Nonlinear Control Design

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So let's just formalize a little bit what we have seen for these elementary examples into some definitions of passivity that you find in the literature and that are useful in passivity-based control. So we consider a generic system again just a black box with inputs and outputs and we denote E as its energy we're going to assume that it is bounded. We are going to say that the system is passive if the energy balance equation looks like this ok so available energy equals initial energy plus whatever I supplied into my system. We're going to say that the system is output is strictly passive if the energy balance equation looks like this. So not only I have the initial energy and the supplied energy but some energy was dissipated and just you know went somewhere else I don't know where but just into heat or whatever as I explained before.

So there is this component of dissipated energy with a minus sign that means that I have some damping in my or friction if you if you want in my system right so we call it output strictly passive. So notice that we have a coefficient here that is positive and we have the square of the output in this integral that's what this is. And then we are going to call it input strictly passive if we have a similar balance equation but instead of these we had before with the output square here we have the actually the input. Actually this is the kind of system that will be very useful in later it's the kind of system that you want to have when you are doing some control because it means that you have dissipation in your in your system and you are able to stabilize it.

A very important a very important statement actually the fundamental theorem about of passivity is that when you take two systems that are passive and you interconnect them in feedback like here so this system is let's say is passive from this input to this output and this one is passive from here to here and you interconnect them in feedback you will again recover a system that is passive from from the new input which would be the the vector of these two guys to a new output that would be the vector of these two guys. And you can keep interconnecting blocks in feedback and you will conserve the passivity property and and that's where we see this this can be very useful to to construct your your controllers always trying to maintain this this passivity property or rendering the system passive if it is not in the beginning. So if the system sigma one and sigma two are passive then the connected system is passive meaning the the map from from u to y is passive. So we write that as using this this funny arrow like this that's in LaTeX that's maps two. So that's the it's we are seeing the system as a map right as an operator that maps transfer once again transforms inputs into outputs so that's why we would write it like that.

So once again the the electrical circuit let's see what happens to this this circuit we already saw that we have a resistive element we actually have this this integral of I squared R right in sitting there in the with a minus sign so meaning that that's that system is satisfies this this balanced equation right so my circuit my RLC circuit is output strictly passive. So what can I do with with a system that is output strictly passive? Well I can for instance interconnected with another system that is passive in this case if I just use another resistive element this resistive element you can consider it as input is strictly passive because it's just a constant so if you if you put the so the input here I mean the output on this side will be of course just RC I so you can you can see that this is this is of course input strictly passive right because the input and the output are basically the same just escape by some constant so it comes to connecting this over here right so I'm adding this this resistor into my circuit and now I will have a new a new input I will be measuring the the voltage from from here from these terminals so this is what I'm doing in in in the from the block diagram perspective I have this output I pass it through this new resistor the current and I inject everything back into into my circuit and now I have a new a new input so this is this is how we deal with with passivity and we go constructing passivity blocks right so we start with some passivity block we make a feedback interconnection with another passivity block and we define a new passive map from a new input to probably a new output in this case to send off all what we did here was adding to other a resistor so what will happen with the Kirchhoff's law so is that there will be this new term here appearing so the voltage this in the new resistor that I added and it will just go to the other side and we add up to to RP so what I did with this is of course we I added more dissipation into my system I already had some but I added more with so I use P for plant right there was already some dissipation there but let's suppose that this is a control term so I added I chose RC and that's that's like I'm modifying the circuit with this so I'm adding dissipation into into into my system that's what I have done with these with this feedback here of RC right so now my new energy balance equation will look like this and I will have that the available energy equals the initial energy of course so kinetic potential energy in the capacitor plus kinetic energy in the inductor that are that were already in the system and I just started the resistor to increase dissipation of the in my suit I could probably also add a capacitor and modify the potential energy right that could also modify that yeah yeah you can yeah you can if you so if you connect a system that this is just you will definitely conserve the passivity property that you have that you had initially right so with this feedback you will probably be adding more passivity or just conserving what you have so if you have passive and you interconnect with passive you will have passive right but if you if you add some two let's okay let's see this way imagine you don't have RP so initially you don't have the dissipation so you don't have this term here that that system would only be passive right but then when you inject this dissipation you are adding you are enforcing the passivity and then you will have output district passivity right this is the output so this energy equation is telling you that the system after the interconnection is up with the strictly possible yeah so you can you can enforce the passivity property with with the so the the energy of the secret is the same with or without feedback that's that's the energy but by adding a purely resistive element we would just add the dissipation we added friction imagine if it were a pendulum previously I showed you an example without friction then you come and add friction now you have output district passivity so you enforce the possibility the addition of an inductance would change the kinetic energy you could also go and do that and you could also add a capacitor and modify the potential energy so you can do a lot of things you can modify reshape the energy the way the way you want and then add dissipation the way you want we'll see how we by manipulating these things you can you can control the system very nicely the resulting system is still passive because once again the fundamental theorem of passivity tells me that the interconnection of passive systems is is is possible okay so what happens with if I now I interconnect to systems that are different in nature right a mechanical and electrical system let's let's go actually the motor this is supposed to be a motor it's it's electromechanical bullets let's just suppose it's just an electrical system so before we said that the input to the pendulum was was torque right but this torque is actually coming from somewhere probably from from my hand or or or preferably from some actuator from from a motor and this motor so this motor has an output which which is going to be the torque that is going to be injected into the pendulum but it obviously needs to have an input so that or transforms the energy to move the the pendulum and that input will be typically a voltage right you apply a voltage into the motor you make it spin and and generate a torque that moves the pendulum right we all know that if we look at the equations we will have the following we have the pendulum equation with a torque that is being injected and this torque is is coming from the motor right so the motor sees this torque so the this this term appears in both in both equations here with positive sign here with negative sign because in the equation of the motor so this is the question of the motor this is supposed to be the the inertia and this is the resistance in the the friction sorry in the in the in the rotor and this is the the generated electro what do you call it magnetic feedback torque or something is called from the torque that the motor is generated generating and and the motor sees the torque tau L as a load right because for the motor it's a load with the the the the pendulum is a load so you have this this term there in there so this is how these two systems get to be interconnected and we go and look at the energy balance equations for for these for these guys so basically we integrate on both sides of both equations we will have an energy balance equation for the motor which will be of course these the energy by the energy available at some moment will be equal to the available energy at the beginning whatever was dissipated in because we have friction in the in the rotor and then the supplied energy so basically the energy that is entering into the motor well there is of course what I supply into the motor to make it spin that comes that comes here but there is the load yeah okay so so I inject some some energy to it but some energy goes into moving the the pendulum so the supplied energy will look like this and for the for the pendulum the pendulum is so happy because it only sees energy coming in so there is supplied energy there and there is some initial energy and and there is then available energy so basically the available energy equals whatever is supplied through I mean coming from the motor plus the potential energy that was over there already now the nice thing about this is the whole thing when I the system interconnected will also have some energy right and this energy will be basically just the sum of of these plus that so now we have the sum of the the energy in the motor plus this energy in the in the pendulum that will be the energy of the whole thing and what will happen is that if the pendulum didn't have friction it doesn't matter because this guy will will just contribute there and the energy balance equation for the for the closed-loop system for the interconnected system will look like this now we have this term of energy dissipation and we have this term of supplied energy coming from all the way from from the input voltage that this is here and now my output is still there okay so now I have an interconnected system and I have passivity actually I would have output strict passivity due to this from the map V to Q dot and this is so once again this is because the motor is output strictly passive with input being this difference the pendulum is passive so I'm interconnected a passive system with an output strictly passive system so the resulting interconnection will be output strictly passive due to due to this guy right so I recover this this term in the total energy balance equation so again the feedback interconnection of two passive systems yields a passive system a passive system with input strict passive feedback yields an output strictly passive feedback yeah in in interconnected it will become strictly passive that's the case that they showed you with the but the electrical circuit with the just with the additional resistor right the output is Q is Q dot yeah the output is Q dot yeah there is no possibility with respect to Q the output is Q dot velocity and yeah one thing to remember is that passivity is conserved when you interconnect the systems in feedback okay or is that feedback yeah feedback if you interconnect them in cascade so the output of one goes as input into the other and so on but there is no feedback loop you don't have passivity necessarily okay from from here to whatever output you choose that we are talking about interconnections in feedback form I will not do input output stability let's let me just tell you a little bit about this because I want to show you how we can use passivity for control but for that I first need to tell you about a couple of things in the steel of passivity theory so in linear systems we have this this concept of positive realness which I believe was introduced by Popov Romanian mathematician who studied a lot equations linear equations right and was figuring out properties of these linear equations and one one property that I think he actually came up with this with this concept of positive realness is the following we say that the system a transfer function g of s for a linear system with with a B C and D matrices is said to be positive real if the real part of the transfer function is larger equal larger equal than 0 the nice thing about this is that a positive real system is is passive ok and there is another important concept in for linear systems which is called a strict positive realness so a strict positive realness is the property that the transfer for the real part of the transfer function with s shifted a little bit right so epsilon is a small number but but positive that should be larger equivalency ok so basically you are asking in this case you're asking in the first case you are asking that the the Nyquist plot is is somewhere here right so in all this zone and in the the other case it has to be like sorry the other the other place right so the positive real in the first case you are asking that the the Nyquist be here right there and in the other case you are asking that the

Nyquist be strictly separated from the from the vertical axis and we call it strictly positive real now strict positive realness is a very strong property of for a linear for a system to have it basically means that the system is strictly passive in the whole state I didn't speak about state strict passivity but you can just think of these as having the in the energy balance equation instead of having q dot you have here the the whole state right so X so both you know we're both input and output circuit passive now the nice thing about strictly positive real systems we will see that with the apron of functions it will probably be clearer is that we have this very nice lemma that is known as Kalman Jacobov which pop off essentially because I think this the story goes that pop off proved the one implementation Jacobov which probably approved both Kalman proposed different proof each of them work separately on this on this result I think Kalman published it first but Jacobov apparently had it first but then as many good Russian mathematicians just put it in the drawer and want to solve something else so anyway so it is called the KYP lemma it's a very famous lemma and it says that a system is a strictly positive real if and only if so assist a linear system like that if and only if yeah there exists a matrix P matrices L and W such that all these equations hold all these qualities hold so in the case when there is no feed through I mean directly from the input to the output which it would be this term here then these equations just boil down to these equations here ok so we have that ATP plus PA equals minus Q and and this is structural condition here now this one may be quite familiar to you because it just you know that if a is Hurwitz for a system you can you can always find for any for any Q positive definite you can find P also positive definite so such that you have these these equations called the Lyapunov equation right but here we have something else we also have a relation between so B is the is the thing that is multiplying the inputs and C is what defines the output defines the output yeah so this condition gives a structural property between between the structural property of the system and actually this condition means that that the relative degree of my system of G is is what most it can be one it will be zero if there is this feed through but if we are in this case down here then it can only be one so this is a relative degree condition and this is basically a condition that says that a should be should be called what's right so should be stable yeah so that's that's another thing to know passive system necessarily is of relative degree either one or zero you cannot system of higher relative degree than that cannot be possible that's me back in the day with professor Yekovovich I'm very proud of this this visit just after my PhD and this is the piece of paper that he used to announce me as professor Antonia Lori I had just to finish my PhD so yeah it's sitting there I will not do these slides now I will do it in there and the other set of slides so I will just pass that yeah one one nice thing about passivity is you can also use it as I said you can use it for linear or for nonlinear systems because again it's a concept of input output systems right so for our for nonlinear systems yeah let me go here this is this is like a nonlinear version of of the KYP lemma and basically it says well this this this statement I took it from Holly but the result is by the original papers are if you're interested you should look at the hill and Moylan I think he was student of Moylan but I'm not sure I'm talking about series of papers in 76 and 80 and there is probably one more intersections so he essentially introduced passivity for non for nonlinear systems and the the this expressions as you can see they are really like a nonlinear version of the previous ones so you want to have storage essentially you want to have a storage function or call it an energy function such that the inner product of the partial of V with times F is less or equal than zero and you have these these inequality here so basically the first one is like having ATP plus PA equal to zero or less or equal than zero and this is if we had an equality here this would be the condition that we saw I think it was yeah probably was like this right and in this case as you can see we have a quadratic term here so this is like having ATP plus PA equals minus yeah minus y square right and here we have again the basically the structural condition you already saw this okay so yeah this is this is the nonlinear version of this of this thing here well mostly of what is below right of this and this formulation here is again is from Halil but the original result goes way back to the 70s as we will see we can use these to also use passivity for nonlinear systems not just for linear systems but it's nice because you can kind of in a way see linear systems as if they were nonlinear if they were linear because of this input output properties and think of energy terms and so on you