So, this U, we have already written here. So, you can see this h 1, 1 will get cancelled. So, you can write p naught is equal to 6 mu u by alpha h 1. If we take outside, so it will be 1 minus so h 2 divided by h 1 plus h 2 plus c. So, this you can write as.

So, 6 mu u by alpha h 1. So, this h 2 h 2 will get cancels, so you will get h 1 by h 1 plus h 2 plus c ok. So, from here, you can see you can find constant c as p naught; this h 1 h 1 will get cancelled. So, it will be minus 6 mu U divided by alpha h 1 plus h 2 ok. So, now, we can find the pressure distribution. So, we can see p x minus p naught, after the putting the value of c, we will get this expression; alpha h 1 minus alpha x minus 6 mu U divided by alpha h 1 plus h 2 minus 6 mu Q divided by alpha h 1 minus alpha x square.

So, now if you put the value of Q and this h 1 minus alpha x, anyway we can write as h ok and the Q value you know, it is h 1 h 2 U divided by h 1 plus h 2. So, you put all these values here. So, we will get this as 6 mu U divided by alpha. If you take it common, so you will see this is h. So, you will get h into h 1 plus h 2. So, in the numerator, you will get h 1 plus h 2 minus h minus 6 mu U divided by alpha h 1 h 2 divided by h square h 1 plus h 2 ok.

So, now if you take outside 6 mu U by alpha, then we will get h square h 1 plus h 2. So, here you will get h 1 h plus h 2 h minus h square minus h 1 h 2 So, now, you can see that here we will take common h from here and here and this and this term. So, you can see that we can write now this as 6 mu U by alpha.

Now, you can see if you take common from these two h, then you can write h 1 minus h and from here, you can see minus h 2 if you take common, then you we can write h 1 minus h from these two terms ok divided by h square h 1 plus h 2 ok. So, now, next, we will write. Next now, let us simplify it.

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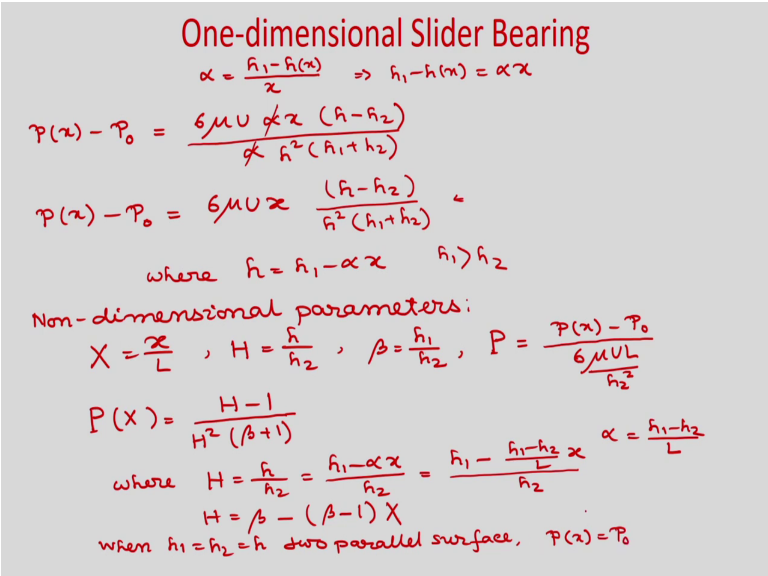

So, you can see that alpha is equal to h 1 minus h divided by x ok. So, from here, you can see that h 1 minus h x is equal to alpha x. So, from here you can see that now we can write p x minus p naught is equal to 6 mu U ok and 1 if you take h 1 minus h common and you put alpha into x and we will have h minus h 2 divided by. So, we have 1 alpha and h square into h 1 plus h 2 ok.

So, you can see now we can simplify, and we can find the pressure distribution from this equation. So, it will be 6 mu U. This alpha will get cancel. So, you will get x h minus h 2 divided by h square h 1 plus h 2 ok, where h is equal to h 1 minus alpha x and h 1 is greater than h 2 ok.

So, now, we have found the pressure distribution inside the slider bearing. So, if you can solve this equation and you will be able to find what is the pressure distribution inside the slider bearing, having the same pressure at the inlet and outlet. So, if you use some suitable non-dimensional parameter. So, you can write this pressure distribution in non-dimensional form. So, we will use these following non-dimensional parameters ok.

So, we will use x is equal to x by L, capital H as h by h 2, beta which is known as tapered ratio which is h 1 by h 2 and now, in this case capital P is the non dimensional pressure difference that is p x minus p naught divided by 6 mu U L divided by h 2 square ok. So, you can see from this expression ok, now we can write this P which is function of x as H minus 1 divided by H square beta plus 1 ok.

If you do the mathematical manipulation, then you will get finally this expression, where you can see that H is h by h 2 which is h 1 minus alpha x divided by h 2 and we can write that h 1 and now alpha, we can write as h 1 minus h 2 divided by L right. So, if you put it here. So, you will get h 1 minus h 2 divided by L into x divided by h 2.

So, now if you divide by h 2, then H you will get as h 1 by h 2 that is beta minus here you can get h 1 by h 2 that is also beta minus 1 and x by L is x ok. So, in this expression, you can see that if h 1 is equal to h 2 is equal to h. So, what will be the pressure distribution? So, obviously, you can see that this pressure difference will be 0; that means, inside the lubricant will have same pressure p naught ok. So, that means, when h 1 is equal to h 2 is equal to h; that means, you have two parallel surface ok.

So, that means, the gap is uniform. So, p x will be p naught. So, everywhere the pressure will be same; that means, the gauge pressure will be 0. So, let us find now the load capacity of the bearing because there will be a load which is acting on the normal direction. So, we have the pressure difference p x minus p naught and if we integrate this over the area flow area, then we will be able to get the load capacity.

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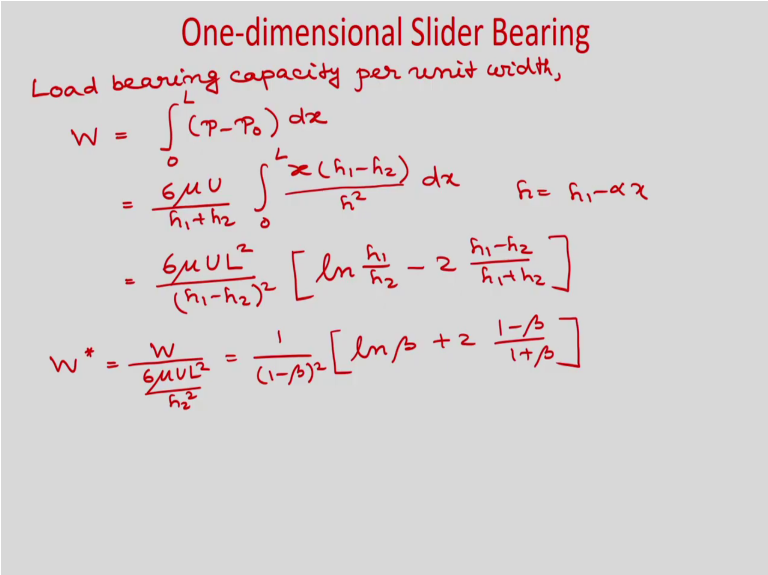

So, you can see that load bearing capacity per unit width, we will calculate now. So, W will be just integral 0 to L p minus p naught into d x ok; per unit width we are calculating. So, now we can write. So, p minus p naught we have the expression. So, you can put the constant outside the integral and you can integrate 0 to L x h 1 minus h 2 divided by h square d x ok; where, h is h 1 minus alpha x ok.

So, you perform this integration. Finally, you will get as 6 mu U L square divided by h 1 minus h 2 whole square ln h 1 by h 2 minus 2 h 1 minus h 2 divided by h 1 plus h 2 ok. So, in non-dimensional form if we write this, so let us say W star, then we will get W divided by 6 mu U L square divided by h 2 square.

Then, we can write it as 1 by 1 minus beta square because here from here, you will get and you will get ln beta plus 2 into 1 minus beta divided by 1 plus beta. Now, let us find what is the shear stress acting on the bottom plate ok and the force required to move the plate, to move the bottom plate.

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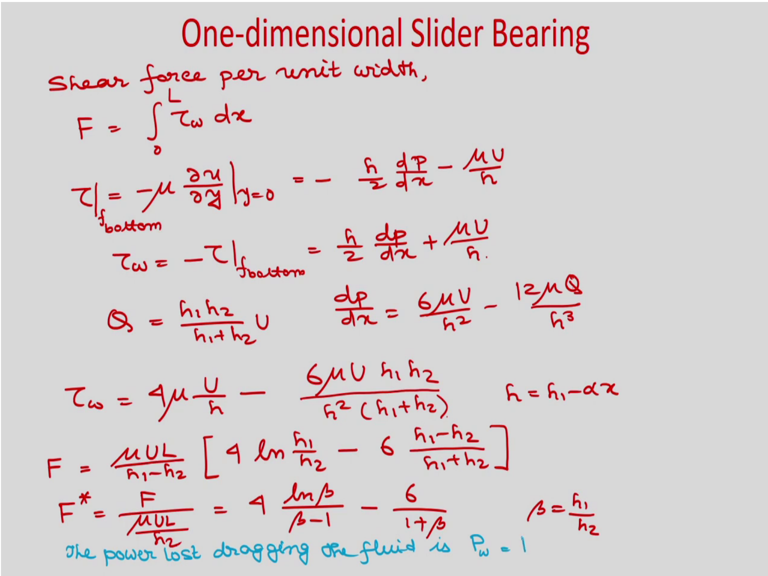

So, we can write shear force per unit width F is equal to 0 to L tau W into d a and per unit width we are writing, so d x ok and we know that tau W. So, we have tau W is the shear stress on the wall. So, obviously, if we calculate the tau that is nothing but minus mu del u by del y at y is equal to 0. So, in the fluid side ok. So, in the fluid side at bottom ok.

So, if you see. So, we have the velocity distribution from here, you will get this; you will get this as minus h by 2 del p by del x minus mu U by h ok. So, now, if you want to find the what is the shear stress on the wall. So, it will be obviously tau W will be minus tau f bottom ok because this is fluid side. So, obviously, it will be equal and opposite on the wall. So, tau W will be just minus tau. So, it will be h by 2, this you can write as d p by d x; d p by d x plus mu U by h ok.

So, now, you substitute the value of this d p by d x and invoking the value of Q, we can get. So, Q is equal to h 1 h 2 divided by h 1 plus h 2 into U and d p by d x, we have the expression 6 mu U by h square minus 12 mu Q by h cube. So, you can see this Q, you substitute it here and put it here, you will get tau W as 4 mu U by h minus 6 mu U h 1 h 2 divided by h square h 1 plus h 2 ok. So, now, if you put it here and if you integrate finally, h obviously you know that it is h 1 minus alpha x.

So, if you integrate it; finally, you will get the shear force per unit width F as mu U L divided by h 1 minus h 2 4 ln h 1 by h 2 minus 6 h 1 minus h 2 divided by h 1 plus h 2 and in non-dimensional form if you write then, F star will be F by mu U L divided by h 2 is equal to 4 ln beta divided by this h 1 minus h 2, this will become beta minus 1 minus 6 by 1 plus beta ok; beta is equal to h 1 by h 2.

So, this is the F which is the shear force per unit width and if you want to calculate the power lost that in the fluid, then you can multiply with the velocity and you will get the power lost.

So, the power lost dragging the fluid is; so, power is P w is equal F into U. Now, whatever in non-dimensional form we have represented, the pressure difference, the load, bearing load, the force, per unit width, so that let us plot with respect to either the non-dimensional length x or the taper ratio beta.

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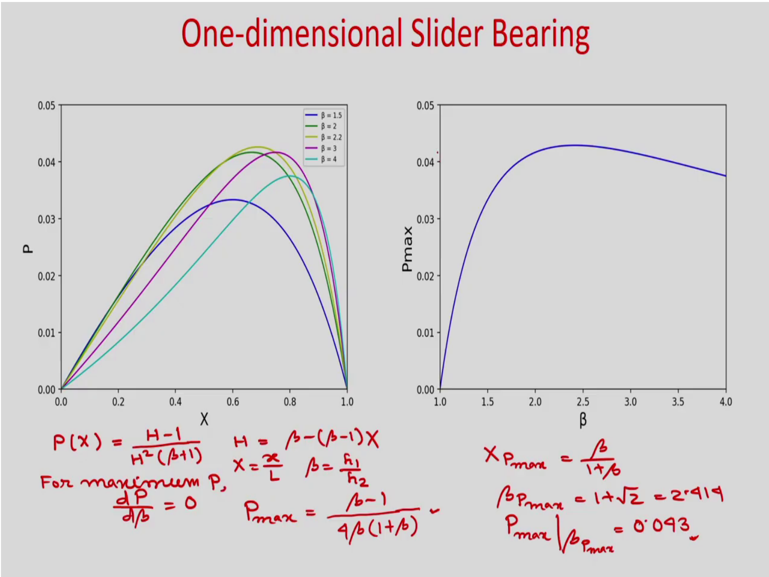

So, first let us consider the pressure difference. So, in non-dimensional form, the pressure difference we have written as P X is equal to H minus 1 divided by h square beta plus 1 ok, where H is beta minus beta minus 1 X and you know that X is non-dimensional coordinate x by L and the taper ratio beta is h 1 by h 2 ok.

So, in this figure, we are showing the this non-dimensional pressure difference P versus the non-dimensional X coordinate X. So, obviously, it will X will vary from 0 to 1. So, we have plotted for different taper ratio.

So, let us consider at beta is equal to 1.5. So, if beta is equal to 1, obviously this will be parallel surface ok. So, p minus p naught will be 0 and beta is equal to 1.5, so you can see this is the blue color. So, the pressure increases along X, it reaches one maximum point, then again it decreases and becomes 0 at the exit ok.

As you increase beta, you can see the peak increases and at beta is equal to 2.2, so this line you can see that there will be a maximum P, after that if you increase beta, the peak decreases ok. You can see for beta is equal 3 and beta is equal to 4, this peak increases, this peak decreases.

So, now, if we find that at what beta this becomes maximum. So, we can write for maximum P, d P by d beta is equal to 0 and you can see that this P max from here, you will get as beta minus 1 divided by 4 beta into 1 plus beta ok. And the location at which it will have that is X P max; it will be beta by 1 plus beta. So, you can see the location, where it is becoming maximum so that will be beta by 1 plus beta and that is P maximum value at different beta, you will get from this expression beta minus 1 divided by 4 beta into 1 plus beta.

Now, if you want to find that at if you want to find P max versus beta. So, now, this we are plotting ok. So, we are varying the taper ratio from 1 to 4; obviously, 1 is parallel surface. So, you can see this P max, gradually it increases, then it becomes maximum, then again decreases. So, with increasing beta, you can see that after a getting a optimum value, it decreases. So, if you want to find these beta at which this P max happen.

So, this will be 1 plus root 2 and it is 2.414 and the value of P max at beta, where P max is happening, so that is 0.043. So, you can see that at beta P max 2.414. So, at 2.414, this somewhere it will become maximum and this value corresponding P max value is 0.043 ok. So, it is just slightly above here. Now, we have calculated the volumetric flow rate and if we express in non-dimensional form and if we plot with beta.

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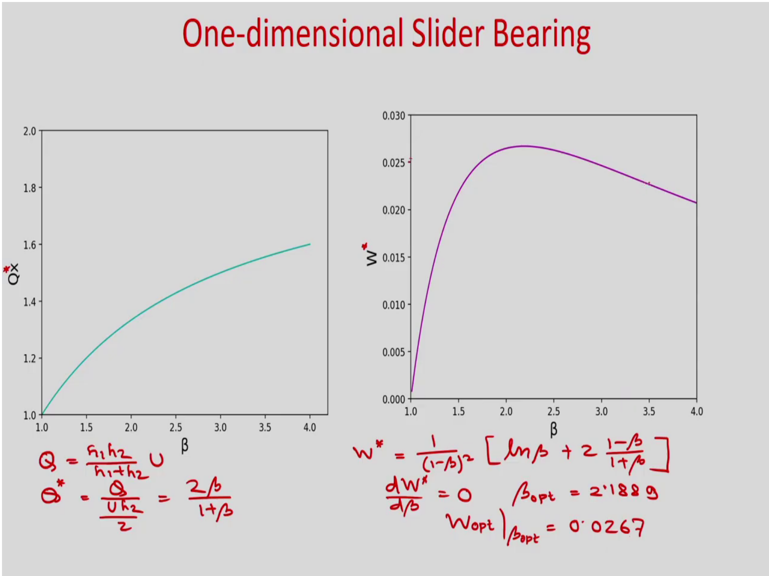

Then, we will get, so Q we have h 1 h 2 divided by h 1 plus h 2 into U and Q star will be just Q divided by U h 2 by 2, then it will be 2 beta by 1 plus beta ok. So, you can see if we plot this Q x star, this is your Q x star versus beta. So, you can see gradually, this volumetric flow rate will increase as taper ratio increases.

Now, we have this non-dimensional load bearing capacity. So, that we have expressed at as 1 minus beta square ln beta plus 2 1 minus beta divided by 1 plus beta ok. So, now, if you want to find that at what beta this W star will become maximum.

So, we can write d W star by d beta will be 0. So, you can see that beta optimum, you will get as 2.1889 and omega optimum at beta optimum, at this value, you will get as 0.0267. So, we can see that with increase of taper ratio beta, this W star gradually increases, it reaches a maximum, then again it decreases.

So, at beta optimum, you will get the W star value as 0.0267 slightly above this point and you can see there is an optimal film ratio beta that determines the largest load capacity. So, at beta optimum, you will get the largest load capacity; but if you increase further this taper ratio, load capacity will decrease ok. So, two large taper ratio act to reduce the load capacity ok.

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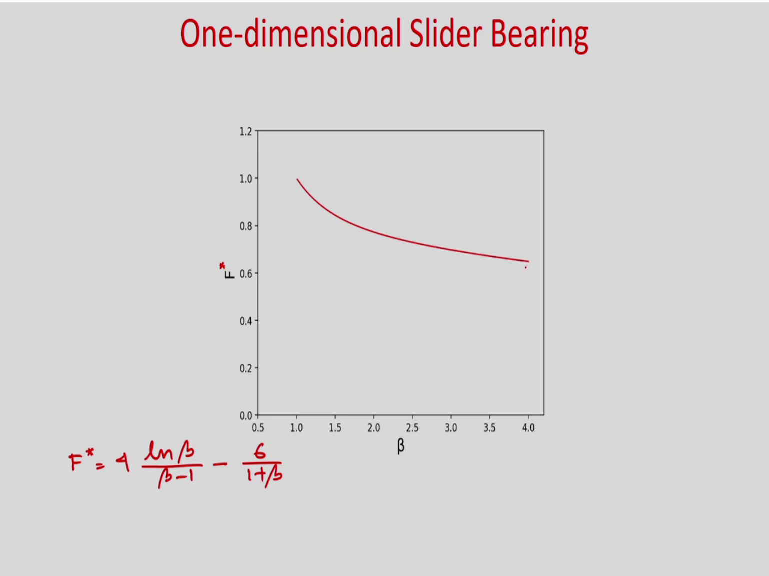

Now, the shear force per unit width in non-dimensional form, we have represented as 4 ln beta divided by beta minus 1 minus 6 1 plus beta. So, this if we plot with beta, so you can see that it gradually decreases ok. So, it gradually decreases ok.

So, in today's class, we considered one-dimensional slider bearing. We assumed that in third direction, the width is very very large compared to the length of the bearing plate and we considered one small angle of the bearing plate with respect to the bottom surface.

With these, first we calculated the velocity profile, then we calculated the volumetric flow rate and then using Reynolds's equation, we found the pressure distribution. And then we calculated the load bearing capacity, the force required to move the bottom surface, then we represented all these parameters in non-dimensional form.

Thank you.