Nonlinear Control Design

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Week – 1

Lecture – 1

So, why do we do non-linear dynamic systems and control is because very very important most systems that we see around us are non-linear and if you are doing, if you want to study stability, if you want to understand the behavior evolution asymptotically or the transient behavior, you need to understand how to analyze these systems. So that's sort of the first relatively big half. We learn techniques. So everything we do in this course is deterministic. We are not doing anything stochastic here. So it's a deterministic course and we of course want to introduce a little bit of design.

So we, especially in the second half of the semester, you will see more and more of doing design, methods of doing design and then examples of doing design. So that by the end of this course, if you have some example system, for example if you have a robotic system, if you have an electrical system, anything and you have a fairly good fidelity model for this, then you can actually use some of the methods that you have learnt here in order to do a control design. Mostly of course this is a very fundamental course for control theory practitioners, control theorists and control theory practitioners, industry folks. So of course a lot of SYSCON students show up here because it's a core course.

But it is also of lot of interest for anybody who is working in the general area of dynamical systems. So I get a lot of students from aerospace, mechanical. I myself from an aerospace background. So a lot of what I do is apply these theories to aerospace systems, things like satellites, attitude control, spacecraft on orbits. My own background is that I did my aerospace engineering PhD from the University of Texas at Austin.

So it's one of the nicer programs in aerospace in the US I would say and Texas was also a very good place to be. And we had a lot of exposure to not just doing a lot of theory but we also got to interact with folks from NASA, Air Force Research Labs who were sort of coming in with their own projects and their own research which was sort of very real problems. So it would typically work like it works here. The faculty has a project and you typically get put on the project as a PhD student and then you contribute bits and pieces to it. Of course you are not just running the whole thing but we did a little bit of satellite control design for a few of these agencies.

So lot of applications in recent times of course I have seen applications in I have seen people work on applications in smart grid control. I mean in Germany Siemens actively sponsors these projects with the smart grid control because in Europe a lot of this I believe this energy generation locally is very highly encouraged. So a lot of local folks do energy generation and then they supply to the grid but then the problem with these is that everybody is generating at their own pace, at their own frequency. So the parameters and everything is off. So you can't just join things to a grid and expect a uniform frequency.

So here you have in applications for non-linear systems in terms of synchronization. So it is like a multi system, multi agent cooperative system sort of a problem where you are talking about how to synchronize the frequencies, how to synchronize some of the power outputs. So that when you join one thing into the grid, the grid doesn't start to fail. So lot of applications. Now of course there are folks who are doing biological systems, biological kind of applications, reaction rate during this COVID pandemic there were a lot of models.

Again I am not sure how successful those were. But there are applications now, more modern applications to reaction rate models which is sort of the infectious disease spreading models. Then there is, please come forward. Then there is lot of applications to more recently social systems. For example if you are talking about advertising and marketing, there is a lot of applications there where a particular marketer wants to know which agents to influence.

For example, there may be, I mean a lot of you are on social media platforms like Instagram. So you see that there are a few people who have millions of followers. So these folks are influencers. So if they say that buy this toothpaste, your teeth will be perfect forever, then people believe them. So typically these advertising agencies and these marketing managers, they now want model based decisions on who to target, which particular influencer to target or which particular social agent to target so that their campaign goes forward much more efficiently.

So here also there is applications of nonlinear dynamical systems. Nonlinear dynamical systems to look at the evolution. Once I introduce an opinion in one corner of the graph of a very very huge network of social graph, there is what is the outcome. Asymptotically after a long time, you want to see how many new people got influenced to buy this toothpaste and the other end of the spectrum is how to influence, where to introduce the new opinion. So things like these.

So let's see. This syllabus is from as you can see 2021. I will fix it and then upload it to the Moodle platform. Don't worry about it. So the structure is pretty standard what we have here. It's just lecture and credit.

We don't have tutorial and practice included here. Anyway, we may have extra sessions with the TAs depending on how we think we are progressing. If you want to learn more, if you want to sort of, you know, you have trouble understanding some particular bit of material, then we have some sessions, but they are not officially included here. This is also wrong. Like I said 2021, I'm going to fix all this.

You know the sections are Wednesday, Friday 11 to 1230 right here. So this is what is going to be. Office hours, I'll actually put a formal set of office hours. Not by appointment. So this is also something I will sort of introduce.

There'll be a formal office hours during the week. And so that if any of you have doubts, you can just walk in during that time. I will be available during that time. So you can walk in and ask your doubts. No need for an appointment.

If you want, of course, separately to talk to me, then of course you will need an appointment. TAs also, I will make this change soon. We'll just have Maitreyi and Pallavi as TAs. This is one of the key things I sort of expect. Although I know we have newly joined students who probably don't exactly have this background.

But we sort of stretched this. Okay. So the general expectation is that there is a graduate level competence in ordinary differential equations. So you must have seen some kind of a state space ODE course.

Okay. So that even the models shouldn't look like unfamiliar and foreign to you. Because that's what we work with. We start with models. So if you're unsure of our academic preparation, please talk to me after the class.

This is the first class. So usually a lot of people show up anyway. So please talk to me after the class if you have. So I expect some basic MATLAB or Python or some alternate programming experience. Because see, I do some bit of design aspect also in the second half. So there I can assure you if you, unless you do some bit of hands-on coding and things like that at least, you don't get a good feel for whether what you did worked well or not.

Okay. So I expect some basic MATLAB or some programming experience. Again, if you're unsure, please talk to me. And if a lot of you are not on the same page in terms of programming, then of course we may have to chuck it. But I prefer not to. You will also enjoy a little bit of a programming experience.

All right. You see some real systems and then working under the controllers you designed. And then you learn also things like, things that we don't talk about is like how to tune the gains in nonlinear control. I mean, it's still a little bit at home.

All right. Okay, great. The topics. This is the very, very broad overview of the topics. We may choose not to do some of them depending on how much time we have. Yeah. But this is a generally broad overview.

More or less you can assume that we will cover at least 90% or more of this material. Okay. So it's not going to change significantly. Like I said, this is a very fundamental course. So unfortunately I don't have a lot of freedom in, you know, talking about a lot of new areas.

Okay. So if we do have time, of course we will. But usually it will be a fundamental course. So the methods that we talk about are by now classical. Okay. Not classical in the sense of 1800s, but classical in the sense that everybody knows this in the community.

Yeah. We just learn it better here. Okay. So first we start with some nonlinear systems, introduction, examples, some preliminaries. Yeah. Which all of you will require, which will sort of set up notation.

Okay. For how things will look, how the mathematical notation will look. All right. Then we will have, we start immediately with the Lyapunov stability. Okay. So this is the fundamental stability notion for nonlinear systems, like for linear system.

Well, again, these notions are also pretty much valid for linear systems. Okay. So there's no real difference as such. The only thing is that linear systems you are used to working with input output stability.

Yeah. So you hardly talk about, there is notions of internal stability. Right. And there you pretty much characterize it with, what do you characterize it with by the way? How do you characterize internal stability for linear systems? Eigenvalues. Yeah. Just write the matrix X dot is AX and just check the eigenvalues of the system.

If they are on the negative left half plane or whatever, I mean, if they are on the, the real parts are negative, we are good to do. Okay. So, so this is how you evaluate.

But that's an evaluation method. Yeah. That's not a definition. All right. It's an evaluation method and not a definition. So let's be very clear.

Definitions still remain the same. Okay. So you, you see a lot more evaluation methods. When you talk about linear systems, you hardly look at the definition. Most of you, when you, if you did linear systems, you would not have seen definitions. You would have seen tests like, you know, check if the poles are on the left half plane and things like that. You would not have talked about what exactly is stability.

Okay. So Lyapunov stability definitions are the universal standard for what is stability. Okay. How do we characterize what exactly is the notion that we are talking about? So things immediately get mathematical here.

Yeah. So welcome to the course very quickly. All right. Then of course, we talk about Lyapunov theorems. Yeah. These are the tests for nonlinear systems.

Okay. These are the tests for nonlinear systems. Okay. In nonlinear systems, notions of eigenvalues and all that, no questions about it because you cannot write it as a matrix, a constant matrix. Okay. And so if you even, even if you have a linear time varying system, I hope all of you understand that just checking the eigenvalues of a linear time varying system does not tell you anything about stability. Yeah. Even if for all time, the time varying matrix that you have has negative eigenvalues, negative real eigenvalues, it does not guarantee stability.

Okay. This is a well-known fact. Yeah. So for time varying linear systems also, these eigenvalues test do not work, you know, the way you expect them to. Okay. So there is no question of them working for nonlinear systems.

Yeah. I mean, obviously not. Of course, there is possibility of doing linearizations and things like that. We do not talk about linearizations. Okay. We directly test stability of nonlinear systems via Lyapunov theorems.

Okay. That is the idea. All right. Then we talk about invariance theorems, which are, give a little bit more flexibility in terms of sort of an extension for our Lyapunov theorems. Then we will talk about input-output stability. This is sort of the linear system equivalent.

Yeah. Also has its advantages in nonlinear systems. In modern nonlinear systems, there is notion of notions of input to state stability. Yeah. So ISS results and input-output stability, these are important when you are talking about disturbances.

Okay. So any real system is affected by disturbances. Yeah. Whatever I do in theory, you cannot expect that, you know, the real system is going to have a very similar sort of an outcome. You know, you may apply the same control. Yeah. You design a control for a quadcopter and you put it on a quadcopter.

It will not behave how you think it is expected. Yeah. These are all effects of disturbances. Okay. Of course, there is actuator saturation and so many other things, but all of these can be clubbed as disturbances if you want.

Okay. So input-output stability is an important notion. It sort of connects linear system notions. Yeah. Then the design part.

Yeah. Which I am hoping a lot of you will be excited about. Yeah. So the first is Lyapunov redesign, where we use the notions of Lyapunov functions to design controllers. We learn how to do that. Okay. Then we have one of the most, most powerful techniques of designing Lyapunov functions. So once you design a Lyapunov function, of course you can use Lyapunov redesign to get a controller.

Right. So this is called backstepping design. So it is like a, it is a step-by-step way of coming up with a Lyapunov function. So it is a rather powerful method. I talk about it even in my adaptive control course, because backstepping is the sort of central idea for designing controllers there. Then we have feedback linearization, which is one of the oldest ways of designing non-linear controllers. And then finally we have passivity and energy shaping, which I have clearly said is subject to time availability.

Okay. So again, depending on how I feel, I may add some more design elements. Yeah. The, the fundamental analysis elements are standard. There is no change. All of you have to learn this. If you have to even talk about design and talk about non-linear systems, stability and so on.

Yeah. So this, there will be hardly any change in this first section. But in the design part, we can choose to do a little bit more here and there, depending on what our interests are and what is the time that we have. Okay. So these are the references, very standard. Okay. Khalil's book by non-linear systems for the invariance design, very, very good book, very good reference for that.

Vidya Sagar's book is one of the most mathematically precise non-linear systems analysis book. Yeah. Then all the geometric notions, feedback linearization, all of these are best explained in Alberto Isidori's book.

Yeah. Very good book again. And then you have this Krstic, the KKK book. So it is the Krstic, Kanellakopoulos and Kokotovich book for non-linear adaptive control. Since it is so difficult to say, I just call it the KKK book. So usually I will tell you which book I am somehow referring material from. But of course, as you will see, I have handwritten notes and things like that.

So, maybe in the future it will become printed notes. But right now it is handwritten notes. So mostly we will have this sort of, these are the sort of key references here. Okay. Very good books, all of them, very good books. I mean, over time, if you want to be in this area of non-linear control, I think you should own all these books.

Yeah. Over time. I am not saying you have to buy it now. I am saying these are really really good books to have as references for life. Yeah. Good. Thank you.