Nonlinear Control Design

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Week 12 : Lecture 73 : Sliding mode control: Part 3

Hello everyone. Welcome to the final lecture in this NPTEL course on nonlinear control. So we had, we were looking at sliding mode control and we sort of looked at last time a way for chattering attenuation. And after that we started to discuss the notion of equivalent control. What is equivalent control? It is basically the control that you obtain from the fact that your sliding variable goes to zero in spite of the disturbance beyond a certain time. So this is a very important and interesting idea because this equivalent control is obviously not an implementable control and this is the actual control.

But the fact is the actual control in a time average sense must go to the equivalent control otherwise there is no way that the system would remain on the sliding surface. So therefore an estimate for the equivalent control is simply a filtered low pass filtering of the signum function here and that's what is implemented here. And essentially now what we can say is that from here you can see that if you simply match this guy with this guy, as you can imagine this has to match and otherwise you understand there is no way that ideal sliding happens. And therefore once you match these two you understand that this quantity and this quantity have to be the same.

So therefore what we are essentially claiming is that a very good estimate for f is rho low pass filter of signum of sigma. For t greater than equal to tr. And you understand this becomes a very nice way of estimating not just disturbance but any derivative for that matter. Because where was the disturbance? The disturbance was actually in x2 dot. So even if there was no notion of control, you could in fact compute the derivative using this idea.

I mean because of lack of time we are not going to get into that but this is very closely connected to the notion of a sliding mode differentiator. Yeah it's a very nice way of doing a obtaining a differential or in fact obtaining any disturbance signal. Yeah so what have we done? We've taken simply the signum of this sliding mode variable and we are scaling it by this rho which is obviously greater than L and all of that. We've already designed that and so we are simply scaling by rho this low pass filter version of signum of sigma and we are saying that this is a great estimate for any disturbance. And that's great because that is what we will sort of use in trying to look at the notion of a super twisting control.

Because until now you have seen that the chattering is not still completely gone. So what is this super twisting control? So you see that even though the chattering was gone in the first level, in the second level that is even though the chattering was less because we use an integral control for example in this chattering attenuation kind of idea still there is going to be chattering at the next level. So it's not like you're going to completely get rid of this. But super twisting controls are a sort of quite an amazing way of getting rather smooth controls. A very good way of getting rather smooth controls.

So let's sort of look at what is this super twisting control. So what's the idea here? I'm going to go back to the original treatment here. If you see what did we do? We had this half sigma squared as our V and then we constructed our control as minus rho signum sigma. I mean the V is already cancelled the CX2 and all that so we are not worried about that anymore. But we've constructed the V as minus rho signum sigma.

So now we do something slightly different and that's what is the title here. It's a super twisting control and this is due to some really really interesting results from Oetkin etc. So what is the super twisting control? You remember that we had sigma as X2 plus CX1 and we had V as one half sigma squared. And of course you know what was X1 and X2. So if you look at V dot we had taken it is sigma sigma dot which is X2 X2 dot which is U plus F plus CX2.

I am deliberately not writing the arguments of F just to save some space. And what did we do? We said that we will prescribe U as minus CX2 plus a new control V. And so this becomes sigma times V plus F. And this is the point when we had prescribed V as minus rho signum of sigma. Now this is where we prescribe a different control.

We say that our V is not going to be minus rho signum sigma but it is going to be minus rho sigma to the power half signum sigma. Now let's consider for a moment and say there is no difference no disturbance at all say F is in fact 0. Then what happens is that V dot becomes minus rho sigma to the power half times sigma signum sigma which is basically this is just absolute value of sigma so this is just minus rho sigma to the power 3 by 2. This is what you have. This is minus rho sigma to the power 3 by 2.

Okay now so let's see so right right right now so I'm just trying to see how we can do this so I want to write this in terms of the V itself right and so that's my primary question right. So this is actually going to be if yeah so so this is implies sigma is somehow equal to square root of twice V is what I get from here and if I substitute it here the absolute value of sigma square root of 2V if I substitute it here I will get minus rho 2 to the power half so 2 to the power three fourth times V to the power three fourth okay great so so it doesn't matter what I choose my rho as it's evident that this is less than 1 so implies have finite time convergence okay have finite time convergence so it's not like we've done any achieve anything new in terms of the convergence the convergence is still finite time convergence for the variable sigma the sliding variable sigma however something really cool has happened my control has changed yeah let's not worry about U let's just look at V because U is just a smooth term along with V now earlier the control was minus rho signum of sigma so what was the earlier control going to look at look like if I try to plot it let's see we take a different color if right the earlier control was depending on the sine of sigma right it was going to jump between the yeah the earlier control could would be something like this right depending on the sine of sigma it would look something like this right so this is the old control yeah but the super twisting control is rather nice it is not just signum of sigma it is signum of sigma multiplied by the square root of sigma and this makes it rather nice and continuous yeah so this is rather cool yeah why the control is not going to be these jumps anymore at all because even if sigma becomes here what was happening was even if sigma became slightly positive you had this guy if it became slightly negative you had this guy it's just moving from this to this this to this this to this was like a bang-bang here the movement is much more smooth if it's slightly positive then you only have a slightly up curve slightly negative slightly up and so on and so forth right so this you will have as the super twisting control now the only problem is when there is actually the disturbance right it is not really cancelling the disturbance term if you notice the whole purpose of row signum sigma was it was dominating the disturbance term this guy is not doing that and so what do we propose we propose that so if you look at what happens in the presence of disturbance is that the sigma dot dynamics right starts to look like minus rho sigma to the power half signum sigma because and and because the CX 2 is already cancelled the sigma dot will just be this plus the disturbance yeah so you see the disturbance is not really compensated okay not really compensated so in the presence of disturbance you will not get convergence to the sliding surface so what do we do we actually use the idea of this estimator right what is it we of course we have this assumption that f dot is also less than equal to some L bar right and then we construct our V as minus rho signum sigma plus a variable W where the W is now an estimator right where the W is now an estimator of F right okay so W is somehow F hat why does this work is exactly from the notion of the equivalent control exactly coming from the notion of the equivalent control right if you look at this what is the equivalent control we say that the estimate is simply a low pass filter right and what is the low pass filter it's something like this right so here the only thing we've done is we've not put in a low pass filter type term we've simply kept it at this B signum sigma and this also works right so this is from the notion of equivalent control yeah you can actually estimate and you can actually estimate the quantity F okay you can actually estimate the quantity F and that's the whole sort of really cool idea okay that's the whole interesting interesting idea that of course you can always have a you know there's no problem if you have a minus K W as well no problem right but this works and this is becoming this will be an estimator of F and and that is essentially what you apply on top of your super twisting control and this will also compensate for the F itself okay this will also compensate for the F itself of course we are not specifying what B is so we will just say B large enough yeah because if you remember here also this row was large enough to compensate for F F dot and so on and so forth okay so again we have shown these without proof so obviously you cannot say that this is a complete rigorous treatment but the idea is the control obtained from super twisting is going to be much nicer much cleaner right than anything that you will get from the typical classical sliding mode control okay the final sort of notion that I want to introduce is the notion of second order sliding mode right or higher order sliding mode you can always have higher and higher order sliding mode but I am simply going to introduce motivate this using the notion of a second order sliding mode alright so what is the notion earlier we constructed sigma as x2 plus some CX1 okay and we said this is first order yeah because it essentially it is what does it do it sigma goes to 0 finite time and slides on sigma equal to 0 for infinite time right until both states actually go to 0 right you remember the picture the picture was drawn probably in the first page itself here first and second right right you actually reach the sliding surface and then you keep sliding for infinite time until you reach here right so you are moving in a one-dimensional surface for infinite time right and this is why this is a first order sliding mode right and this what we say is when when it when a behavior like this happens for a dynamical system that it starts to move or restrict itself to a one-dimensional surface then we say that there is partial dynamical collapse right which means what that you were you have started off with a second order system two state variables but you collapsed and started moving on a dimension one curve and that's a partial dynamic collapse when you do second order sliding mode you actually do complete dynamical collapse so the target is complete dynamical collapse which means what implies x1 and x2 both go to 0 in finite time okay and how does one do that one does that by choose non-linear sliding surface right what is this non-linear sliding surface it's very straightforward right I choose my sigma exactly motivated by the super twisting sort of a expression instead of x2 plus cx1 I take a c absolute x1 to the power half sigma x1 right the purpose of having this super twisting type expression right I am going to use this call this a super twistings type expression because this is exactly what we used in super exactly what we used in super twisting control right the purpose of having this is to make sure that this is a nice continuous function right otherwise if I just use c signums x1 then it's not continuous right and that's you cannot even categorize that as a surface right so as I said the purpose is to ensure continuous sigma right because if it is not continuous then sigma equal to zero is not a surface right so therefore the idea of sliding mode sliding surface sliding manifold is not possible right if this is not continuous so it has to at least be continuous and that is what this sort of a term does right now what do we know about this guy what do we know about this guy right if sigma goes to zero in finite time say then what do I have I have x1 dot equals minus c absolute x1 to the power half sigma x1 right all I've done is substitute for x2 as x1 dot right now if I take v as half x1 squared I know exactly that v dot is going to be minus because we did this analysis right here right we did this analysis exactly for the similar case here right instead of sigma there will be just x1 right so I'm going to get v dot as minus c x1 absolute value to the power 3 by 2 and that is just again just pulling it out from here that is just going to be this guy right minus 2 to the power 3 4 c v to the power 3 4th and therefore what you have implies x1 goes to zero in finite time right because obviously this is less than one right so that's exactly the condition so that's exactly the condition that v dot has to be some negative constant multiplied by v to the power alpha where alpha is less than one that's exactly the finite time convergence condition therefore x1 goes to zero in finite time and x2 is simply x1 dot and therefore x2 also goes to zero in finite time right and so we have what we we will have what we want right this is what does it mean we will have finite time collapse right we will have complete dynamical collapse because in implies have complete dynamical collapse right why because we ensure that both states go to zero in finite time yeah and stay there right so it's not sliding on a one-dimensional surface it goes to a zero-dimensional surface which is origin right goes to origin in finite time and we are done goes to origin in finite time and we are done and we are done so that's the very interesting notion now what is it that we need to remember we need to of course find out what u to use for sigma going to zero in finite time right that's an important question what will be the control right now if you see if i try to use v equal to sigma squared to arrive at a control i'm going to get pretty complicated things right because i have to take derivative of this guy and all that i have to take derivative of this guy and all that right because if i so sigma dot uh complicated not that it's not possible but yeah it's fairly complicated the it's fairly complicated right because once i in order to do sigma sigma dot i'll have to take derivative of this guy right so however it is well understood that u equal to again minus rho signum sigma works for large sigma okay but again with a lot of chattering right this control also will have a lot of chattering okay so notice that this looks similar to before but actually it's different because sigma itself is different right okay but again this control like i said before also has a decent bit of chattering right even though you have sliding your smooth your non-linear sliding non-linear sliding surface but the point is only the sliding surface is non-linear the fact that if you go across it this way or that way across even a non-linear sliding surface your control is still jumping right then the other option is of course you can or use super twisting yeah so whatever be the order of the sliding mode uh the all the sliding you can always switch to a super twisting control right because then you get rid of the chattering issue uh in a very very nice and efficient way right because this is definitely going to jump across the sliding surface right even if the sliding surface is linear non-linear it doesn't matter um it's still going to jump it is still going to move along around it and if it moves across it it's the control is going to jump and therefore there's going to be high frequency chatter on the other hand the super twisting control because it's not going to be rho sigma sigma it is going to be something like minus rho sigma to the power half sigma sigma yeah you are going to have a much more smoother performance okay and that's sort of what you're looking for all right so that's sort of all we wanted to discuss in sliding mode control we did first order sliding mode we did second order sliding mode we sort of understood how using the notion of equivalent control you can actually obtain the value of the disturbance itself we understood the limitations in the in the sense that there is a lot of boundedness uniform boundedness assumptions we understood that there is chattering issues that we need to resolve super twisting control is one of the really really cool developments in sliding mode control area which actually gets rid of chattering very efficiently right and this can be combined with the function the disturbance determination scheme to actually get to the sliding surface yeah so of course we did most of this without proof and with a very nice double integrator example things are significantly more complicated if you do more elaborate things for more general systems and of course there is enough literature and books and texts in this area so I have particularly been referring to the book by Urey Stessel and others yeah uh on sliding mode control yeah there is quite a few authors all the most of the pretty heavy lifters in the area are sort of co-authors in this book and so it's one of the better written books most of the material that we've covered is from first chapter yeah and so as you can imagine this is what you covered is literally just the first chapter introductions to sliding mode and there is so much more than this in in sliding mode control yeah that we have not actually covered here so that brings us to the end of this NPTEL course I really really hope that you enjoy learning through this course and I also really hope that some of what you learned ends up being useful for you in the real world and I would be very happy to hear here and on youtube you know please tag me please tag the course on your social network pages and I mean so that more and more people can actually take the course learn from the course and there is more interest in the topic in general yeah so that's all from me thank you so much again thank you.