Nonlinear Control Design

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Week 8 : Lecture 41 : Passivity in control systems: Part 1(Prof. Antonio Loria)

Yeah, so Srikanth asked me to give you a short lecture on passivity. So what we will do is I will revise some concepts about passivity, definitions and such, and then do something about passivity-based control. So this is the plan for part of the talk. I will speak about passivity systems theory and also passivity-based control. That will be like the second part of the talk.

So passivity, if you have studied stability in the sense of Lyapunov, passivity is some kind of dual of Lyapunov or complement. You have to leave behind the story about a model with a differential equation and so on. Passivity is really a black box perspective on system analysis. So for instance an electrical network transforms voltages into currents, a radiator transforms current into heat, etc.

So a falling object that transforms gravitational force into speed of falling and so on. So you have a system that has inputs and has outputs and it transforms the inputs into outputs. So let's look at passivity through the examples of electrical networks because it kind of started that way. Basically you have in an electrical network you have this input here, a voltage that you apply to terminals and then you have a load and you have this current flowing through the load. You can see that as a black box system.

So you have the voltage that goes in and the current that goes out. You can say that there is a difference of potential that is here and a difference of potential as you know is just the difference between two charges of opposite signs. And that will produce the current. So considering that the load is a conducting element it will actually oppose to probably to the passage of these currents. So owing to the loss of magnetism the charge Q plus will, I mean this is a convention, will move through the load towards the charge Q minus and this movement of the charges is actually the current.

So as we know the current is the derivative of the charge with respect to time. How these charges essentially are moving inside the wire. In terms of energy we say that there is an electrical potential energy that is transformed into kinetic energy. So again we apply an input voltage that's a potential energy that we have there because we have some potential energy stored probably in a capacitor or in the adapter and it is injected into the circuit and it generates kinetic energy, generates movement, meaning the charges are moving if there is this current that is being generated. So the key thing here is that there is transformation of energy right so hence the dynamic system.

Now if we assume that the load is purely resistive and we apply a voltage at the input then we have, what we are doing is we are supplying some energy into the system right so there is a supply of energy this is this is an important keyword in passive systems. Naturally the resistive element will warm up right so then some energy will be lost so again there is there is voltage there is an energy transformation there is current generated it goes through the load but some of the energy that is generated or transformed actually will dissipate in the form of heat because the load is heating up. Then we speak of systems that are dissipating energy right so it's energy that is kind of lost well not lost because well it's lost in this in the sense that you are not using it but it is there right it's just transformed and it's transformed in the form of heat. Maybe you are too young to know the tungsten incandescent bulbs now that they are all lens but there was a time when they were resistive elements and they could get pretty hot so that would be an example in which you have this energy loss right. Another part of the energy maybe perhaps recovered for some purpose nowadays we see a lot of research applied research into going in this direction how can we recover energy from these type of systems.

In France for instance they are working in Alstom the company that makes trains they are trying to figure out how to recover energy from the breakage of the trains when they break when they come to stop they want to recover the energy that is suspended. This is a classical example otherwise of a passive system right of energy transformation. A dam you have some water here there is a difference of level in the water so there is a difference of potential right so there is some potential energy stored due to the difference in the levels of the water and then when you open the valves and the water flows here that you create a movement because there is this transformation of energy once more of potential into kinetic it goes you know the story it goes into a turbine it makes it turn etc and you convert it to electrical energy and so on. So this is just one more example of how you can transform energy and that's what we want to do essentially when we do when we deal with passivity and passivity based control. So to study all these in a more formal way what we do is we need some some tools right some definitions and some some theorems and so on.

So what we are going to do is to start with this energy transformation equation which recovers all what I have been saying so far and this energy balance equation what it says is that the energy that you have available at some time t is the same as the energy that you had at the beginning of your experiment whenever that beginning was and minus or the energy that was dissipated that you lost into into heat in the bulb or whatever and of course the energy that you have available also depends on the energy that you put into the into the system right so you have some supplied energy for what you had at the beginning and what was dissipated somewhere in the way. If there is no dissipation well ideally we can think of that of course in reality there are no systems that are lossless you always lose some energy but mathematically you could say that if you don't have dissipation then you you would have an equality here and we would call it lossless system. Otherwise this energy balance equation is telling you that in a passive system you can only recover as much energy as I mean the maximum energy you can recover from it is of course smaller than whatever you supplied into it plus whatever was was already there right so very very simple inequality. So in the example of the circuit we have that we have this energy inequality and this energy balance equation satisfied due to the energy dissipation in the current right so one cannot pull out more energy out of a passive circuit than what was fed into it that's that's a very clear statement right so let's let's look a little bit closer into into circuits at least that's what people did many decades ago and came up with this interpretation of passivity of linear systems there is a whole theory in the frequency domain but I will not go there so let's imagine our circuit is an RLC circuit so it has a resistive and inductive and capacitor elements each of these systems is of each of these elements have a key role in the in the passivity of the of the circuit so first of all if we apply Kirchhoff's law we can see that the we have this this equation here so the the the voltage equations law of Kirchhoff it says that the voltage of the input is equals to the voltage to the sum of the voltages in each of the elements right so the voltage in the inductor the voltage in the capacitor and the voltage that is dissipated in the in the in the resistor element right so this is the the Kirchhoff's law of balance of voltages if we multiply by by the current on both sides so everywhere we obtain the power balance equation right so you can see the first you could see it as a force balance equation and the second one it's a power balance equation right so you are multiplying the current by the the voltage now what we are doing here I said that the the current is the output and the the voltage is the input right so this is you should see it in a more general way as the product of the input and the output that's that's a power balance it gives you a power balance equation so the power in your in your circuit is equal to the power dissipated through the resistor element the power in the capacitor and the power in the in the inductor right if we integrate that on both sides so we integrate this on both sides we put the integrals everywhere we will obtain this right so then we have the integral of power well it's it's energy right so we have the energy balance equation that this is on the slide so this is the energy in dissipated in the through the resistive elements due to the passage of the current this is the energy in the inductor and this is the energy in the capacitor now the as you can see I'm using the letters V and T here to assimilate these part of energy in the inductor as a kinetic energy right because yeah the the energy stored in the in the inductor is considered to be kinetic energy and the energy in the capacitor is potential energy right so it depends on the charges on the on the circuit so what but we have we have stored in the capacitor and yeah so all that is so this is a dissipated energy this is this is potential energy and this is kinetic energy now you have the integral from 0 to T so that gives you the energy available at any time T and here we have the energy that was available initially both in the capacitor and the inductor so to say and on the other side you have the integral of the input and on the output the product of the input on the output this is called this expression here is called sometimes inner product okay so the inner product of E and V and yeah it's written so this is written sometimes like this maybe you have seen it so that would be just this this integral there yeah so this is the energy balance equation as I said if we rearrange all the all these terms we put these and this guy together over here we will call that available energy at the time T and these two guys which come from here and here that will be the initial energy that was in my circuit and then the society already said is the dissipated energy right and this is what I supplied I just put it on the other side I guess there should be a minus there no maybe not no this one days these one went on the other side and then go to minus so this is the the energy balance equation right so what I can pull out of my circuit at any moment equals what was there in the beginning minus what was dissipated plus what I supplied into into my message very simple the nice thing about passivity is that well basically somehow any system I would say probably is possible in some way right but it just depends what do you mean by passivity as you have you have seen everywhere here we have an input and we have an output so when we talk about passivity we just need to be clear passivity what do you mean from from which input to what which output okay but then you take any basically any system in real life and there will be somehow some map mapping there between some input and some output and you should have this passivity property so in engineering another typical example of a passive system is a pendulum right so so let's see a little bit about about pendula how this this passivity works so let's say we have this this pendulum here with there is some torque applied to it here and it moves with certain velocity Q dot and then it acquires a position an angle that we are calling Q with respect to the to the horizontal axis you can define it the way you prefer but I'm doing it this way and assuming that the mass is right there at the all concentrated here then we have this center of mass there and the gravitational acceleration acting there so there is a force applied and L is the distance from the joint from the axis of rotation to the to the center of mass so the force balance equation for the system as we see it so for the for the circuit now the force balance for this for the system is equal to this right so we have mass times acceleration then we have mgl sine of Q which is the gravitational force right we can view to the to the gravity that is acting on the on the mass here and we have some torque that we are putting in in our system to move it right so this is a force balance equation we we apply some force and then there are forces that are natural to our system one force that comes from kinetic energy and one for the comes from potential energy so we have this force balance equation and we can also compute the energy balance equation so the energy balance equation will be given by this expression here it's going to be this the addition of kinetic energy and potential energy right so quadratic function on the angular velocities and this term of potential energy now the the energy balance equation can be can be obtained as follows the total time derivative of the the energy equation equating the force balance equation is this let me recall what I'm doing here so if we take the derivative on both sides of this of this equality here everywhere right the derivative of that will obtain this equation here so the derivative of the energy equals Q dot times times tau the details are not here but you can see that actually what we have is that so the derivative of that so the derivative of this with respect to t is going to be mgl sine of Q right times Q dot so when you take the derivative of that you will obtain you will obtain and from the derivative of these you get m times Q dot times Q double dot that comes from here so that's going to be tau minus mgl plus sine of Q times Q dot and all that is yeah something like that sorry all these all should be divided by no this is this is fine right this is Q double dot is yeah divided by m anyway this I think there is a wrong sign there but the the thing is that these should go away with that and you are left only with this with this term here is there a sign wrong there here is minus right oh here yeah so yeah anyway normally you should get you should get these so the thing is that all the the nonlinear terms go away and you only have on one side the derivative of the of the energy and on the other side you have the you have Q dot that we are going to call output and you have tau that we are going to call we are going to call input yeah so we have this energy balance equation once once more as we had before current integral of current times times voltage equals equals energy or energy the derivative of the energy equals current times voltage in the electrical circuit now if you integrate on both sides of this equation you get of course the energy balance equation notice here the available energy equals the initial energy plus the supplied energy so the supplied energy was applied through the torque that you injected into your in your system right so this is the external input if you are trying to control the system that will be your your control input in this in this pendulum as you see there is no this is an ideal pendulum without friction which obviously does not exist but if we add friction what will happen is that we will recover from from the same energy equation we will recover yeah this dynamic equation with some damping here okay so this this is a friction coefficient that is dumping our our our system right so like if we just let it go it will go down and then eventually it will stop oscillating and just go to to the natural equilibrium due to this due to this dumping in the previous example if you just it's an ideal pendulum that you you can just push and it will just keep oscillating forever or or maybe just small oscillations but of course that doesn't does not exist yeah the energy balance equation for this system with the dumping now will be these so the derivative of the energy equals minus B Q dot plus square plus Q dot times top so once again the the output times the input the dissipativity term and the energy the derivative of the energy if we integrate that then we get again this energy balance equation right so available energy equals initial energy minus what was lost due to the due to the friction and the of course the supplied energy so we can get the maximum energy that we we can get at any moment it amounts to maximum the initial energy plus what we supply into it so the moral of the story is that passive electric element is a device that does not generate generate of course rotation because we know that it's not possible to generate energy but just to transform it but let's say a passive element is a system that does not generate energy but it only consumes it okay so at best it doesn't we don't lose anything but yeah normally it consumes energy right so in electrical circuits theory we say that the voltage source is an active device right it's the one that is injecting energy actually probably taking it from somewhere else and and the resistance is a passive device right because it is it is the one that is actually dissipating energy passivity is an energy transformation concept you want to see a system as a black box it is just transforming inputs into outputs and in that way it's just transforming the some kind of energy into some other kind of energy and the in this in this process there is necessarily some energy that this is consumed and probably dissipated so overall is an input output perspective into into analysis of systems okay so we don't care so much about about what is inside when we don't want to do use passivity to control the system of course we want to look into the into the model but we will see how these concepts help to design controllers from a very engineering viewpoint trying to manipulate this this energy so but other than that in this in this theory the system is really a black box and what else we have here since we regard the nature of the inputs and outputs and also the laws of physics then a passive system may be modeled by a transfer function so of course there is a lot of theory and passive systems for for linear systems using transfer functions using the frequency in the frequency domain but we can also but it's it's a it's way more general because again it's input apples on your black box you can have whatever right it can be nonlinear it can be discontinuous time varying when you can you are just you don't even basically care what's inside as long as you just have inputs and outputs right so it's really very general a very general theory you