**Plastic Working of Metallic Materials**
**Dr. P. S. Robi**
**Department of Mechanical Engineering**
**Indian Institute of Technology – Guwahati**

**Lecture - 24**
**Drawing of Rods, Wires and Tubes**

See this module will be discussing with the drawing operation, drawing of rods, wires and tubes. See in our earlier module we have discussed about for the mechanics of metalworking we have taken an example of strip drawing okay. So but here let us come to the practical case of drawing operation.

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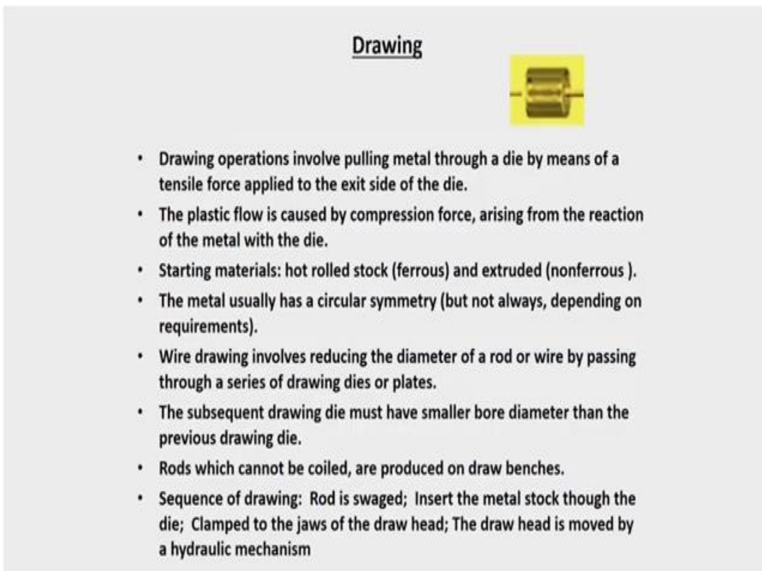


What is this drawing operation? See a drawing operation it involves pulling metal through a die by means of a tensile force applied at the exit side of the die. So you are passing this material through a die generally a converging die and then you are pulling throughout so at the diameter at the exit through a small orifice when you are pulling it out, dimension of the part which comes out will be almost similar to that of the orifice diameter of the die.

So here you are just applying a tensile load. But as I have mentioned in earlier in the mechanics of metal working also, though it is a tensile load you are applying when the workpiece material is coming in contact with the die material and it exerts a force on the die material. So that the nature of the stresses in the deformation zone, they are compressive in nature. So you will find the compressive stresses there.

The plastic flow is caused by compression force arising from the reaction of the metal with the die. The starting material are generally hot rolled stock which are steels and extruded nonferrous materials will be the starting material. You will find it is large application especially in drawing a fine wires and other things. Generally, with circular cross section, but it is not that true you can have any cross section also.

But normally it is for electrical applications and other things you will need very fine wire and very small diameter wires and other things. These are obtained by this wire drawing operation. The metal usually has a circular symmetry but not always depending upon the requirement. Wire drawing involves reducing the diameter of a rod or wire by passing through. It is not 1 die but it generally will be passing through a series of dies or maybe sometimes the die itself maybe if several dies may be kept in a single plate also.

So whatever it be normally wire drawing operation involves a series of drawing operation. Because you may have to reduce it from a larger diameter to a very fine diameter. So subsequent drawing die certain entries and the output of 1 die will be the input of the next die. So the second die should have a bore diameter much smaller than the which should be smaller than the previous die.

But output die will have a velocity which is higher than the input to the die. And similarly the next stick phase also you will find the output velocity of the pulled wire which will be higher. So you may have to have that a continuous increase in the velocity of the wire when it passes through the die is taking place. So and this finally when it comes to know it may be a coiled. So that rods which cannot be coiled or produced on draw benches.

Coiled means okay you can have a bobbin and then if you just regret coiled on to that. If it is very difficult for that then you may use a draw bench. If the diameter is very large, coiling of these wires drawn wire may be a problem or if it is a rod then coiling will be a problem. So in that case you may use a draw bench. So sequence of drawing operation it involves the rod is first swage.

Swage remains its reduced with the cross section at the tip so that is what. Otherwise you know, you will find that if it is conical die, the diameter of the bell it is large but when it has to pass through the die. So initially what has happened you may do a swaging operation by which you will reduce the tip diameter and form a conical shape. And then that part will enter into the die and comes out and two things are there.

When that happens since it is swage, the strength of the material at that tip portion where it is swage will be higher. So you can pull it through with the sufficient strength and other things. So rod is first swage then they insert the metal block through the die. After swaging that is you are inserting the metal block through the die because swaged path it will enter and come out and then it is clamped there to the jaws of a draw head.

So when you are pulling it, so the drawing head so it has to be clamped. So it cannot be done by manually. So you cannot pull it because the stresses or the total rod which will be required will be very large. So in that case it has to be clamped to drawing head so maybe if it is larger size it will have jaws holding jaws and then it will be pulled. And the draw hat is moved by a hydraulic mechanism.

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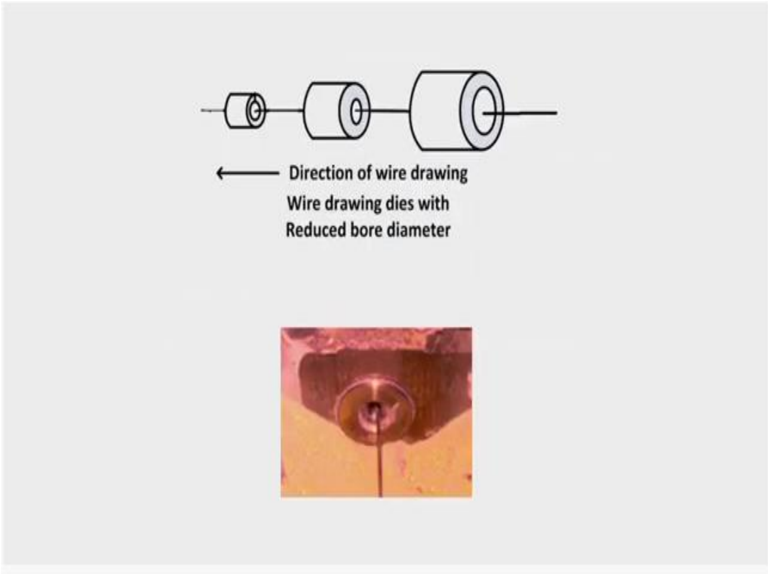


So this is the case like you may have a number of dies, one session is here, second session is there, third session is here. So once it enters the first die it comes out with a higher velocity, it enters in the second die whose diameter is very large and then it very small and then it comes out from the second die, it goes like this and your pulling will be taking place. So this is a typical photograph of a drawing die okay. So here you are just pulling in this direction so and it comes out. So this is the typical converging die is there.

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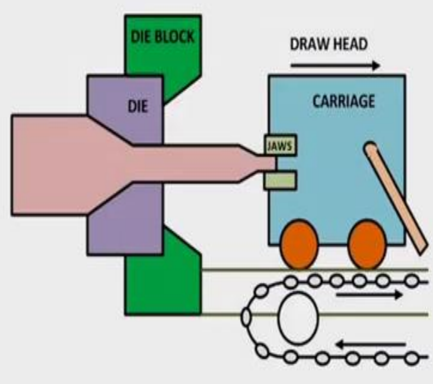

Figure : Schematic of a Draw bench

So schematic of a draw bench we can see that how a draw bench looks like that. Sometimes you know the diameter may be very large and then the stresses or the total rod drawing rod required may be very, very large. So sometimes you may have a draw head may be a big carriage itself, it may be a vehicle also. All those things are chain pull they are working on a chain pull roll. So it can have any type of things are there.

So this is the die, this part is your die. So which will be held in a die block. So this is the die block and it will be say resting on the die block and your billet you are just inserting. So this part is what you are initially doing is by swaging operation, you are reducing the cross section. And so that now it will facilitate the entry into the die or through the die opening and comes out. So that you can hold it by means of this jaws of the draw head.

It may be a simple motor mechanism, if it is coiling it is very simple or if it is very large size where it cannot be coiled then you have a carriage which will be used. So based on that you will find it okay. And then here as it moves here the metal gets drawn through the die and with a reduced diameter.

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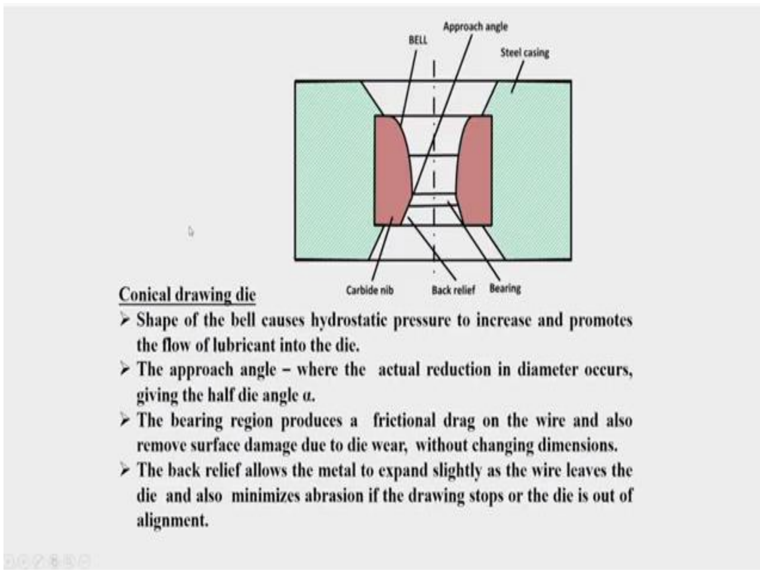


So when you look at that the die for drawing generally it is a conical drawing die which is used and this many times when the size of the wire is very small you know it becomes very small in diameter. This is a exaggerated view of a die assembly okay. So you will find that because there is a very high worker hardening which is taking place when deformation is taking place in the deformation zone.

Work hardening is taking place; the hardened material is having a relative motion with the die. So that means it will after too much of drawing operation the die may get worn off. So in generally what happened? The die part will be made of a carbide which is very hard, carbide in some cases know it may be it can be even diamond also. So for some but diamond is very costly so normally a carbide is the material which is preferred. So tungsten carbide is a material which is preferred for this die material.

So it will be like a say you call it as carbide nib, this part we can easily take it out and replace it with another die if you wanted. So that is the biggest advantage with this. It will just rest into that. So and then because it will the metal will be filling up this area and it will be moving in this direction. In this direction since it is moving it will be resting on the steel casing okay. So only thing this steel casing should have sufficient rigidity so that nothing will happen.

So it should be able to withstand the reaction forces. So now from here the metal the workpiece metal and this and then that is reduced into that and that through this material it flows into that. So when you look at that the entry of the metal see this itself is not as simply a tapered hole so you will find that at the entry there is the better shape is there. This is the type of a shape bell.

So this shape is such that it facilitates some hydrostatic pressure to increase and promote the flow of lubricant into it. So shape is such that you will be adding some lubricant into the material for that so that the friction at the interface workpiece interface is reduced okay. So when your the shape is such that now some it promotes the flow of lubricant into the die okay and at the same time it also creates the hydrostatic pressure.

There is a lubricant if it is a liquid lubricant then it can always create to some cause some hydrostatic pressure. So that is always good for this deformation. Now after that after the bell shape now you will find that there is a approach angle or the actual reduction takes place. This is the actual reduction which is taking place here okay. So this is the approach angle which is mention okay.

So it is here the major deformation is taking place. So that it reaches the final shape or final size okay. So when you talk about the die angle it is basically this die angle okay. This is the die angle when you called it as 2 alpha. This is 2 alpha is what is the die angle when you talk about it. So it is not at the bell region but at the approach angle. Then this is followed by a bearing region you see that.

There is a bearing region it produces a frictional drag on the wire and also remove surface damage due to die wire. So sometimes you know if it is not there, there may be some surface damage. So when it is passing through this region, so it will be like a parallel region. So any surface damage due to the frictional drag and other thing is taking place because it passes through that bearing region. So that will be taken care of okay.

That it will reduce that any type of otherwise any type of die wire which is taking place, surface damage which is taking place that will take care of that, but it cannot be very large. If it is very large your drawing rod has to be very large. Because your total contact area will be very high. So it should not there should be an optimum size for that. Now finally there is a relief angle okay. See because this relief angle allows the metal to expand slightly as the wire leaves the die.

Because of the spring back effect when it just comes out there will be an expansion. Because it was pulled in a tensile form. But when it comes out and other things there will be going to be a small increase in the diameter. So that to avoid that part so you are having this is a divergent type of region which is giving. So that is so it minimizes the abrasion the drawing stops or the die is out of alignment. So in that case when it will be you can easily take it out, push it back and other thing. So that is why to take care of that only this back relief is there okay.

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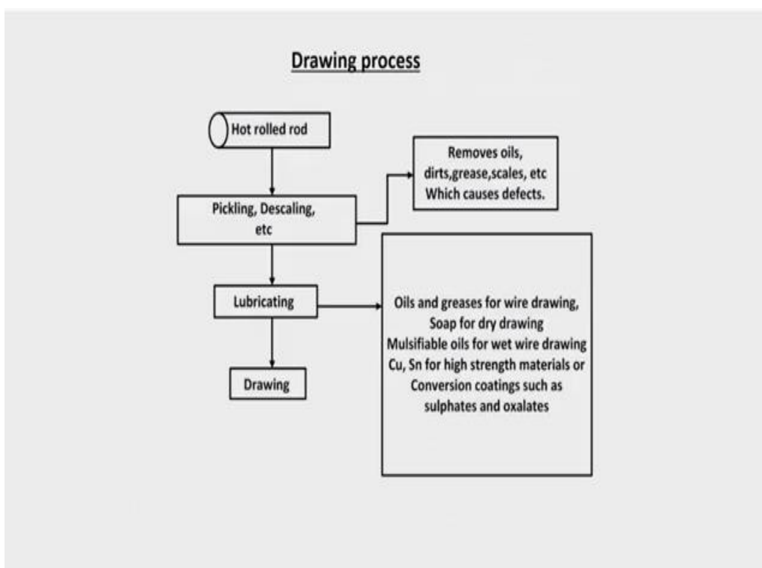


Now if we look at the drawing process the material which you are getting is normally obtained in the hot rolled form. So that hot rolled form means that in general residual stresses and other things are minimized and the material is soft. If it is cold rolled you know the material will get to work hardened and then your drawing stress will be very large not only that there will be a maximum limit for the reduction which can takes place.

So if it is a cold rolled piece, then the amount of deformation or reduction which is possible will be very less. So then further if you wanted to do give a large deformation large reduction you may have to do the process annealing or any type of annealing, so that the material is softened. So to avoid that generally for wire drawing operation the material which you are going use is the hot rolled rod.

Of course it will be at (()) (13:48) method but it will be hot rolled, so the material is very soft. It is like annealed material type thing it will look though not exactly like that. So that rolled piece you will do the pickling of and descaling all those things are there. So surface treatments are done. Because in the hot rolled operation itself there may be some oil, dirt, grease, scales, etc which may be there on the surface.

So to avoid all those things you have to do the surface heat treatment. Basically the pickling and descaling and other things. If it is steel know you have to do this descaling because that otherwise it will get embedded into that during the deformation stage. And but anyway the surface has to be cleaned. So there should not be any amount of dirt and grease or scales. So for that purpose only the surface treatment is done.

Then followed by lubricating, so if it is for small wire drawing then generally people use so oils and grease for wire drawing operation. Now if it is dry drawing you are doing then you may use a soap. So it will pass through a soap now the surface gets embedded in that. So there will be a good lubricant when it passes through that.

If it is just wet wire drawing, then you will say emulsifiable oil sorry there is a spelling mistake, emulsifiable oils are there which is commercially available in the market and then those type commercial lubricants also you can use it. Sometimes copper and tin coating is done for high strength materials. You may give a coating itself. So that this is very soft and other thing. So it will act as a lubricant and other thing.
Very thin layer for high strength material or conversion coatings such as sulphates and oxalates are also used. These are the type of lubricants which are being and then after that you will be doing the swaging operation and then you are doing the drawing operation, wire drawing operation.

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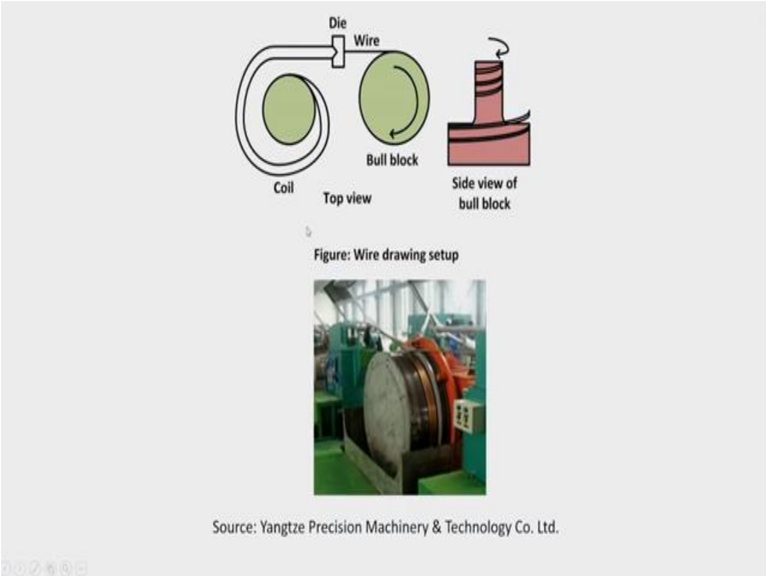


So these are the wire drawing setup and other things you will see that. So may be if it is wire drawing, it will be a small size. So it will be a round on a coil and from that you directly take it may be passing through that you will be doing all this treatments and other things here okay. So it comes out of that and then it is pulled by means of a bull block. Bull block is this type of a block which will be rotating along the vertical axis and or horizontal axis.

See here this I have taken from the net from Yangtze Precision Machinery and Technology company limited, China. So one of their product I have taken from the net really and then it shows that. This bull block will be rotating in this so that it will so because of the torque on this there will be a tensile force here and then it will be just coming out of the die and it gets round over it.

So ultimately what to do is that you get it rounded on that. So if there may be a bobbin which can be removed and then you can take it out also. This is for small wires. But if it is for larger blocks and other things then what happens is that you will be the raw head will be used. Sometimes it may be very big in size depending upon the forces.

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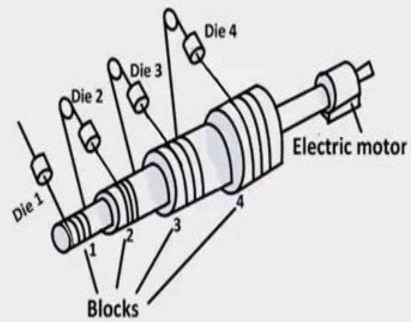

Figure : Stepped-cone multipass wire drawing

Now because many times see either you can have it in a serious arrangement which is kept there from one end to the other when it passes through that as we have shown here see in this case. So here now what happens is that, it will just a series of dies are kept and it will be pulled. Otherwise you can have it on a stepped-cone multipass wire drawing.

See as I said when it passes through 1 die say may be die 1 when it passes through that, the velocity of the wire at the exit side of the die will be higher than that the inner side of the die okay. So here it will be round around it and now what happens is that this it will just wind around it and then 1 or 2 turns and then it passes through this pulley and then again goes to the second die.

Now when it passes through a second die, the diameter at the outlet of the second die will be smaller than the inlet to the die. So here you need a higher velocity so that is taken care of by means of the diameter of this block okay. So this is a stepped-cone block, so it is a second care of that whatever be their based on this reduction what is going to the velocity here and by having the diameter here we can have the velocity which is equal to this the output in the second one okay.

Similarly, from that it goes as input to third die and then when it comes out of the third die, the velocity will be still higher to take care of the diameter is adjusted and so on this series of arrangements are there. So in a single block a stepped-cone block you can have all these things found. So that it will just pass through this after taking 1 or 2 rounds it will just pass through this.

Again taking 1 or 2 rounds, it will pass through this and like that it will run a continuous basis and at the end okay you may get it round on a bobbin or whatever. This is for the wire drawing operation. So this is much easier. So you need only a single motor, of course the motor capacity has to take care of sufficient torque which is required.

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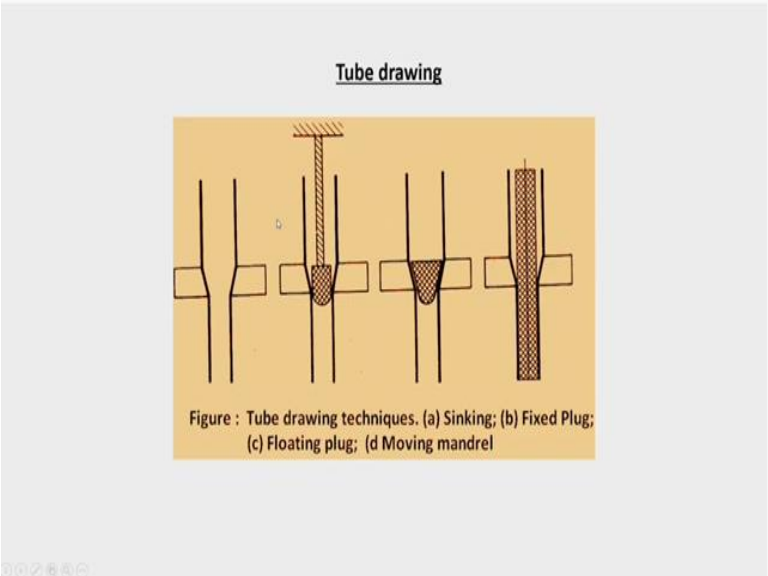


Now it is not that only rod set, you can also obtain tubes by drawing operation. So tube drawing operation is also there. So in tube drawing there are 4 types of tube drawing operation. So one is the sinking operation, this is the sinking. This is a fixed plug, drawing through a fixed tube drawing using a fixed plug and tube drawing using a floating plug and tube drawing using a moving mandrel. So we can have 4 types of what you call it as drawing tube drawing operation. So in this the first is the sinking operation.

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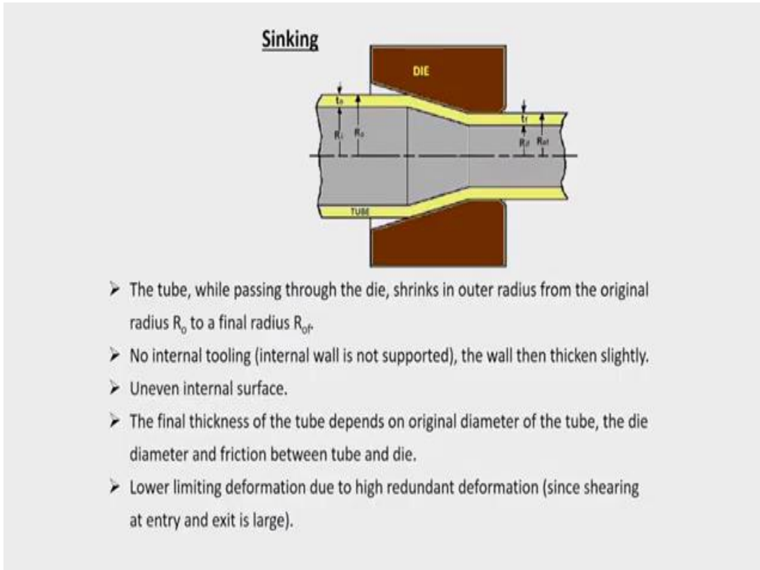


So we will just look at it one by one. So it is this is in a horizontal case what was shown earlier in a vertical case. So this is a simple thing. You have the die here and the converging die is there it comes out. So initially your diameter is very large. So you need to have one thing which is important in all these cases is that you have to have a hollow material as the input material. Your starting material should be hollow.

So that you may have to do by any other technique some other techniques you have to have a hollow tube itself. But when it is drawn through this die, a converging die you will find that so initially your diameter $R_0$ and $R_i$ as the outside and inner diameter of this. When it comes out of that you will find that the diameter has change but length has increased okay. So initial thickness is something, final thickness is something.

The tube while passing through the die you will find that there is a shrinkage at the outer diameter from the original radius $R_0$ to a final radius $R_{of}$ is there. There is no internal tooling which is required. So inner wall is not supported. So it is just free. Only the outer wall is supported so that will take care of the outside diameter of the tube which comes. So basically at this throat region when you see that or bearing region when you see that it will take care of only the outside diameter.

So inside diameter there is no support and you may find that because the metal is flowing into that there is going to be a slight increase in the thickness of the material after the tube drawing operation okay. Slight increase will be there. Because the inside is not supported this inside is not rough supported by anything and this free, you will find that the inside surface is nonuniform or uneven we can say.

The thickness is not uniform. Sometimes it may get misplaced on one side you may find the thickness is different and other side may be one side thickness may be low and the other side thickness may be high. That is one thing plus internal surface roughness will also be very high because there is an uneven metal flow and other thing. Due to some misalignment and other thing.

Even in any case you will find that internal surface is not good for this case but outside will be good. So the final thickness of the tube depends on original diameter of the tube. The die diameter and the friction between the tube and the die so this is the thing. So here the final thing what is the friction between the tube and the die. This is a one important thing and your original diameter and the die diameter.

So that is what you know. But you do not have any control on the insert diameter so that is the major drawback with the die sinking. So lower limiting deformation due to high redundant deformation. Here the redundant deformation is very high because there is a shearing at the entry. The shearing at the entry and the exit is very large in this case. So very high amount of redundant deformation is taking place.

So you have a limitation on the actual reduction which is possible because of this okay. This is a second drawback of this. The first drawback is that your internal surface is uneven and another thing you need a very high rod for the deformation for that.

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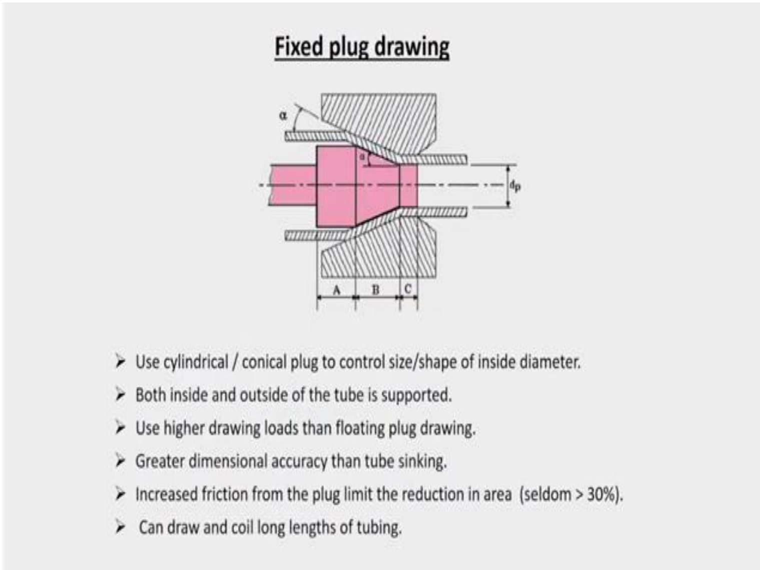


The second type of tube drawing is drawing using a fixed plug okay. So here the internal surface and the external surface both are supported. So external support is external surface of the pipe is external part of the tube is supported by the die. So here it is supported by the die and the internal part there is a fixed plug which is stationary okay. In this case these two things are there, so you will find that these 2 surfaces are supported.

So you can have a uniform thickness in the material your drawn tube you can have a uniform thickness okay. Here you will find that the total load is lower than the tube sinking which always may not be true also. But normally it is found to be but you have a very good surface finish. So greater dimensional accuracy is of 10 to compared to tube sinking. But you will find that the load in this case is higher than the floating plug drawing which you will be discussing soon.

So this is one and but you have a uniform thickness but the drawback is the thing is at this fixed plug is there. So it has to be fixed somewhere. So your there is a limitation on the your starting material the tube material which you are coming**.** So the length which can be obtained there is a because you cannot have this plug which is fixed at some other place and then having a long length it is not possible.

So that is one major problem. So increased friction from the plug limit the production area. Here you may find that the reduction which is possible is almost maximum which is possibly is around

30 percent. In exceptional case only it may be going high. But here the thing is you can draw and coil long lengths of tubing that is there. But somehow if we can take care of this we can get a longer.

But that limitation is there because your fixed length of the starting material or a tube material hollow material which you are going to start that problem is there actually. So it cannot draw very long length. So it not can it cannot draw and to coil long lengths of tubing because of the limitation on this fixed plug. But otherwise you get a very good surface finish.

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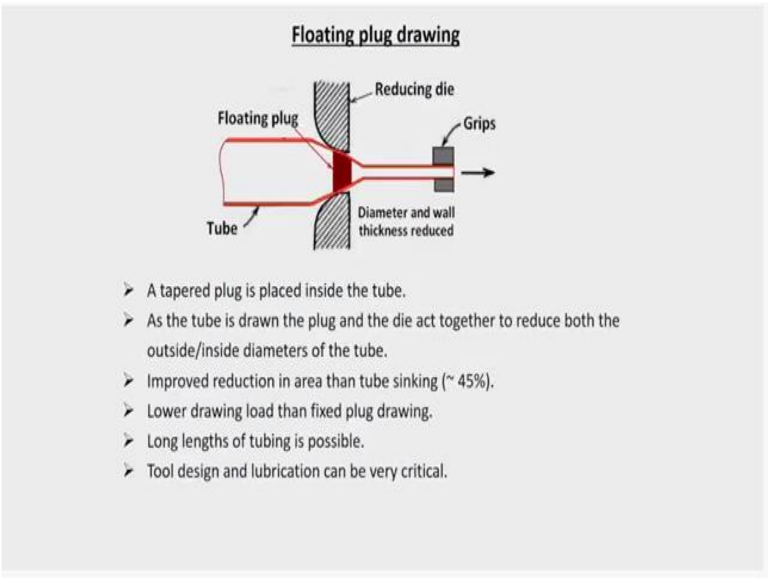


Now to overcome that part a very schematically drawn. There is a floating plug drawing. So drawing tube drawing using floating plug. So in this case also in every case you need to have this is a sectional view. In every case you need to have hollow tube as a starting material okay. But then the tube diameter is getting reduced. So here this floating plug is kept here. Normally this will be in a vertical condition.

But if you want a very long length then it can be kept horizontal. Because in that case there will be a problem for this floating plug because so normally it is kept at you know vertical position. A tapered plug is placed inside the tube instead of a fixed plug you see this will be like floating here. As the tube is drawn the plug then the die act together. This die and the plug act together to reduce both the inside and outside diameter of the tube.

So both reduction is taking place. So this will adjust itself. So that is why it is called as the floating plug. It will be floating at some intermediate position. You need to have a very clear cut design and other tool design is very important here. And that also depends upon the type of lubricant, the type of surface finish of the die material, the surface finish of your workpiece material all those things is a very complex part analysis is required for this purposes but anyway you are going to get it.

So here the advantage is that you have because you can completely you can reduce the large surface area. So that will reduce your frictional area or frictional forces which are there. And then you can get a higher reduction also. So here you can go up to even 45 percent the reduction can be obtained in one go.

So that is the biggest advantage with this. Now because the frictional area is there the frictional forces are there. So total rod necessary for the tube drawing by floating plug is much lower than the fixed plug drawing because the contact area is less. And here the biggest advantage is that there is no restriction on the length of the tube which can be drawn. So because the length the restriction the limitation of the length is depending upon what is the maximum size of the tube starting materials starting tube material which is there.

It is only on that rolling. So you do not have any limitation on that. So it can just come out and you have the tubes. But here the main problem is that the tool design, the design of the floating plug itself is very important and then tool places the friction is coming. The friction is coming at the die and the workpiece material. Then the friction is coming at the workpiece material and the plug.

So 2 types of this here. Now if you look at the balance of metal flow during this operation you will find that there is a frictional drag also is coming and then sometimes it will get nullified. So you need only very less for the drawing tube drawing operation by using a floating plug and but that also depends upon the surface roughness of the plug as well as the surface roughness of your die okay.

So that and the type of lubricants you are going to use it. So it is a very complex thing. So you can always do that a proper design you can reduce the total length which is necessary for the drawing operation.

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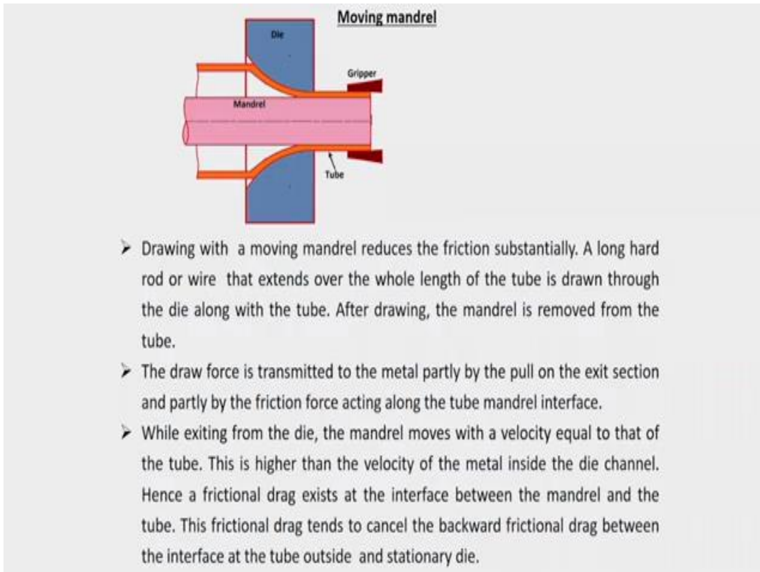


The last case is a moving mandrel okay. So drawing with the moving mandrel reduces the friction substantially this is another. A long a mandrel is there which is generally a long hard rod for a smaller diameter it may be wire also that extends over the whole length of the tube. It is drawn through the die along with the tube. So you will see that in the mandrel and the piece is held together initially you insert it and with the gripper you are fixing it and you are pulling this to together.

So metal will just flow into this region and it just plastically deform and then its size gets reduced and other thing when it is just when it passes through this the size of the tube gets reduced and the gripper will just pull it in this direction. This is just a schematic of this. So you will be pulling in this direction with the gripper moving okay. So after that the drawing operation the thing is that the mandrel is fixed to that.

So you have to remove it that is one major problem with that. But you have a very good control over the both the outside as well as the inside surface. The thickness will be more or less even in this case. That is the biggest advantage with this. The drawback is that you need to spend you need to have another setup for removing the mandrel from there. The draw force is transmitted to the metal by partly by the pull of the exit section and partly by the frictional force acting along the tube mandrel interface.

While exiting from the die the mandrel moves with a velocity equal to that of the tube. So that is a mandrel will be moving with the velocity equal to that whereas since it is moving this also this workpiece also will be moving around that. So there will be accumulation of material here and that will create a frictional drag okay. So this is higher than the velocity of the metal inside the die channel.

So that is another advantage. So hence a frictional drag exist at the interface between the mandrel and the tube. This frictional drag tends to cancel the backward frictional drag between the interface at the tube outside and stationary die. See that way it will help in the drawing operation. So you require only very less amount of drawing rod okay. So that way this moving mandrel by the tube drawing using a moving mandrel is always advantageous.

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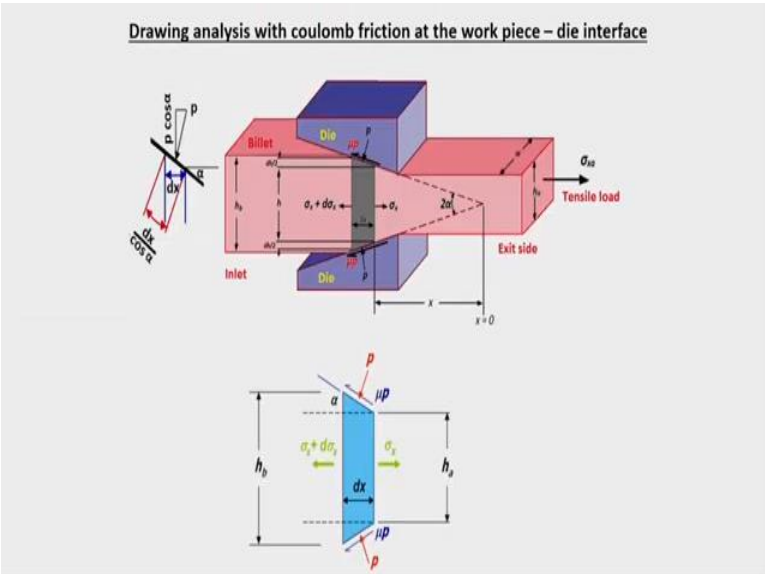


Now let us come to the analysis so your mechanics of this metal working. So maybe this is similar to that what we discussed in the mechanics of metal working. But there now we just discussed for the case of a slab method where homogenous the rod necessary for just to homogenous deformation of the material is there. Inside a frictionless die and were redundant deformation also is not considered.

So here let us take the case because normal case in practical application there is going to be a friction at the workpiece material and the die interface. So you have to consider the friction also. So let us take the case similar to that where we discussed in or module 3 only difference is that here we are assuming that there is going to be a frictional force at the interface. Because the metal is moving forward so that will result in a frictional force which is directed in this opposite direction okay.

So there is a die pressure because the metal is just moving here and it is accepts a compressive stresses at them. So you will find a die pressure P and because that is normal to the surface the frictional force will be equal to $\mu P$ on both the surfaces. This is also for a slab okay. So $\mu P$ is there. Now if you just take this element if you isolate the element and then draw this free body diagram, then you will find that the stresses are like this $\sigma_x$ along this direction.

Along the negative direction, in this direction you will find that there is an increase in the change in the stresses $\sigma_x$ + d $\sigma_x$ is there across this length dx. And then you have the force P acting in this direction and then you have the frictional force $\mu P$ acting in this direction. So all these things are there which is equal to your initial height of the strip is $h_b$ and final height is $h_a$. So this geometry we can look at it.

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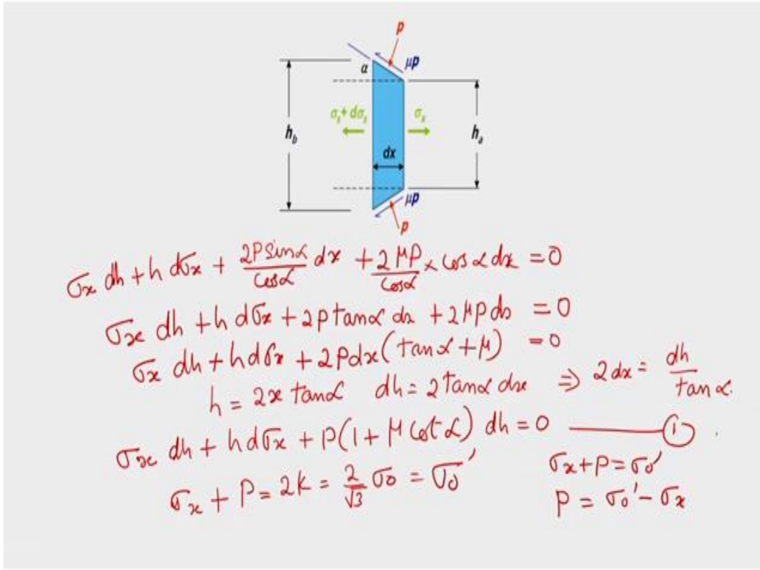


Say now come back to the previous case the equilibrium includes the term here this $\mu P$ is there. So if you just under equilibrium the sum of the total forces in the x direction is equal to 0 and if you neglecting so when you write that equation and then if you neglect that higher order terms like d sigma x dx term so that way that equation we can get it as sigma x dx the horizontal forces.

Under the equilibrium the total force in the x direction if you write. So sigma x dh + h d sigma x + 2p sin alpha/cos alpha dx + 2 mu p/cos alpha x cos alpha dx because these are all resolving this p and the mu p along the x-axis direction okay, dx is equal to 0. So that on simplification we can write it in this way that is sigma x dh + h d sigma x + 2p tan alpha dx + 2 mu p dx = 0. So this on simplification we can write in a simple way that is sigma x dh + h d sigma x + 2p dx into tan alpha + mu = 0.

$$\sigma_x dh + h\, d\sigma_x + \frac{2P\ sin\alpha}{\cos\alpha} dx + \frac{2\mu P}{\cos\alpha} \times \cos\alpha\ \ dx = 0$$

$$\sigma_x dh + h\, d\sigma_x + 2P \tan\alpha\, dx + 2\mu P dx = 0$$

$$\sigma_x dh + h\, d\sigma_x + 2Pdx(\tan\alpha + \mu) = 0$$

So if you look at the earlier case looking at this equation we can also write that h = 2x tan alpha. So that means dh = 2 tan alpha dx or said to that implies that 2 dx = dh/tan alpha. So if you substitute into this you will find that equation comes as dx dh + h d sigma x + p into mu cot alpha 1 + mu cot alpha into dh = 0. This is your equation number 1.

$$h = 2x \tan\alpha \qquad dh = 2\ tan\alpha\ dx \qquad 2dx = \frac{dh}{\tan\alpha}$$

$$\sigma_x dh + h\, d\sigma_x + P(1 + \mu \cot \alpha) dh = 0$$

Now if you apply the yield criteria which also we have the Von Mises criteria and because this also Von Mises criteria and Tresca criteria for plane strain condition it is the same. So and since here it is a strip drawing operation where the width of the strip is much higher than that of the thickness of the strip. So we can approximate it into say what we call it as a plane strain condition.

So in that case you know the previous thing also we had also I am not going into the derivation here. So you can write it as sigma x + p = sigma 0 dash. So that is so that basically this will come that is sigma 1 - sigma 2/ = 2 K okay. So from that only we are getting it and then we are assuming these are the sigma x and p are the what do you call it as principal stresses. So considering that so you will get it as equal to 2 K = 2/root 3 sigma 0 which is equal to your sigma 0 dash okay.

$$\sigma_x + P = 2k = \frac{2}{\sqrt{3}} \sigma_0 = \sigma_0'$$

So if you just substitute that. So and then so or you can say that sigma x + p = sigma 0 dash or p = sigma 0 dash - sigma x. This is the if you substitute into this equation 1. The equation 1 if you substitute.

$$\sigma_x + P = \sigma_0'$$
$$P = \sigma_0' - \sigma_x$$

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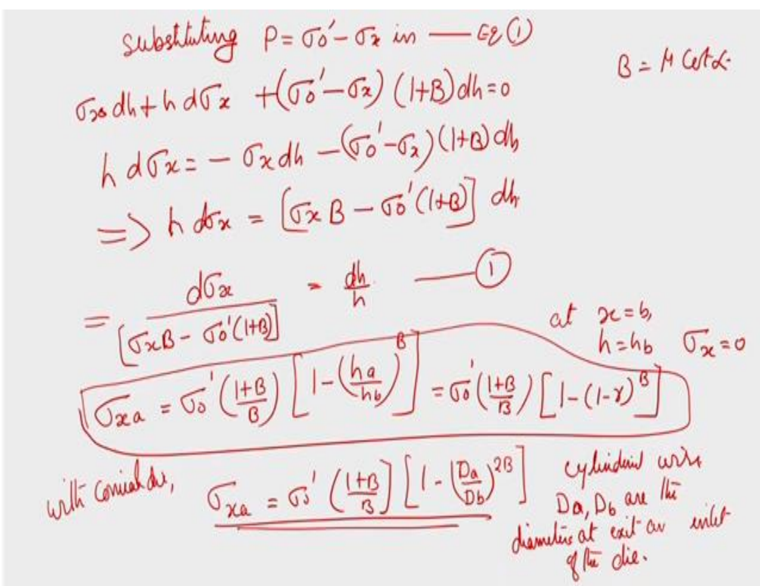

You can write it as sigma x dh. So I will write substituting

$$P = \sigma_0' - \sigma_x$$

in equation number 1 + h d sigma x + sigma 0 dash - sigma x into 1 + B dh = 0 where B is equal to mu cot alpha okay. If you use because this is a constant so you can always take that as B or from this now we can say h d sigma x = - sigma x dh - sigma 0 dash sigma x into 1 + B dh okay. So that implies that you are getting h d sigma x is equal to if you do a simplification of this you can just get it as sigma x B - sigma 0 dash into 1 + B into dh.

$$\sigma_x dh + hd\sigma_x + (\sigma_0' - \sigma_x)(1 + B)dh = 0 \qquad B = \mu \cot \alpha$$

$$hd\sigma_x = -\sigma_x dh - (\sigma_0' - \sigma_x)(1 + B)dh$$

$$hd\sigma_x = [\sigma_x B - \sigma_0'(1 + B)]dh$$

So from that you can get the differential equation as d sigma x/sigma x B - sigma 0 dash into 1 + B = dh/h. Now if you integrate this if you integrate this equation and applying the boundary conditions. That is at h = hb or x = b, h = hb so in that condition you will find the sigma x horizontal this one is equal to 0 and you apply that after integration you will get this relationship as the draw stress.

You can answer very simple thing always in a similar way which we used for that in the mechanics of metal for strip drawing without friction. If you just directly integrate it, you will get the draw straws sigma x a = sigma 0 dash applying the boundary condition we can get the constant also into 1 + B/B into 1 – ha/hb raised to B. So that is equal to sigma naught dash into 1 + B/B in terms of reduction we can get 1 - 1 - r raise to B, we can write.

$$\frac{d\sigma_x}{[\sigma_x B - \sigma_0'(1 + B)]} = \frac{dh}{h}$$

$$at\ x = b, h = h_b \quad \sigma_x = 0$$

$$\sigma_{xa} = \sigma_0' \left(\frac{1 + B}{B}\right) \left[1 - \left(\frac{h_a}{h_b}\right)^B\right] = \sigma_0' \left(\frac{1 + B}{B}\right) [1 - (1 - \gamma)^B]$$

So if you are using so this is the relationship you are getting for drawing operation. Now instead of this $h_a$ and $h_b$. So like if you are using this is for a strip drawing, but if you are just considering that it is a cylindrical drawing operation which is taking place. The pieces are cylindrical in shape then this ha/hb raise to n know so that is your reduction which is taking place. So you can just substitute that ha/hb same relationship and with conical dies.

So with conical dies you can get this relationship as sigma 0 dash into 1 + B/B into 1 – ha/hb in terms of diameter, if you take the cross section area. Because here in the strip we are using a unit width. So if you take the actual cross sectional area this height and this one you know it can be reduced if it is a diameter then you can just have this relationship Da/Db it will be Da/Db so raised to because 2 it will come pi/4 d square is coming. So in that case it will be 2B cylindrical piece wire.

$$with\ conical\ dies, \sigma_{xa} = \sigma_0'\left(\frac{1+B}{B}\right)\left[1-\left(\frac{D_a}{D_b}\right)^{2B}\right]$$

So where $D_a$ and $D_b$ are the diameter at exit and inlet of the die okay. So that straight relationship we can get it. Now there is another method for wire drawing the stresses necessary for the wire drawing operation which was proposed by Johnson and Rowe okay. So let us look that. So it is in this case.

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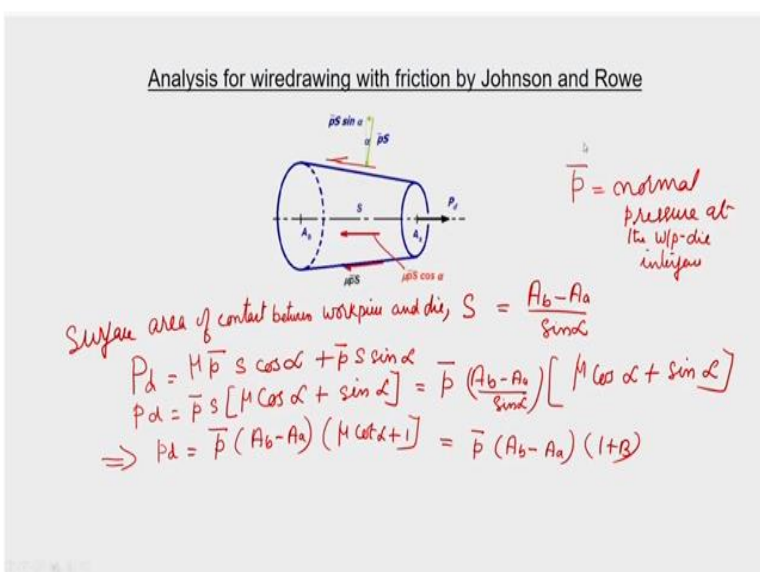


So here this is the deformation zone because it is a tapered cylindrical piece. So this is assumed to be the deformation zone inside the where the metal is radial deformation. So your initial cross sectional area Ab is there and final cross sectional area is Aa is there okay and you are pulling this so this is a free body diagram of the region inside the deformation zone. So all other things will remain same and with the friction and other things also coming.

Friction is not drawn here okay. But in any case you know that we can draw the friction also okay. But here only thing is that the surface area of the contact between the wire and the die. So that surface area between workpiece and die. If you write it as S, so we can write it as in terms of Ab – Aa, Aa and Ab is clear in this figure by sin alpha depending upon your die angle. So that way you can write it.

$$Surface\ area\ of\ contact\ between\ workpiece\ and\ die, S = \frac{A_b - A_a}{\sin \alpha}$$

So how much it is coming. So this is a straight relationship so if you just draw the balancing of horizontal components of the frictional force and the normal force. So you have to do only this you just balance it is Pd is equal to the horizontal component of your frictional force and your die pressure okay normal pressure. So we can write it in this way Pd = mu p dash S cos alpha + p dash S sin alpha.

$$P_d = \mu\, \bar{p} s \cos \alpha + \bar{p}\, s \sin \alpha$$

So where p dash is the normal pressure which is from okay. So from that now we can just write it as Pd = p bar into S into mu cos alpha + sin alpha. So that is equal to if you substitute for S. So p bar into Ab – Aa/sin alpha into mu cos alpha + sin alpha. We can write like this or we can just write it as Pd that is your raw force Pd is equal to your pressure die pressure into Ab – Aa into mu cot alpha + 1 or we can write it just p bar into Ab - Aa into 1 + B we can write.

$$P_d = \bar{p}\, s[\mu \cos \alpha + \sin \alpha] = \bar{p} \left(\frac{A_b - A_a}{\sin \alpha}\right) [\mu \cos \alpha + \sin \alpha]$$

$$P_d = \bar{p}(A_b - A_a)[\mu \cot \alpha + 1] = \bar{p}\, (A_b - A_a)(1 + B)$$

But remember that this is not the average pressure. See p bar is the normal pressure at the workpiece die interface, do not confuse with average pressure and other things okay. So this is the thing. So this is the expression you are getting. So for the drawer stress in this.

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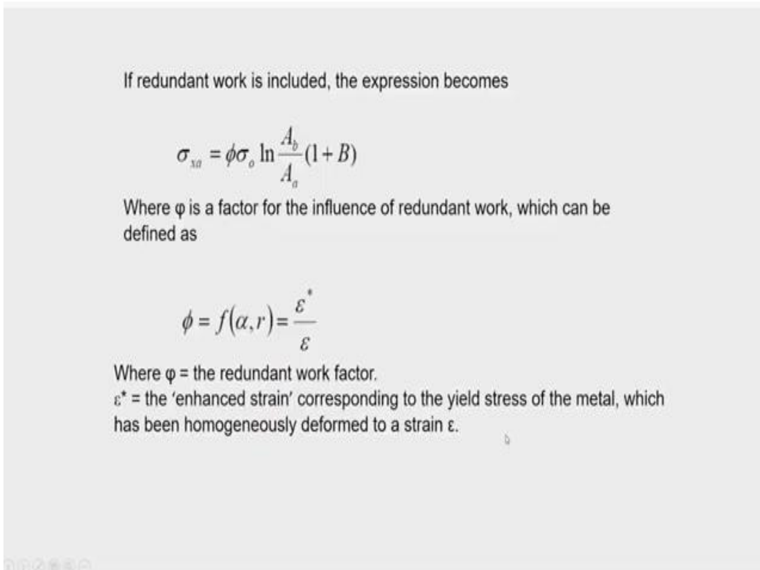


And the thing is that you know.

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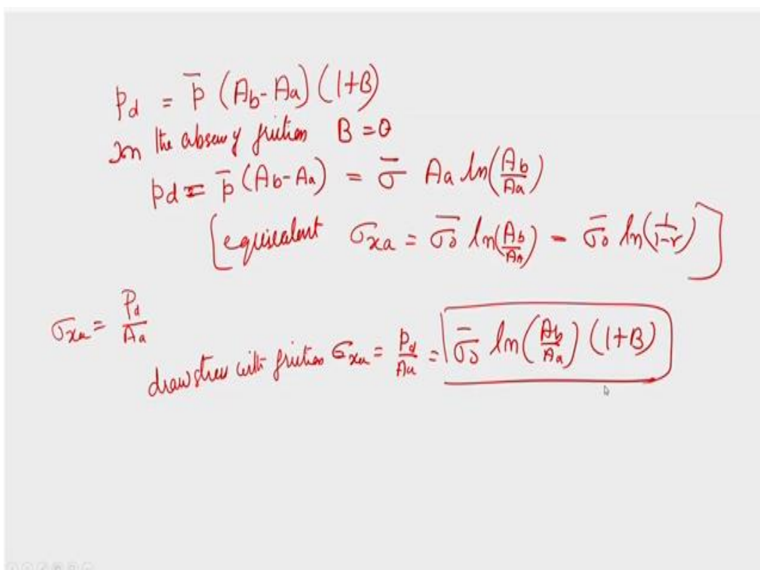


So this relationship for the draw force like if you look at that if there is no friction if you assume that then this B term will be 0. Because mu cot alpha so that will be 0. So that means in the absence of friction B = 0 and then you will get it as pd = p bar into Ab – Aa which is we can write it as with average stress okay of your flaw stress of your material during the deformation which is equal to Aa log Ab/Aa.

$$p_d = \bar{p}\,(A_b - A_a)(1 + B)$$

$$\textit{In the absence of friction } B = 0$$

$$P_d = \bar{p}\ (A_b - A_a) = \bar{\sigma} A_a \ln(\frac{A_b}{A_a})$$

So this is equivalent to the case of slab method you know without friction you know which is so this is equivalent to if you recollect that relationship in our mechanics of materials where I discussed about the slab method. So you can you will remember that xa = sigma 0 bar log Ab/Aa or that is equal to your sigma 0 into log 1/1 - r you got. So this is equivalent to this one that the same relationship.

$$\sigma_{xa} = \overline{\sigma_0} \ln(\frac{A_b}{A_a}) = \overline{\sigma_0} \ln(\frac{1}{1-r})$$

And since your sigma xa is equal to your draw force divided by your area Aa. So we can write that the draw stress with friction is equal to sigma xa = pd/Aa = we can say sigma 0 bar into log Ab/Aa into 1 + B. So it is more or less similar to that case okay. So that is the biggest advantage you are getting. So in both the case if you use this also you are getting the similar type of relationship for the drawing of the wires okay.

$$\sigma_{xa} = \frac{P_d}{A_a}$$

$$\sigma_{xa} = \frac{P_d}{A_a} = \overline{\sigma_0} \ln\left(\frac{A_b}{A_a}\right)(1 + B)$$