Nonlinear Control Design

Prof. Srikant Sukumar

Systems and Control Engineering

Indian Institute of Technology Bombay

Week 7 : Lecture 37 : Backstepping method for control design: Part 4

So, we have already started the design right. So, we have already started a little bit of design. So, we will continue with the design ideas itself. You will also have a tutorial session over the weekend where you will cover a little bit of back stepping and more examples and control Lyapunov functions. I think there is still I am sure a little bit of uncertainty about how to do these things. Coming back to what we were doing last time, we were doing back stepping yeah.

The idea is that it is a very very nice way of designing CLFs stage wise right. So, if you start with a scalar system right and you basically well not a scalar system, but whatever you know I would say first order system yeah. And then you know that you have a CLF and a stabilizing controller here. Then if you add an integrator to this sort of a system of the same dimension, then you can extend the existing control Lyapunov function to a control Lyapunov function of the new system ok.

And that is what we did. We realized that if you actually you know make sure that the state is exactly equal to the desired feedback everything works nicely, but since we cannot do that we do next best thing which is basically try to drive this variable to zero yeah. I also emphasized and please keep this in mind that this is different from the tracking problem yeah because tracking here usually you have a function of time right. It is some trajectory that you have designed which is a function of time not a function of state. So, this is not the trajectory tracking problem or anything.

If you on top of this want to solve a trajectory tracking problem there will be some additional terms here ok. So, anyway we have not gone to the tracking problem yet. So, we basically essentially just proved that this new function is a nice CLF ok. It is a very nice in fact it is a quadratic right. These are the sort of Lyapunov functions we are very comfortable with right the quadratic Lyapunov functions right.

So, we essentially have added a quadratic term in this error ok alright great great ok. Now, we also looked at this example ok. Once we have done this proof all is good yeah. We are now we now also looked at an example which is actually a non-linear system yeah also does not look like a pure integrator right. The important thing to remember is this does not look like a pure integrator, but we were still able to use back stepping.

So, you so this adding just a pure integrator is not sacrosanct this is not like you know this has to be the case. You can it can be slightly different the key thing is you require that the control be able to cancel these terms out right. So, you want this system to somehow you know the control to be able to cancel these non-linearities out ok that is sort of the idea. So, what did we do? Now, one of the big concerns was how do you even start with a you know Lyapunov function or a CLF for the first you know first integrator. You take the simple case yeah we took half x squared and we saw that if you take k 0 is minus k x then you are good to go ok just stabilizing control.

Then we use this k 0 and the new xi which is omega in this case to construct a new variable which is the error between the state and the desired value of the state right and that is it. It gave us the control Lyapunov function ok and once we had that we just took kept taking derivatives and the control showed up right and once the control shows up here we already know that this v is a CLF we do not have to verify it again for every particular case we know that this is going to be a CLF right. So, either at this stage you can use the universal controller which like I said is a more complicated formula. So, which is why you know you might want to do something different so we do like a Lyapunov reshaping type of a thing ok. So, you essentially choose u so that this whole thing becomes negative definite ok and we took a few guesses our guess and we basically cancelled these guys and then you introduced a good term right and we also saw what is the physical meaning of this I mean how do you know control practitioners denote these terms in the controller.

So, you have a it is actually a PD controller with a feed forward term ok. So, it is a PD plus feed forward controller very standard even in the SIS CON department you do some experiments with 2DOF and this and that I mean there is always a you the typical controller that you are experimenting with is a feed forward or a PD plus I controller. So, and you know that the purpose of the integral is to in the linear case sort of do the job of the feed forward terms ok. So, we discussed this great. Now, what I want to do is I want to go back a little bit and look at this.

So, this is from the control Lyapunov function lectures by the way. Let us look go back and look at this example again let us revisit this yeah. If you remember the purpose was to sort of find a CLF for this system very simple double integrator system right a lot of you know mechanical systems have this structure you can actually reduce them to this structure yeah. So, this is pretty relevant actually. So, if you look at this system and you we tried to construct a sort of control Lyapunov function initially if you remember.

So, I am just trying to remind you what we did the stuff in the blue was what we wrote initially and tried and wanted to check as a CLF and we took a derivative and we ended up with this much ok. Here the control vector field term that is the Bx was x2 and the drift term was the x1 x2 right. So, this is the LF 1 v and this is the LF 0 v ok. So, we so what do we want we want that whenever this guy is 0 we want this guy to be negative for all nonzero states right. Now, if this guy is 0 we know that x2 is 0 which means this guy also turned out to be 0.

So, this was not a good CLF right there was an issue and then it almost seemed like I arbitrarily gave a CLF I said this is a CLF ok. Of course, we verified it I did not tell you any motivation for how I came up with it ok. I just wrote this right and I said let us try this as a CLF we took the derivatives again yeah the Bx that is the control vector field terms came out to be this guy and the drift vector field terms came out to be this guy. Now, if this was 0 you wanted it meant that x2 is minus x1 right. So, x1 is minus x2 whichever way you want to write it and then the first term Ax became minus x2 squared right which is essentially negative right whenever the state whenever you have nonzero states ok.

So, this was a valid CLF ok. Of course, I gave you another sort of trick also that whenever your Lyapunov function does not turn out to be a CLF you can always try adding mix terms yeah which also worked out this was also a valid CLF ok no problem ok. But I am not I do not want to focus on this guy I want to focus on this guy does this remind you of something now what if you look at this and you sort of look at this and you look at this yeah yeah this is how did I get this I because I know back stepping yeah because I knew back stepping beforehand yeah. So, I had some more additional information over you which is why I know that this will work yeah. So, if I if you simply go and go back and take this example in fact yeah very simple and now I say that I want to construct a CLF for this system what do I do as usual I focus on this system first right.

So, what is this I will say that my x2 desired or what you have been using as k0 x is equal to what is the desired x2 I would like minus k x this is no x k x1 minus k x1 let me say I just take minus x1 just to make my life easier I just make I keep k to be 1 no problem technically you can choose any k right, but I say I want k to be 1 ok. So, then what is my error term. So, what is my and what is the v0 what would be the v0 just half x1 square right this whatever I mean we have been doing nothing too fancy because I know that if I take v0 dot it is x1 x1 dot and if I take and x1 x1 dot is x1 x2 right. So, this gives me v0 dot is x1 x2 and if I did substitute for x2 as minus x1 I get minus x1 square good to go no problem ok, but of course I cannot make x2 to be exactly minus x1. So, I use the back stepping idea I create an error.

So, what is my v for the entire system what would be my v for the entire system now absolutely yeah and I know because I already proved I do not have to do any further work that this is a CLF. So, this is a valid CLF I already know that this is a valid control so I do not have to do any additional work and what is this what is this is exactly that guy right we just we just saw this right here yeah half x1 square plus half x1 plus x2 square. So, the motivation for writing this was exactly back stepping because I know that this will work for this particular system. So, it is actually rather powerful you can do this kind of you can play these kind of games for a lot of systems I am not going to go into any further examples right now anyway you will see a few more in the tutorial hopefully which is planned for weekend, but what I will do is I will go to the next design methods and there whatever examples we find we try to solve a few examples we will try to do the same with back stepping also. So, the next sort of module is passivity based design.

So, we want to use passivity for control design. So, whatever examples we find here what I will do is we will also try to do the same with the back stepping idea and see how things are different. So, that way you have a comparison point because the ideas are sort of connected there is an integrator idea here also. So, a lot of these ideas came about because of aero mechanical systems to be honest. Then later on although again most new traditionally especially in India you will find control engineers in electrical engineering and maybe chemical engineering process control and so on.

More recently in aero mechanical engineering aero mechanical programs you find controls folks it is not it was not there that far back of course, you had in aerospace always guidance navigation and control, but you know when I was doing my undergrad in mechanical control was like a you know almost negligible honestly speaking in mechanical engineering. At least there were no you know researchers in the area maybe there was of course, a course the standard frequency domain course, but what I am saying is a lot of these methods which are by now classical have come about from mechanical system ideas. The motivation is aero mechanical systems later on they have tried to see if these conditions are also satisfied by electrical biological systems and then these methods have been applied there also. But so, it is very interesting that somehow we you know aero mechanical folks have lost contact with controls for a while, but anyway it is back so we are fine I think. All right so, until now you have seen two methods of design one is CLF I say this as a separate method because you have already seen that once you have a CLF you can do that is take derivatives try to you will get a control term and you try to choose the control so that you get a v dot negative definite.

So, this is a pretty good method in itself if you can already guess a CLF. Now, then you have a back stepping method which is actually an idea of a CLF, but you are just extending it to you know higher order systems. Again I did not mention this, but you can imagine that it is not difficult and again the ref is k k k book yeah you know which one right it is the kristic karnalokopoulos kokotoevich book on adaptive control yeah not easy I practiced several years. So, the k k k book is a reference you can go look at it yeah this method can easily be extended for let us see systems of this kind I am going to say f x 1 plus g x 2 x 2 dot is f 1 sorry g 1 f 2 x 2 plus g 2 x 3 x 3 dot is f 3 x 1 x 2 x 3 plus or if you want to make it you know I am sorry if you want to make it simpler I mean this will also work, but yeah it is actually it will not it will work for several stages right I did this two stage thing, but you can see that again I could have added a third stage and added another term to the error yeah there will be a third term with x 3 plus something yeah I can go on doing this forever yeah it will look very very complicated of course, I am not saying it is going to look simple, but in reality again in the typical aeromechanical system context we are working with what at most sixth order system yeah it is not that far that difficult yeah you can actually do this by hand, but these kind of systems are called I mean these are triangular form yeah or I mean I think there is also strict feedback form these are called triangular form or strict feedback form systems and so on why because you can see what is happening right these drifts are depending only on the previous states right and the sort of the additional terms are depending on the next state yeah this is like this is controlled by this guy this is controlled by this guy this is controlled by this guy and up and up so backwards you keep designing right yeah in reality it does not look like this is controlling because these are states, but that is how we have done back stepping right we have created a desired x 2 then created an error desired x 3 create error desired x 4 create an error and you can do this yeah so kkk book actually has a you know proper structure on how the design will look very messy looking, but it will work yeah great. So passivity based control alright so we have already seen that if you have a chain of integrators now I am going to say if you have a chain of integrators you can do back stepping back stepping gives you a way of constructing a clf for a chain of integrators very powerful once you have a clf you can do so many things yeah great.

Now we are going to look at slightly nicer systems this is somehow systems having some intrinsic good property until now we have not assumed anything, but the strict feedback structure and all that yeah which is not so difficult not such a very stringent assumption honestly most systems have this kind of a structure it is not that difficult yeah because if they do not then life is really really hard for you yeah. Now typically whatever systems you can think of and realistically you will find that this sort of a you know strict feedback form is there in you know maybe it is not a linear strict feedback form there may be some you know typically what you will see is there will be some thing pre multiplying these right there may be some non-linear pre multiplication to this and all that yeah that would be the complication but otherwise you will have some strict feedback form which is still doable workable but for passivity we need a little bit more assumption on the system intrinsic property itself. So what is it we are going to now define passivity first great so consider this input output dynamics so now we have an input output system ok. So not just a states and control but also an output yeah because passivity requires there to be an output so x dot is f x u again we are not assuming you know explicit dependence on time things become way more complicated so this is just x dot is f x u and there is a y which is equal to h of x yeah notice that we are assuming that the output and input are the same dimension this is also a requirement otherwise it is difficult there may be more generic versions but this is the more established version yeah where the input and output are the same dimension typically the dimension less than the number of states right typically your number of actuators will be less than the number of states yeah it will be unusual if you have more then they are over actuated systems ok. Of course standard assumption is that f is locally Lipschitz and h is continuous yeah standard assumptions ok.

Now this system is called passive if there exists a C 1 storage function v of x which is positive semi definite such that if you take v dot which is as always defined as partial of v with respect to x f of x then this has to be less than equal to u transpose y or the inner product of u and y ok. It is a weird looking definition yeah it is sort of weird looking definition so I hope you sort of appreciate that well we will try to see what is what may be a physical sort of interpretation for it ok but what you are saying is that you have a storage function which is like a Lyapunov like function right because we are not saying that v is positive definite we only want it to be semi definite right so it is not a Lyapunov candidate but it is a Lyapunov like function and it is C 1 function of course right. So what we are saying is that if you take the derivative of the v along the system trajectories yeah then it is upper bounded by the inner product of the input and output notice that the input is appearing in both places ok. So this is as you can see a very intrinsic system property this is not has nothing to do with how you choose control or strict feedback form or anything like that ok but a lot of mechanical systems possess this property which is the cool thing when we will look at it let us not worry about that ok great. Just passivity itself is not enough for us to give stable controllers we also need another property which is called the zero state observability ok.

This is very much like the observability that you know from linear systems the definition itself but just generation to non-linear systems is not a big deal yeah. What is it? The system is called zero state observable if no solution of x dot is equal to f x 0 can stay in the set h x equal to 0 other than the trivial solution ok. As of now I am just reading this ok. What did I say? I am saying that if you make the control to be zero ok if you do not apply any control forget the control because typically in observability even in linear systems control plays no role yeah. If the system x dot equal to a x and y equal to c x is observable then x dot equal to a x plus b u and y equal to c x is also observable ok.

I hope you know this anyway because your controllability matrix is what c a c a squared c it does not have b anywhere right b is irrelevant here ok. So, same in the non-linear case also right you make you remove the control, control is playing no role ok. So, what is the point that we are trying to make? We are trying to and what does it what do you how do you define observability in a linear system anybody linear system observability definition not condition. Condition is this whatever the the observability matrix condition, but what is the definition? Yeah what is the definition? It is the same word that I just said. No that is again one very very special situation.

If you if your all states are measured then obviously system is observable typically your observations or measurements are less than the states number of states right pretty obvious right I can give take the simplest of example you can take whatever you can take a drone right states are position velocity angular position angular velocity what is your observation? You just have the three positions or three velocities for a gyroscope yeah or three velocities three linear velocities and three angular velocities you do not have position measurements typically or good position measurements. So, measurements less than number of observations. So, basically the way all these definitions are stated yeah observability control observability stated in a sense it says that you can reconstruct the state from the observations this is the thing can you reconstruct the state from the observations or not that the whole idea. So, basically how do you then you try to formalize it in you know many different ways yeah but the basic idea is reconstruct state from observation and what do you mean by reconstructing states in for most systems governed by ODEs all you need is the initial condition right once I give you initial condition entire state is reconstructed again we are talking theoretical if there is noise and all obviously there is filtering and all that is a different matter but we are not talking about the practical case we are talking about the theory if it works in theory the practical case will also work with some perturbation some oscillations yeah but the point is you just have to reconstruct the initial condition ok. So, given a set of observations can you reconstruct the initial conditions that is the question that you ask ok this is also very similar yeah here you say that if you look at the set H equal to 0 all the states where H x equal to 0 ok we are saying no solution of x dot equal to f x 0 will stay in this set except the equilibrium except the 0 state or except the 0 trajectory ok. Thank you.