Nonlinear Control Design

Prof. Srikant Sukumar

Systems and Control Engineering

Indian Institute of Technology Bombay

Week 8 : Lecture 47 : Passivity based control: Part 6

Okay, the next one is of course cascade connection with passive systems. Okay, so this is exactly what was happening here. Okay, so what I am presenting is just a simplified already distinct version of that. So what did you have here? You had a nonlinear system which of course by under several assumptions could be written as a linear combination of some nonlinearity, nonlinear drift and some outputs. Okay, and then there was this linear system which is of course you have some nice properties that is passive with respect to the output and the input V, yeah, and there is this feedback interconnection. Okay, so that's exactly what is specified here for a more general setting again even a nonlinear setting.

Yeah, if you see now I have a system that is not controlled anymore. Okay, I have completely removed the dependence on the second state here. There is some differences. I am just trying to highlight what the differences are from here to here.

Okay, if you notice this dynamics first you see that there is this drift term here. Right, this no longer depends on the Y on the X state. Right, here it does. It depends on the Xi state. Right, so I have sort of removed the dependence.

It's not required, I mean but it's okay, I mean we don't necessarily need to do that. You could also have considered F sub A Z, X but again just to keep things simple we have taken this sort of situation. Okay, so of course we will assume that this system is you know asymptotically stable. Right, which is what? If Y equal to zero there exists W such that this is, in fact we are actually we are only assuming stability not even asymptotic stability, just less than equal to zero. Okay, we are assuming that there exists some Lyapunov like function or Lyapunov function.

Yeah, in fact I should say maybe more carefully this is positive definite function. Yeah, such that partial of with respect to Z FAZ is less than equal to zero. Okay, this is essentially the Lyapunov stability condition. Okay, so if I don't have any outputs then you have the passive system which is actually connected with it. That's exactly what you have here.

This system is passive. Right, that's what we assumed with all the strictly positive real and all that. Yeah, so this system is actually interconnected with this guy. Yeah, because of this Y. Yeah, this is the interconnection.

This Y goes in here. That is essentially the interconnection. Okay, so that's what we assume that there is this passive system. Now not necessarily linear but nonlinear. Okay, and we are seeing that this is interconnected to the Z system.

How? Y gets fed back into the Z system. Okay, exactly the same setting. Yeah, just that here you have nonlinear, there you have linear. So the linearity here is also not required honestly speaking. Yeah, not required.

So what are we saying? We are saying that the passive system output drives the Z dynamics. Okay, so what are we going to do? Of course if you have passivity you already have a storage function. Right, for the system VX and such that what happens? You already know that for this guy you will have V dot. Right, because that is essentially passivity for this system. Okay, notice again that this system has no connection to this system.

Yeah, not yet. Of course we will introduce it via the control. But as of now, no obvious connection. Okay, but we will make it an interconnection means back and forth. Yeah, that is otherwise it remains a cascade.

Yeah, here you see this way, this way. There is connection both ways. That's what we will do through the control. Okay, great. What are we now saying? We are now saying I will construct a new function U which I am going to claim is a storage function for this complete system.

Okay, a valid storage function. Let's see. So what is this U? It is just the W that I had from the stability of this guy and the V that I had from the passivity of the second guy. Okay, just added the two. Yeah, almost like back stepping.

Yeah, reminiscent of back stepping. Had some function for the first system then had a function for the second system, added the two. Yeah, that's it. That's all. I am now going to carefully take partials because I have to compute the U dot, the total derivative.

What do I do? I take partial of W with respect to Z and then Z dot which is this guy, the whole thing. Right, and then I take partial of V with respect to X and then X dot. So what is that? This doesn't have any del V, del Z and this doesn't have any del W, del X. Right, that should be evident because this is only a function of the X state because this is a storage function for this system. This has only Z states because this is a talking about asymptotic stability of this system.

Right, so no dependence of these functions. So these are actually, you know, functions on different state space. Yeah, great. Great. Now what do I know? I know by stability of the first guy that this is less than equal to zero.

Right, excellent. Almost ignore this. I also know that this whole thing has to be less than equal to U transpose. This is the passivity assumption. Yeah, so what do I know now? I'm only left with this guy, right, because I can ignore this. This is less than equal to zero.

Anything less than equal to zero I can ignore in V dot. Right, so this is actually from equality I go to less than equal to and then I keep these two terms. This is this. Yeah, notice what happened. I have a sort of feedback passivation type of situation now.

What will I do? I will simply choose, so if you see I can take Y transpose common outside in both these terms. So I have only this term left and this term left. Right, so I take Y transpose common and I have U from here and the transpose of, sorry, this guy from here. Okay, what will I do? I will simply choose my control U to get rid of this guy and introduce a new control V. Okay, introduce a new control V.

What does that give me? It gives me V dot is simply less than equal to Y transpose V. Right, so with this new control V and the output Y that was already there, this entire system is now passive. Right, now this entire system is passive. Okay, and as soon as you have passivity you know what to do. Right, you can construct V which is say minus KY and hopefully Y equal to zero implies X and Z equal to zero, you are done, you have a asymptotically stable equilibrium.

Excellent, yeah. Exactly what you did. Look at what you have. What is the control? Exactly this. Del V del X times this Fi. Same del W del Z times F, whatever was multiplying the Y.

Yeah. Just you know the Lyapunov function for the first system, the partial, so basically LGV if you assume Y as the control, that's essentially what you chose. Yeah, you think of this as a controller then partial of W with respect to Z times F or LGV as we sort of know in conventional terms, yeah, is exactly the feedback passivation term that you had. Yeah, exactly what he did too. If you think of Y as the control, this is just LGV, right, and that's what he chose as your feedback passivation term here and then of course a new control term.

Okay. Again he may have arrived at it differently with more assumptions or you know more complicated sounding assumptions but actually it's the same thing that we are dealing with also, okay. In fact the linearity here is also not required, right, as you can see we worked with a nonlinear system as long as you have a passive system and the output of the passive system drives this nonlinear system, yeah, in this way, yeah, linearly of course there is a linear parameterization of course, yeah, cannot have Y square and all, otherwise these terms cannot be combined, you see that's the structural requirement, yeah. So if you have a passive system which is driving a nonlinear system in this particular way, yeah, then and this nonlinear system without the Y is already stable, then this entire system is also passive, okay, so stable system in cascade with passive system also passive, okay, so or if you want to say it differently if a passive system is driving a stable system then entire system is also going to be passive, okay, so very cool result, very powerful result, in fact we will see a nice example of a very, very, of course if you also have zero state observability which none of these guarantee by the way, zero state observability nobody guarantees that you have to verify for that particular output, okay. So notice in this case zero state observability will mean that Y equal to zero implies not just X equal to zero but also Z equal to zero, okay, but if you remember Antonio actually spoke about the zero state detectability, yeah, zero state detectable if Y equal to zero implies that X converges to zero, okay, and in all these cases, all these results zero state detectability is enough, not zero state observability is not necessarily required completely, okay, zero state detectability is more than enough, okay, so this is actually rather nice, yeah, that you can actually work with zero state detectability, yeah, please keep this in mind, yeah, you can even write it down in your notes, but anyway it's in Antonio's notes which I have already posted on Moodle, all these ports, all of these, all these three are now posted on Moodle, yeah, so anyway so zero state detectability is enough, zero state observability is not required and if you see zero state detectability is rather easy to achieve in these cases because if Y is equal to zero you know that the system is anyway converging to zero, right, Z is going to tend to zero as T goes to infinity, okay, by asymptotic stability assumption, so I am done, this system is zero state detectable, okay, not necessarily zero state observable, I don't know that you can't say very easy, yeah, but it is definitely zero state detectable and that is enough, okay, feedback interconnection passive systems passive, passive system in cascade with a stable system also passive, that's what we just did, okay, where is it useful? Attitude control of spacecraft, okay, of course I do very very simple setup here, I don't explain anything, yeah, I'll not, of course I don't have that kind of bandwidth in this course, but this is one of the most important problems that space engineers work on, which is the attitude control, that is the orientation control of a satellite, so why is orientation control required, should be pretty evident, so you have remote sensing satellites or you have navigation satellites like GPS satellites, they all have some kind of antennas or some instrument that has to be pointed somewhere, for example in most cases in your GPS type satellites or even in remote sensing satellites, you want the antenna pointed to a particular point in earth, for example maybe towards India, yeah, but the satellite is rotating on the orbit, right, it's going on the orbit, evolving on the orbit if you may, yeah, so obviously it's, if you do nothing, if you don't put any actuation, you don't do any attitude control, the antennas are not going to remain pointed towards, you know, a fixed point on the earth, right, because it is going to do this, if you move and the antennas don't move, then it's just going to start pointing towards something else, right, so the simple task is you have to do attitude control regularly, yeah, because it's revolving on an orbit, right, same with you know if you have solar panels, so satellites need solar panels to generate power for their equipment, right, or else if they are in two years of service, you can't expect to be sending a battery or anything, right, not a choice as of now, yeah, so they rely on solar power, so now the solar panels also need to be pointed towards the sun, so there is lot of equipment on the satellite that has to be pointed towards, you know, a particular point and therefore attitude control is one of the key problems for space engineers, okay, so what is attitude control? You make sure that the, there is a frame, of course, I mean, again I don't talk too much about it, but I can make a small picture I guess, so not sure if I have a picture here, no, so if you have a satellite say or what we just typically just say rigid body, yeah, the same ideas work also for quadrotors and stuff, if you want to do orientation control of a quadrotor, same equation, same ideas will work, there is no real difference, it's a rigid body, as long as you think of anything as a rigid body, the same equations and everything will work, okay. So usually you have two frames of reference, one is what is called actually 3, but I am going to deal with 2, one is called the inertial frame of reference, yeah, usually I denote it as N or this is Newtonian because it's a Newtonian frame of reference, it is fixed to the earth, and then you have what is called the B frame or the body frame of reference, okay, this is the frame that is actually connected to the spacecraft body, rotates with the spacecraft body, yeah, and your aim is to, stabilisation means that I want to align the body frame with the inertial frame N, in typical set point regulation or tracking you will have another third frame which would be say an R frame, yeah, or a D frame whatever you want to denote it, yeah, you sort of want to align the B frame to the R frame, yeah, when I say align the frame, it's same as aligning the body because the body is connected, body and the frame are moving together, so if I am starting like this, yeah, and I want to end up like this, yeah, this is a B frame, this is an R frame, okay, so these are all like standard, you know transformations, attitude or orientation transformation, it's a very important manoeuvre as you can, as we have already discussed, we don't assume any movement of the origin, we assume that the origins are all fixed to the same point, we don't consider the movement of origin because these two problems are disjoint problems, you can solve them separately, that is positioning the origin and then reorienting are two distinct problems, so we work with them distinctly, like in quadrotors you have translational control and rotational control, okay, so we do that, this is the rotational control problem, okay.

So one of the big challenges is how to represent rotations, so again this is not something that I can delve too deeply into in this course, but rotations belong to what is called the typical rotation matrix, actually I should not have called this R, let me call this say some D, the typical rotation matrix between any two frames, common notation can be you know, yeah, is belongs to a, not a space, I can't call it a space, a manifold called SO3, okay, which is basically just the space of, or again sorry, the manifold of orthogonal matrices, 3 by 3 orthogonal matrices, okay, not just that, actually a little bit more, yeah okay, so anyway that tends to get hidden sometimes. So this is the space we are working with, okay, again I keep saying space, I apologise, it's a manifold, okay, whenever you say space it means a vector space and it is linear, by nature vector spaces are linear, superposition principle applies, sum of vectors is in the same space, yeah, sum of two rotation matrices is not a rotation matrix, okay, you can't just add two rotation matrices and get another rotation matrix, which is why they are not a vector space, it's not a linear space, it's a manifold, but we unfortunately cannot cover all that, the whole point is this leads to as you can see 9 state variables, right, eventually the representation, whatever it's an SO3 or whatever with some, it has 9 variables, right, so 3 by 3 matrix, right, with these constraints but still a 3 by 3 matrix, you can't reduce the number of variables, so it's 9 variables, so again engineers being engineers they like to work with less variables, so initially they started out working with these Euler angles, yeah, yaw pitch roll angles, the problem was that there is a lot of singularity in Euler angles, yeah, that is some you can, once you reach a particular configuration you can no longer represent anything beyond that, yeah, because there is singularity in Euler angles, again not going into any detail throwing words, but these were the challenges, so aircraft folks still like Euler angles because their rotations are smaller, typical aircraft, commercial aircraft not doing twists and you know flips, right, so actually the phi, theta, psi or the Euler angles are pretty small, right, for any commercial plane you can imagine, I mean I would not imagine anything more than 15, 20 degrees ever, yeah, so aircraft folks still work with Euler angles, fighter jet folks cannot work with Euler angles because they are trying to do crazy flips and stuff, spacecraft guys can definitely not work with Euler angles because they are definitely exploring all 360 degrees, as soon as the spacecraft is ejected out of a, you know, the launch vehicle, yeah, into the orbit, it's basically tumbling, it's essentially like you threw something with your hand, right, you can't control, it's going to be just flipping and tumbling, you know, all over, right, and then if you want to stabilize, so how do you even deal with the angles, you have to deal with parameterizations which do not have singularity, so Euler angles are a problem, so therefore we spacecraft folks move to quaternions which are four variables, yeah, instead of three, therefore Euler angles were three, they were four, and what we are looking at here is basically a modification of the quaternions only, these are called modified Rodriguez parameters, these are only three, yeah, they have no singularity, yeah, and they have, I mean well, wherever they have singularity is not where you are interested in operating, so you are fine, okay, so modified Rodriguez parameters is one representation of rotation matrices, okay, any rotation matrix can be written in terms of this row variable, okay, that is what the whole idea is, so modified Rodriguez parameters are pretty good, relatively singularity free, and they are only three variables, okay, so they are all these parameterizations of rotation matrices are based on some ideas of projection, so this is also based on some idea of projection, quaternions are simply based on the idea that any rotation, any rotation to initial to final configuration is not actually, you don't have to think of it as three rotations, it is actually one rotation about one principal axis, it's called Euler's theorem actually, any rotation between initial and final configuration is actually a single rotation between around a particular axis which is called the principal axis, and you have a principal angle about it, so this is the Euler's theorem, based on the Euler's theorem you have quaternions, yeah, and then you have modified Rodriguez parameters which can be derived from the quaternions, okay, but the simple idea is all of these help you parameterize the rotation matrix, so basically what I am trying to say is that this rotation matrix say between the body frame and the inertial frame can be written as a function of this row, okay, this 3 by 3 matrix can be written as a function of row, yeah, this expression is also readily available, alright, what are the other things? Orientation means I have orientation and angular velocity also, so there is an angular velocity which is in R3, thank you, linear space, there is a control which is what the thrust, typically the thrust you have a, thrusters are typically used in what is called attitude controller, reaction control systems, so these are basically, these are only jets that are firing, yeah, you must have seen some visualizations, yeah, this fire jets to reorient the spacecraft, okay, so this is the thrust and this is the inertia matrix J equal to J transpose positive definite, inertia matrix is constant in this model, okay, unlike the robot model inertia matrix is constant because the inertia is written in the body frame, all the equations are written in the body frame, okay, so this is also in the body frame, everything is in the body frame, okay, so more details on this are in a dynamics course, yeah, which we teach also later on at some point, but remember that the model is written in the body frame therefore, the inertia is actually constant, yeah, again for the fan if I took the frame as the one that is rotating with the fan and I wrote all my equations on that frame, then inertia is a constant, because my frame is rotating with the fan, therefore no change with respect to that, okay, so that's the idea, we have the kinematics equation and the dynamics equation, don't ask me how this comes, this is not a matter of again discussion in this course, just take it on face value, this is the equation, rho dot, all of these equations are derived from the equation for the rotation matrix, okay, and the rotation matrix derivative has a very simple equation, yeah, this is the equation for the evolution of the rotation, how rotation matrix changes, it is just actually I keep writing this R BN just for your convenience, typically we don't write the B and N, R is usually evident from what you want to work in, so this is how the rotation matrix evolves, the derivation of this is very simple, I'm not going to cover it, yeah, from this you get all these equations, okay, because these are just parameterizations of the rotation matrix, right, so once you know how the rotation matrix evolves, you also know how these guys evolves, okay, so this is the what is called a kinematics equation, the evolution of the parameters, yeah, or you can think in your head in terms of Cartesian as angle derivative is angular velocity, right, connected to angular velocity, so that is what it is, somehow angular derivatives are connected to angular velocity, yeah, that is the kinematics equation, and the angular velocity derivatives have some dynamic terms, okay, this is actually very very easy, this is just the Newton second law, right, this is DDT of, this is what this equation is, DDT of J omega is equal to U, okay, so when you take derivative and so why this turns out to be like this, is that remember that this vector J omega is in the body frame, not in an inertial frame, okay, so this is a vector in the body frame, yeah, it's in a vector in a rotating frame, if on this fan, rotating fan I put a vector, right, which is fixed with respect to the, not necessarily fixed but it's whatever, written with respect to the body frame, the rotating frame, it's a vector in the rotating frame, okay, so when I take the derivative of such a vector it always has two components, one is the change of the vector in the frame that is J omega dot, and the second piece is the inertial change, yeah, this is omega cross J omega, yeah, you would have seen this in your high school, this is I think, I don't know, the terminology used is called transport theorem, you sort of, how do you take derivatives of vectors in moving frames, so typically this in high school or in undergrad there is, I am assuming there is in physics this is usually taught, yeah, yeah, you may not remember the form, this particular form but it is taught, if you go back and you look at, you know, even your high school physics problems on Newtonian mechanics, you will see that you did this, yeah, so basically it's like how do you take derivatives of vectors in a rotating frame, yeah, if I give you a vector which is in a rotating frame, not in a fixed frame then how do you take it derivative, this is how you take it derivative, you first find the derivative with respect to the rotating frame and then you are taking the derivative somehow of the frame with respect to the inertial frame, that is omega cross J omega, okay, so that is what this is, it is just Newton's law written in a moving frame, okay, so very interesting but again not, I am not delving into too much details because this is not the intent of this course, yeah, but that's it, simple. Kinematics is angles, derivatives related to velocity, angular velocity, dynamics is angular velocity derivative related to, you can think of it. Thank you.