Nonlinear Control Design

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Week 12 : Lecture 71 : Sliding mode control: Part 1

Hello everyone. So welcome to the fourth lecture of nonlinear control. This is the final week of this course and we have been talking about control methods that allow for some kind of finite time convergence behavior. So we've already covered a decent bit of material on finite time stability and finite time control design and we had just started off talking about sliding mode control. So as we mentioned sliding mode control though has features of finite time stability and control as you will see subsequently. The way it is sort of posed is in the form of a variable structure controller and the primary requirement as we mentioned is for some kind of a disturbance rejection.

So this is somewhat of a lumped disturbance, lumped nonlinearity if you may and the aim for or objective of a sliding mode control design is to develop this controller that will actually help achieve asymptotic convergence or asymptotic stability. While there is a disturbance that is acting on the system and the assumption is of course that the disturbance is uniformly bounded for all time and for all values of the state x1, x2, importantially in some domain. Now as we already mentioned it is a rather difficult requirement to have like bounded disturbances for all states. So even if you look at basic polynomial or even linear examples of such functions you will find that there is no boundedness for all values of states.

However one can always claim that in a compact set of the states this nonlinear function can be bounded and there are results which are more modern in sliding mode theory. So this sort of helps you to get more advanced results in sliding mode. But what we will look at in this series of lectures is the more classical version where you are actually assuming uniform boundedness on this nonlinearity. So this is the sort of double integrated type of a system that we start off with and we are looking to design a sliding mode control. If you look at a more basic asymptotic controller or basic asymptotically stabilizing controller these controls as you know and as we have been studying until now will typically ignore these kind of disturbances and so your control will typically be designed as u is minus k1 x1 minus k2 x2 for some positive gain k1 k2.

And this in the absence of disturbance obviously will drive your states to zero. However in the presence of disturbance you would expect something like a oscillatory behavior. I mean you will expect that you will get some kind of a residual set kind of behavior. You will never expect you will never get to exact convergence. So that's the whole point.

No exact convergence in the presence of these disturbance functions. So then the question is what more can be done. So first is we introduce a sort of a nice differential equation if you may that we want to follow. So that's what we say. We introduce what we typically call as a sliding mode but we will define these things a little bit later.

But suppose we want some kind of a compensated dynamics and this is essentially desired. This is essentially desired. This is not actually the case but this is actually a desired dynamics and we say this is x1 dot plus cx1 equal to zero for some positive c. And it's easy to see easy to understand that this quantity is actually x2 itself because of how our dynamics is. So therefore that's how it's chosen.

So this is actually x2 plus cx1. And what one can also understand from here is that from here it's evident that x1 goes to zero exponentially which means x2 also goes to zero exponentially. So that's sort of what we understand. So if you are able to follow this kind of a dynamics even in the presence of disturbance f you understand that this will guarantee a syntactic convergence of both states x1 and x2. So we call this sort of a function as sigma x1 x2 and you can see it is usually a function of all the states.

Both states in this case. And this is what is called a sliding mode. Essentially what does it do? It gives you for this two dimensional system because you have two states. For this two state system we reduce the evolution to a single one dimensional line because x2 plus cx1 is a straight line in the state space in the phase plane. It's a straight line in the phase plane.

So I hope that's clear. Something like x2 plus cx1 I can even draw something like this. Let's say here. So x2 plus cx1 equal to zero would be something like this. So this is what will be.

It's a straight line passing through zero. So this is what is the sliding surface. Depending on whatever the value of c is the inclination of this line may change and so on but essentially that's what it is. So this is what is a sort of a sliding mode if you may. Now how do we ensure that our system follows this? So first we write the dynamics of the sliding variable sigma.

So what is sigma dot? It is x2 dot plus cx1 dot and x2 dot from our dynamics is just this guy. It's u plus f x1 x2 comma t plus cx2 because x1 dot is in fact x2 and you have some sigma zero equal to sigma sub zero. So you now are working with a different looking dynamics. I mean you're just working with the sigma dynamics. And now our aim is to push the sigma dynamics to zero.

That's what we want to do. So aim push sigma to zero because if sigma is going to zero you understand that x1 and x2 are both going to zero. So what we will try to do is we will try to make sigma go to zero in finite time. So how do we do that? We take a v which is half sigma squared and we get a v dot which is sigma sigma dot which is nothing but sigma u plus f x1 x2 t plus cx2. Now I hope you understand that this disturbance is obviously a disturbance so it's not known to us.

It's not like you can cancel it using the control. However we can certainly cancel this guy. We can certainly cancel this guy. So what is it that we want to do? You already know that you want to have something like a you want to follow from our finite time stability. What do you want? You want for finite time convergence you want v dot plus kv to the power alpha less than equal to zero.

So I mean and you can choose alpha to be anything. You can choose alpha to be anything. That's our call. But alpha has to be within zero one and k has to be positive.

That's our requirement. So based on that if I actually just try to substitute that here what I would do is I would simply try to choose u as minus cx2 plus some v some small v which we don't know yet. If I do that then what do I get? I'll add a page here. Then what do I get? I get v dot. So v was half sigma squared.

Let's remember. And v dot is sigma times v plus f x1 x2 t. Now we'll do a little bit of an inequality. We already know that this is going to be less than equal to absolute value of sigma absolute value of v plus absolute value of sigma absolute value of x1 x2 t. And we know that this is less than equal to L. We know this is less than equal to L.

So what do we do? So we know that this is less than equal to absolute value of sigma times absolute value of small v this additional control plus L. So now what do I do? How do I work this out? How I work this out is I take my v as the small v that we have here as some rho sine function of sigma or also written as rho signum of sigma. So sigma is just the sine function. So signum of x to 1 for x greater than 0 minus 1 x less than 0 and obviously 0 when x is 0.

No, actually this is fine. And signum 0 can be anything in the minus 1 1 range. So signum of 0 can be anything in this range. So if we take actually as minus rho then what we get is v dot is equal to absolute value of sigma times rho. Let's see if I want to do this. I think I probably jumped a few steps ahead and I probably did not need to do that.

What we will do is we will choose the v in advance here and these I think I will move to later. So basically what we are doing is we are actually selecting the v term first. And what happens is when I substitute this in the v dot what I will get from here is v dot is minus rho times sigma times signum of sigma plus sigma times f x1 x2 t. And this quantity sigma times signum sigma is already equal to the absolute value of sigma. This is what we want to use that sigma multiplied by sign of sigma is actually the absolute value of sigma itself.

So what we have here, I think I can erase these safely and redo this again. So this is v dot is now I do the bounding it's because this remains the first term remains the same minus rho absolute value of sigma plus sigma times l. Why because absolute value of f less than equal to l and so there will be an absolute value of sigma here as well. Now if I take the absolute value of sigma common then this is absolute value of sigma with a negative sign rho minus l.

So this is v dot. So if I take rho as well I mean for example as equal to l plus two then v dot turns out to be minus l plus half turns out to be minus half absolute value of sigma. And that's basically saying that this is v dot is less than equal to minus let's see let's see let's be careful here. I'm going to be very careful here one over root two and one over root two and this is minus v to the power half. And this is very similar to what we wanted. If I take k equal to one so same as finite time convergence with k equal to one and alpha equals half.

So we are allowed to choose alpha anything from zero to one therefore we've chosen alpha equal to half and k equal to one so obviously we have finite time convergence so we know that the sigma dynamics converges to zero in finite time. So sigma goes to zero in finite time and the cool thing is this happens in the presence of disturbance. Alright so you've actually rejected the disturbance. Why do I say it happens in the presence of disturbance? Because we did not neglect the disturbance in the analysis. We actually put in a term that is the rho has this l value which is basically going to compensate for the disturbance.

So we have something that compensates for the disturbance using the bound itself. So we have a disturbance compensated convergence. That's pretty interesting. That's very interesting. So what is our control now? So our actual control is u is if you may it was here.

The control was here. It's minus CX2 plus V and V is chosen in this way. So therefore our control is minus CX2 minus rho signum of sigma. So that is X2 plus CX1. So that's the control. As you can see the control has switching on the sliding surface.

I hope you understand that this control switches on either side of the sliding surface. So how will this work? This will be actually evaluate to equal to minus CX2 minus rho when X2 plus CX1 is positive and this will evaluate to minus CX2 plus rho when sigma is X2 plus CX1 is negative. So there is actually a switching across this sliding surface. There is a switching along the sliding surface. So the sigma equal to X2 plus CX1 equal to 0 is actually the sliding surface.

So there is a switching sort of a thing happening. So therefore you can imagine that what will happen is if you look at the plots for example, what you will see is the sliding variable of course will behave in a very very nice way. I mean it will actually converge in finite time. I mean it will just do this.

It will just do this. On some finite time your sigma will converge. So you expect some really nice plots on the sigma variable. However and similarly I mean X1, X2 is obviously converging asymptotically. So this sort of if you made the time in which you have basically this reaching. So this until this time my apologies is called the reaching phase and this beyond this is called the sliding phase.

Now the problem with this controller is pretty obvious. The problem with this controller is pretty obvious. Because of the disturbance I mean if you look at this sort of a plot on what happens very close to the sliding surface. So you will start to see some kind of a zigzag motion happening. So this is the sliding surface and we already know that it probably looks something like this.

So that's the sliding surface and there is of course I mean this is X1, this is X2, there is a sort of a reaching phase which is finite time and then there is a sliding phase. But this is the small zigzag things here. Why because what will sort of happen is that you have because of the presence of disturbances what happens is and you are not exactly compensating for the disturbances as you notice. I mean what you are trying to do is you are simply dominating them in some sense using this L and the disturbances are bounded by L. So it's not like at every instant in time you are exactly cancelling the disturbances.

No you are not. You cannot do that because you don't know the value of disturbances to do that. So what happens is once you get to this sort of place where you are on the sliding surface what tends to happen is you will get thrown out of the sliding surface a little bit and then you will and then the control law switches right. I mean you will go from one side of sliding surface to another side to one and so you tend to do a lot of these high frequency switches. Why because your control law is in the finite time convergence kind of idea at sigma equal to zero the control is not Lipschitz.

It's not smooth. So there is some high frequency activity happening. So that's exactly the thing here. Here the sigma equal to zero is essentially the sliding surface. So on the sliding surface you have non-smooth behavior and because in this case the sliding surface is not actually the origin of or the equilibrium of the system.

So you are still moving on the sliding surface. It's not like you go to the sliding surface and you stop. You continue to move and there is disturbance. So what happens is you tend to overshoot, undershoot and there is a lot of these high frequency chatter happening because of the non-Lipschitz nature of the control. This is actually this phenomenon is actually termed as chattering and in fact the control also, the control also in the same sort of timeline. I mean if you see the control will be really nice sigma comma if I put the control as well and this is the sigma and also the control here.

So the control looks really nice here but then once you reach this place it will be oscillating very fast. There will be very high frequency oscillations. In fact I am making it very nice and clean. It will be much worse than this.

And this is a phenomenon that is not particularly great. So the problem is in first order sliding mode. Why is it first order sliding mode? First order means sliding surface is one dimensional. First order sliding mode leads to chattering. And though there is disturbance rejection and a lot of these nice properties this is not considered a particularly nice property to have. So what we want to look at is what one can do about avoiding the chattering phenomenon and that is what we will look at in the subsequent lecture. Thank you.