Nonlinear Control Design

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So how do we relate passivity to Lyapunov stability when we want to study stability in this sense? So meaning that we have now a model. I have been telling you that passivity is all about inputs and outputs. But now let's say that we also consider the state of the system. And we want to relate passivity with Lyapunov stability. Well we have that if the system is passive with a certain with a storage function or energy function V that is positive definite then the origin for this system without inputs because now we are talking about Lyapunov stability so we disregard the input then we have that we have stability.

Ok so think of again the pendulum without friction it will just be oscillating that's a stable behavior it just keeps oscillating. If there is friction then we will have output strict passivity. I didn't speak about L2 and all that but yeah so forget about this. If I have output strict passivity that means I have friction in my system and then the system will be asymptotically stable if in addition to being output strictly passive it is also zero state observable.

So zero state observability I don't know if you have seen this but it means so zero state observability means that y equal to zero implies that x is equal to zero. So if you have worked with Lyapunov essentially what we have in this situation of output strict passivity is that if V is my energy equation we will have V dot because it is output strictly passive we will have minus y square minus plus y or u or actually usually it's written the other way around plus u y and then we forget for Lyapunov for the purpose of Lyapunov stability we just want to zero the input so forget about this guy so you have only this let's say that yeah due to due to this this property y is going to zero then you just set it equal to zero and check if the state is equal to zero. We will do that for some examples so in case you this doesn't ring a bell but all this to say that yeah output is strictly passive systems output strict passivity essentially leads to asymptotic stability and yeah we if it is strictly state is strictly passive so we have this term here appearing with the whole state then then what then we also have asymptotic stability right away because once again you could use this as a Lyapunov function forget about this guy and you will have the Lyapunov function that is negative definite so the moral of all this is that you if you think in terms of passivity you you have to figure out the the energy function of your system and use that as a Lyapunov function essentially and then try to come up with some quadratic negative terms in your derivative and but the good thing is now now you have an interpretation for this for this terms now internally Lyapunov stability well we saw that the interconnection of passive systems is passive well here because if these systems are passive when we interconnect them this will be passive and due to what I just said over here how passivity relates to Lyapunov stability we will be able to also say things about this passive interconnection this this theorem here is taken by it's taken from voice a known result is taken from from a book by van der shafts it's Springer lecture notes I can send you the computer reference so it just essentially says that if you have these two systems that are passive the the closed-loop system will pass it will be passive so you will have stability now it is left of course to determine stability what will which equilibrium well normally when you are trying to control a system to set point for instance you would like that this equilibrium be a set point that you chose right so the only thing to that is left to do is to to choose to manipulate the energy of these of these systems in a way that the the the energy functions have have minima at the equilibrium that you want to stabilize right so you want the energy of this guy to have a minimum at x1 and the energy of the second guy you wanted to have a minimum at another equilibrium point right if in addition you have you have output strict passivity and the equilibrium is is unique then you will have a synthetic stability okay if provided that you have this this property of detectability yes the ability is to follow in detectability instant y equal to 0 implies that x goes to see so this is a weaker property than observability we just want to to be sure that basically if you if you know how to bring the output to 0 you want to be sure that bringing the output to see also implies that the whole state will go to to see if 0 is the equilibrium you want to stabilize otherwise just just yeah replace the the equilibrium to to be original okay so yeah I'm almost done with this set of slides essentially now we are well they presented you this a bit fast but I will show you how this works in the other in the other slides but what we want to do now is is passivity based control so in passivity based control essentially what you have is a plant that let's suppose that it is passive okay or maybe it isn't but yeah it's let's say it is passive it has some passivity property so you want to design a controller that you will put here in in the feedback loop and as I said when you interconnect systems that are passive in feedback the system the resulting system will be possible if the plant is not passive initially well you apply a controller pro if possible to render it passive right so you will want this at the end of the day anyway what you want is to enforce the passivity or to create passivity in your new system so passivity based control is about designing a controller in a way that the closed-loop system be passive okay that's that's that's what you want to do and if you want in addition really asymptotic stability of your closed-loop system you will want to be this passivity to be strict so yeah if you have seen this this inequalities for for nonlinear systems what we are going to try to do is to design you so that in closed-loop you still have a passive system so to continue speaking about passivity what we want is to to have you you should have one component that makes the system passive and another component that we were going to call you knew just to you know like I did it for the circuit you just need a new input from which to you you define your passive map right because otherwise if you don't have this you close the loop with some function of the state there is no new input then you cannot speak about passivity there is no input new input right so you want to design you so that the closed-loop system with with this part here so that will be these should be less or equal than the new input that you're injecting multiplied by the output okay in that case you will have a passive map from from the new input okay if your you till the in addition injects damping through this output then you will have this inequality here that will give you a strict passivity these on the on the other side is the what is that the energy well not the energy the power in the in the system if you integrate all that you will get the energy balance equation right the supplied energy the dissipated energy and the difference between the initial and final and not final energy at any time and yeah let me finish this set of slides with this very important theorem that says the following this is taken from from a paper by burn CCD or a williams three very big names in its story and it says the following if you have if you have this system and it is passive and you have some energy function with respect to which it is passive and also it is serious state detectable so I already said serious state detectability means that when y is equal to 0 that implies that x goes to 0 if you have these conditions all you have to do is to re-inject the output into it to add to add the damping and then your system will be asymptotically asymptotically stable in closed-loop so the control should it should look just by this like this simple expression just a function that is strictly contained in the in the third in the first and third quadrant right so there so of course just a constant times the the output will do but you can also use a saturation you can many you can use many nonlinear functions right the situation will do would look like this bounded there under and so on so you can use many things so let's see how passivity based control works now so now that we know the definitions of passivity and we know the Kalman-Jakob which pop of statement for linear systems and its equivalent version for nonlinear systems again taken from Halil now I will be working with with these notations so L of V is the directional derivative of V in the in the direction of F right so that's partial of V with respect to X times F yeah that's that's this the output is defined like this so that's that's reminiscent of that yeah for linear systems and this one is reminiscent of this one with so here we have we have Q that is positive definite and here we have a class K function of alpha of the normal alpha so we are asking a strict positive realness in the linear systems it comes to asking this negative definiteness of V dot in the direction of F ok so basically we are asking the the system to be state strictly passive you see we have V dot less or equal than minus alpha plus UY so the energy balance inequality gives me this the supplied energy is larger than the energy dissipation through the whole state plus plus a constant which is actually negative minus V 0 so that's the energy at the initial time and now so that's what we want to work on with for to use passivity to design to the sign controllers it's just a reminder of of passivity for it basically what we saw in the previous slides with different notation to confuse you a little bit why I already mentioned that this integral is known as inner product which can also be written in compact form like this and you know we speak of passivity if it if this integral is bounded from below by a constant a real constant so that's basically the this number here ok output strict passivity we have this guy and the state strict passivity we have this so let's let's see passivity based control in its general form so these methods goes back to the work of Peter Kokovich and Hector Sussman in 89 and I don't know the story very well but Ortega so Kokovich was kind of the big boss in Illinois in in 89 and Ortega and Spong, Spong I think had just been recruited assistant professor and Romeo Ortega was supposed up there so yeah Kokovich came up with with this with this method of passivity based control and Romeo and Spong more or less in the same the same time they wrote a paper that has been cited a lot on adaptive control of robot manipulators and they coined the term passivity based control in this paper it's called feedback passivation or something like that anyway the method is here just for the little story and essentially it consists in the in the following so they they were studying systems that have this form so they these systems basically come from from you have a nonlinear system we are in the 80s in the 80s the people was were mostly working with feedback linearization I guess you have studied that feedback variation so you want to find an output and the feedback such that and transformation coordinate transformation that brings your system into a linear form there are some structural conditions to be able to do that and sometimes you for a certain output you cannot feedback linearize the system completely so write it in in your form but only partially and you end up with a system that has a linear part and the nonlinear part so these guys were came to the system from working with feedback linearization so here what we have is after applying the preliminary feedback and transformation and so on let's suppose you end up with this system so you already applied a preliminary feedback but you say okay I'm going to to add another feedback to to to be to complete the the stabilization task because I didn't manage to linearize this part this is what people knows in feedback linearization of zero dynamics okay with respect to the output psi so meaning that if you partially feedback linearize your system and it became like this it is fairly easy just to design you so that you know if A and B is stabilizable you just bring psi to zero and and that's great but once you bring psi to zero what would happen with with the zero dynamics that is the dynamics when psi here is equal to zero okay so maybe they are unstable and there is nothing you can do about it through by through the the controller priori because well it's just sitting there and there is no control input there so what what can you do so these guys came up with this with this nice method that applies of course to a certain class of systems not just any certain class of systems that for which F satisfies certain properties so we need to lay out some assumptions and definitions and stuff from from this paper to understand what they did first they they they say well now our result will hold for systems for which when we take psi equal to zero we know that the the origin for the for the resulting system is gas and to no minimum phase and the the nonlinearity can be written in this form I sorry I have a doubt about this so anyway the nonlinearity has to be written in the in this in this form so basically you want to from this nonlinearity you want to extract one part they call f0 plus the sum so this is a bit particular of a bunch of nonlinearities that are multiplying certain outputs okay and they call an output feedback positive real if it has a property that there exists a feedback like this so minus cake side to do you know place the poles of this guy here and an extra input such that when you apply this feedback to the linear system here you get a strictly positive real system meaning that the matrix a minus bk will be Hurwitz and the the closed loop system with with this feedback will be strictly positive real okay so well it will satisfy the Kalman-Yakubovich problema with these for the closed loop system right so with with a feedback matrix here so you take a minus bk that should be should be Hurwitz and plus the system should be of relative degree relative degree one now what does it mean a feedback positive real output well there are a lot of definitions it's it's a bit hard to read the paper but it's a very important paper so they call this feedback positive real output essentially if it's an output that such that the the non-linear well such that you can write these non-linearity like that so they say in general terms if f belongs to the span of y so basically y is it's a vector of outputs right there is yi here from there is y so it's a vector of output so you can consider that as if you are dividing you are splitting the the non-linearity into a linear combination of a bunch of other non-linearities in which the coefficients of that are multiplying your your other functions well are also functions of the state okay so it's it's especially this guy here so again you want the output to be composed of y1 up to ym and each of these is multiplying a non-linearity so you have f0 plus f1 y1 plus f2 y2 plus f3 y3 etc so the the non-linearity has to have this special structure and then they say well I am going to call f a stable f PR stories band the composition essentially if you have that the first non-linearity in in in F is is globally syntactical state okay so yeah the origin for this system okay so x your system is x dot equals f0 of x and psi plus y1 f1 plus up to ym fm now essentially what they are doing is assuming that this guy is already gas so they there is not they don't have to worry about that so basically we want we want to worry about the effect of these non-linearities here and what the passivity based control created by kokoto which was is doing is defining defining you in a way that it takes care of the effect of these non-linearity seen in the system okay so that's that's what we what we are aiming at but for that this they need all these these assumptions so as I was saying this has to be gas and not only gas but also uniform globally asymptotically stable uniform link psi what does that mean it means that we should have a Lyapunov function that is positive definite and radially bounded you it's a level of function depends also inside but the bounds do not depend on sir and the V should be negative definite along f0 also independently of time so it's a very strong condition but yeah you can you can have it for for many systems and then then what they say if if we have all these these conditions essentially if the if f I can decompose it like this then the system is globally smoothly stabilizable so it's mostly emphasized because again in the 80s there were already some results using non-smooth controls and and constructing Lyapunov functions that are not smooth etc so the big deal was to do all this so yeah if your system if your F can be decomposed as I said and you have these properties these guys say the following well all you have to do is is go and take that V that you already know that works for for the first piece of the system that is f0 take that F take its derivative the f0 term will give you something negative and then you will have a bunch of other things that you don't know what a priori you don't know what to do with them yeah because f is f0 plus all that so you will have this well that's where this equation comes into play the next thing you want to do is design an additional input in that system that will take care of those nonlinearities and all you have to do is is to add to your Lyapunov function that you you designed for that you have that you have initially for the for f0 right sorry I wrote it as a function of X but it's also function of psi so you use that function right no sorry yeah I think yeah yeah yeah what the psi appears in the nonlinearity I think it should be it shouldn't be there in the in V anyway it's a function V that you use for for for the for the for this part of the system okay now you take the same Lyapunov function you take the derivative you have a bunch of nonlinearities that appear what you want to do next is to add a quadratic Lyapunov function in function of the state of psi which is the system is below and when you take the derivative of that what will you obtain because because you design K for these to be

Hurwitz well since this is Hurwitz it generates a function P right matrix P that you put here and you take the derivative of this new function so the function you initially had plus this new quadratic function here okay you will obtain that the derivative is well is equal is lesser equal than this guy that comes from from V now this one you know that it is bounded from above by a class K function then you have all these guys that you don't know a priori what to do with them and then you have of course this guy that comes from Hurwitz of of these metrics but then because you are the new input that you you are still going to the side now you will get this term here psi transpose times P times B times the new input okay and what will happen next is that you see PB by by construction because because your system here is strictly positive real so the it has this structural property that PB is equal to C C being exactly the the metrics that defines this this output here what you will have is that actually this this term here is nothing else than the sum of all the outputs yi times the inputs VI so it's essentially y transpose the new input so you can see easily that all what you have left to do is say I'm going to take V to be equal to that with a minus sign so because V appears there and V and Y appears here Y appears here so I just have to match this right in this way and all these guys will go away and you will be left only with this but as I said these by assumption generates an alpha term and now and this one comes already from the Hurwitz of that so yeah and all the all the rest you just you just eliminated it and you end up with this very nice inequality so yeah if in case you don't see the passivity here what you have is before eliminating all the all the terms you have this this guy here so essentially you have w dot less or equal than minus alpha 3x right then this guy minus 1 half psi transpose q psi then close this guy plus y transpose V so it doesn't fit plus all the bunch of things that you want to eliminate so if you integrate this and just for the purpose for the sake of argument forget about this guy you see that you have this passivity property between V and Y right so if if this thing that was is bothering you was not there you have a passive system now essentially you have a passive system with something else that this is coming into your system but it's coming into your system through through the right channel because it's multiplying exactly the passive output so the perturbation that this is coming into your system and so multiplying this output that is from which you have passivity so with it with the extra input you can actually go and and kill the the terms that are the perturbation that is coming into your system all