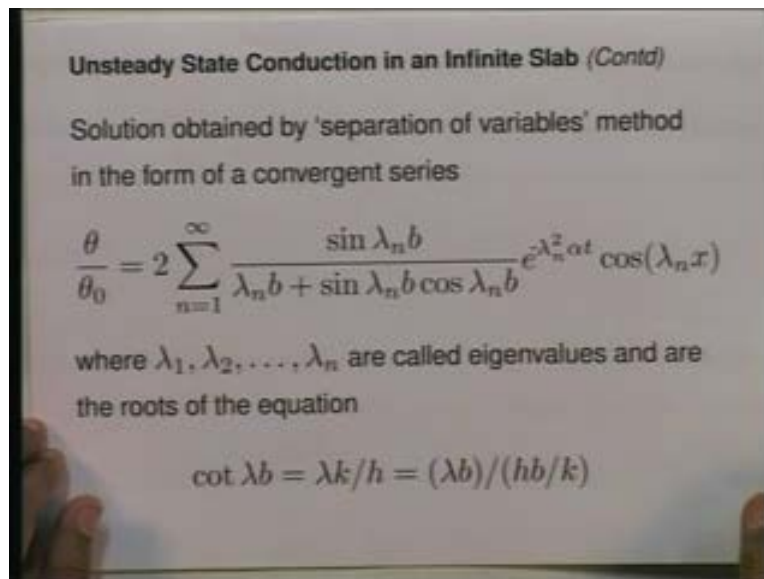


Heat and Mass Transfer
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Lecture No. 08
Heat Conduction-5

Last time when we stopped we were looking at the one dimensional unsteady state problem in an infinite slab. The situation was as follows - you have an infinite slab of width $2b$ initially at a temperature t_0 ; it is suddenly immersed in surroundings at a temperature t_f , t_0 may be greater than t_f or less than t_f . t_0 is greater than t_f it, slab will cool; if t_0 is less than t_f , the slab will heat up. There is heat transfer coefficient h on the 2 faces of the slab, systematical situation about the center line of the slab. So we were looking at that problem which is one dimensional problem in x and in time, unsteady states problem in x and time. We put down the differential equation, we put down the boundary conditions in the 2 faces x equal to b , x equal to minus b and we stated the initial condition and then I did not solve the solution the differential equation; I said the solution can be obtained by a method called the separation variables method and the solution.

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Unsteady State Conduction in an Infinite Slab (Contd)

Solution obtained by 'separation of variables' method
in the form of a convergent series

$$\frac{\theta}{\theta_0} = 2 \sum_{n=1}^{\infty} \frac{\sin \lambda_n b}{\lambda_n b + \sin \lambda_n b \cos \lambda_n b} e^{-\lambda_n^2 \alpha t} \cos(\lambda_n x)$$

where $\lambda_1, \lambda_2, \dots, \lambda_n$ are called eigenvalues and are
the roots of the equation

$$\cot \lambda b = \lambda k / h = (\lambda b) / (hb/k)$$

After doing this separation variables method the solution is obtained in the form of convergent series which I showed you. This is the solution that I showed you last time where $\lambda_1, \lambda_2, \lambda_n$ are called Eigen values of the problem. And I told you that they are the roots of the equation $-\cot \lambda b = \lambda k/h$ which is equal to λb divided by hb/k and then graphically I showed you how the values of $\lambda_1, \lambda_2, \lambda_3, \dots$ can be obtained graphically by intersecting the cotangent graph with the straight line graph obtained for the right hand side.

The cotangent graph is on the left hand side of the straight line, λb divided by hb/k is on the right hand side; the intersection of the 2 lines of this line with the cotangent curves gives me the infinite solution $\lambda_1, \lambda_2, \dots$. Use those solutions in the convergent series and we get the temperature distribution θ by θ_{naught} ; this is where we stopped last time. Now the solution which I showed you can also be expressed in dimensionless form as shown.

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Unsteady State Conduction in an Infinite Slab (Contd)
Solution in terms of dimensionless parameters

$$\frac{\theta}{\theta_0} = 2 \sum_{n=1}^{\infty} \frac{\sin \lambda_n b}{\lambda_n b + \sin \lambda_n b \cos \lambda_n b} e^{-(\lambda_n b)^2 (\alpha t / b^2)} \cos(\lambda_n b \frac{x}{b})$$

Thus,

$$\frac{\theta}{\theta_0} = f\left(\lambda_n b, \frac{\alpha t}{b^2}, \frac{x}{b}\right)$$

$$= f\left(\frac{hb}{k}, \frac{\alpha t}{b^2}, \frac{x}{b}\right)$$

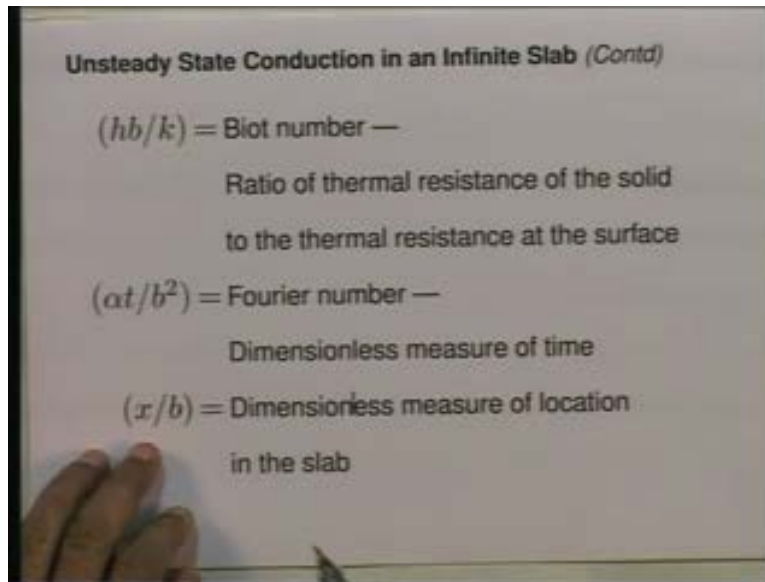
This is the solution: the same solution in dimensionless form θ by θ_{naught} . The left hand side is the same, is equal to 2 times summation n equal to 1 by infinity sine

λb divided by λb plus $\sin \lambda b \cos \lambda b$. That is the same as earlier multiplied by e to the power of minus λb squared multiplied by αt by b squared. You notice I have brought in the 2 dimensionless numbers in the explanation of e and then I have written the cosine as cosine of λb into x by b .

So, what we now have is a situation in which the dimensionless temperature on the left hand side - θ by θ_0 - is a function of the λb s. λb is rather, so function of λb $\lambda_1 b$, $\lambda_2 b$, etcetera. It is function of this dimensionless number αt by b squared which appears in the explanation of e and it is the function of x by b which is the location of the point where we want the temperature inside the slab. x by b tells me a location, x by b can vary from 0 to 1; when x by b is 0 we are talking the center line of the slab, when x by b is equal to 1 we are talking of the face of the slab. So this method of a representation shows us that θ by θ_0 is function of λb s, λb s, αt by b squared and x by b .

Now, from the method that I showed you for obtaining the λb s, you know very well that the λb s depend on hb by k . It is nothing but intersection of the cotangent curves with the straight line that I get on the right hand side of that equation for solving for the λb s. So the λb s really depend on hb by k and therefore I can say θ by θ_0 is also a function of hb by k αt by b squared and x by b . The dimensionless or inwards the dimensionless temperature on the left hand side, the dimensionless temperature of any point in the slab θ by θ_0 which can vary from 0 to 1 is a function of 3 dimensionless numbers hb by k αt by b squared and x by b . Now let us look at the significance of these 3 numbers. Just let me talk a little about the significance of these numbers.

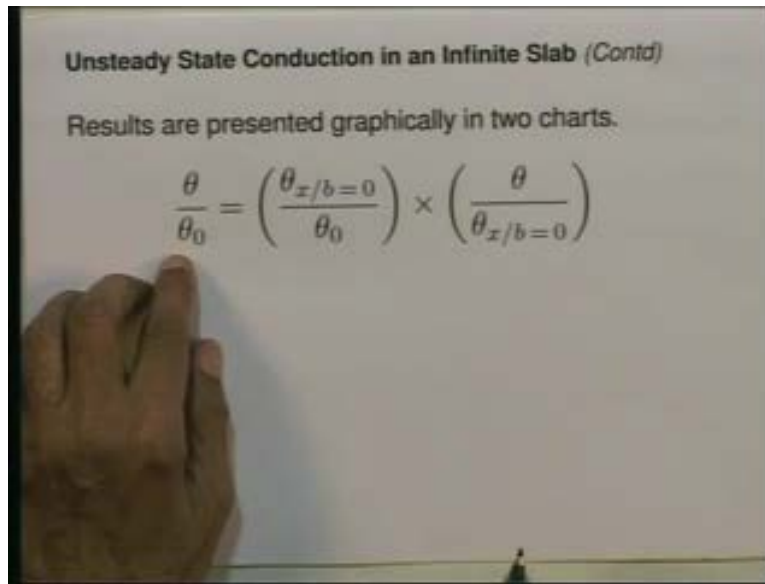
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The first number hb by k is called the Biot number; it is nothing but the ratio of the thermal resistance of the solid to the thermal resistance at the surface it; that is all that it is. That means if the Biot number is less, the thermal resistance inside the solid is small compared to the thermal resistance at the surface. Biot number tending to 0 is situation of negligible internal temperature gradients. So as Biot number increases the thermal resistance inside the solid becomes important and we cannot neglect internal temperature gradients. That is all Biot number is – it is ratio of 2 thermal resistances; the ratio of the thermal resistance of the solid to the thermal resistance at the surface, the 1 upon h at the surface.

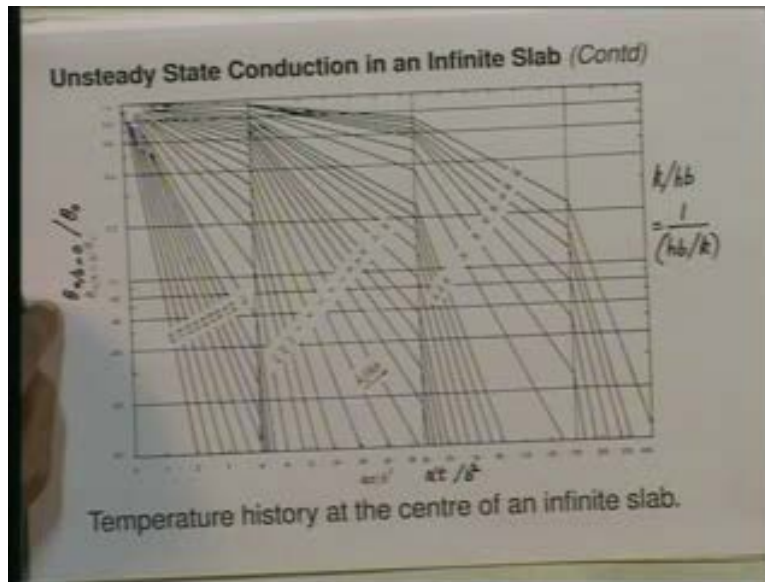
The second dimensionless number αt by b squared is called the Fourier number. It is a dimensionless measure of time because time is appearing in the numerator so the larger the time the larger the Fourier number. And the third number which I said a moment ago is the dimensionless measure of location in the slab; x by b can vary from 0 to 1, x by b equal to 0 is at the center line, x by b equal to 1 is at the surface of the slab. So effectively, once I specify these 3 numbers for a particular situation - particular slab in a particular situation - I can get a value of θ by θ_0 from the solution which I have shown you just a moment ago.

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Now this numerical, this solution has been obtained numerically and is often presented in graphical form in the, in 2 charts as follows. The theta by theta 0 which is the left hand side of the solution it split up into 2 quantities, theta at x by b equal to 0 upon theta 0 into theta divided by theta at x by b equal to 0. theta at x by b equal to 0 is the theta on the center line and theta 0 is the initial difference of the temperature t_0 minus t_f . theta is the temperature at any point in the slab at any time t_f . So we are splitting of this theta by theta 0 into these 2 quantities the product of which gives me theta by theta 0 and the graphical representation is as follows, the graphical representation is as follows. Now let me show it to you.

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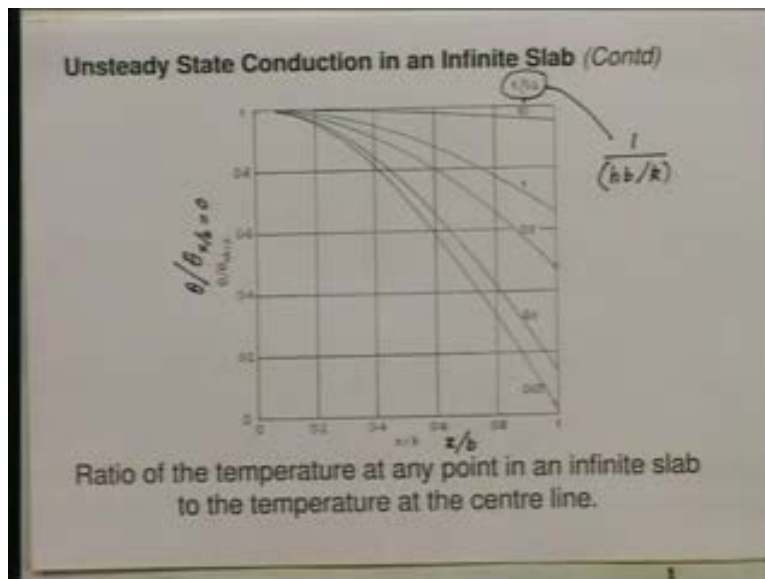
On the x axis I have αt by b squared; I have already told you the dimensionless temperature is a function of 3 parameters, what is it? The Fourier number which I am plotting on x axis αt by b squared; let me write it in bigger letters here αt by b squared. Then the location in the slab x by b is parameter against which we also measure. In this case we want the temperature only at the center so it is θ . Let me write this again in big letters. θ at x by b equal to 0 divided by θ_0 θ naught and the parameter varying from graph to graph which is drawn here is the inverse of the Biot number but we have here is k by hb . k by hb is nothing but 1 upon hb by k which is the inverse of the Biot number. So the inverse of the Biot number is the parameter varying from graph from each line which is drawn on this chart.

So, now suppose I want the temperature at the center line of the slab at any given time. Suppose I have a slab; I want the temperature at the center of the slab at any given time. I know I have the data of the slab - its dimensions, the values of α that is thermal diffusivity of the slab, etcetera. What will I do if I want the temperature at any given time? I will calculate the Fourier number. I know the, I am given the heat transfer coefficient at the surface; I know half the width as a slab, I know the thermal conductivity of slab. Calculate hb by k and therefore 1 upon hb by k .

So given for the given time, calculate αt by b squared. You got your value of αt by b squared, go up vertically to the value of 1 upon the Biot number - the inverse of the Biot number - then go horizontally and you will get the value of θ at x by b equal to 0 upon θ naught. From this you can calculate the value of θ at x by b equal to naught and therefore t at x by b equal to naught that is at the center line. This is how you would proceed to use this chart to get the temperature at the center line at any given time for a given slab with given characteristics of properties and width, etcetera.

The second chart which we use; this is the chart for giving the temperature in the center line. The second chart is for connecting with the temperature at any point in the slab. We don't want just the temperature at the center, we want the temperature at any point in the slab.

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So, the second chart gives me θ at θ divided by θ at x by b equal to naught. Let me again write that in big letters - θ divided by θ at x by b equal to naught; that is at the center line and this is on the y axis. And on the x axis, I have x by b again. Let me write it in big letters - x by b which is x by b equal to 0 is the center line, x by b equal to 1 is the face of the slab.

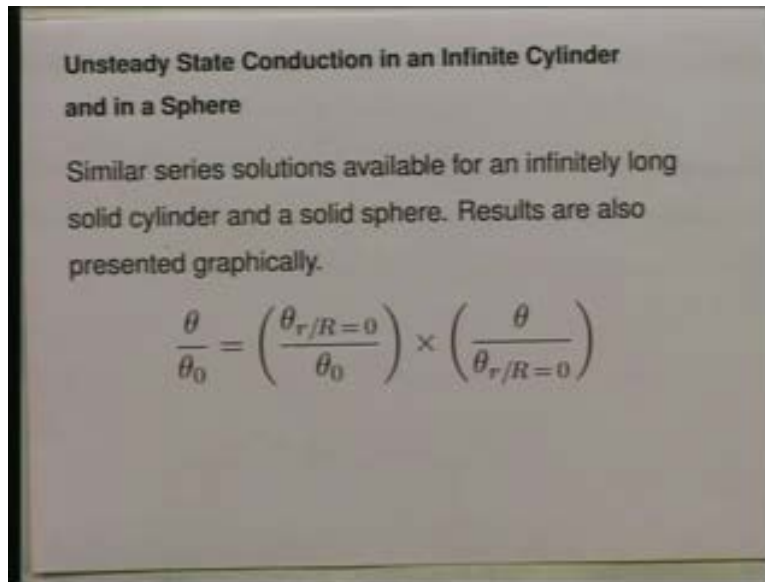
So suppose I want the ratio of the temperature at any point in the slab to the temperature at the center line. Then I will first get the value of x by b ; it will vary from 0 to 1 whatever it is. I will go up vertically, go to the value of 1 upon the Biot number; this particular quantity here is again the same thing that I have mentioned earlier. It is 1 upon hb by k 1 upon the inverse of the Biot number. So calculate this, so given a particular location at the slab, whatever its value, go up to the particular value of 1 upon hb by k , then go horizontally and you will get ratio θ divided by θ at x by b equal to 0.

Now from the previous chart, you have got θ at x by b equal to 0 so if you multiply the value of trend from the previous chart with the value of trend from this chart you will get the value of θ . So this is how we use the 2 charts - the first chart for getting the temperature at the center line, the second chart for getting the ratio of the temperature at any point in the slab to the temperature at the center line. We will illustrate these ideas by doing a numerical problem in a short while so that these ideas will get properly illustrated, I will do one numerical.

Now, let me just for a moment diversify and say a very similar problem can be to that of the infinite slab, can also be posed for a long solid cylinder and for a solid sphere. An identical problem can be solved for the long solid cylinder. We will say we have long solid cylinder of radius capital R ; it is initially at temperature t_0 ; it is suddenly immersed in surrounding which are at a temperature t_f and there is heat transfer coefficient h at the circular phase of the cylinder all along a circular phase of the cylinder. Find the temperature inside this cylinder at any point in the cylinder as a function of time and of course space that is function of R and t .

We can put down a differential equation using our general differential equation in cylindrical coordinates. We can put down the boundary condition; we can put down initial condition and solve it. The solution more or less proceeds along similar lines to what we had for the infinite slab; you get separation variables technique and solve it. And similarly for a sphere; so analytical problem to those for the infinite slab have also been solved, for the cylinder, the long solid cylinder and the sphere.

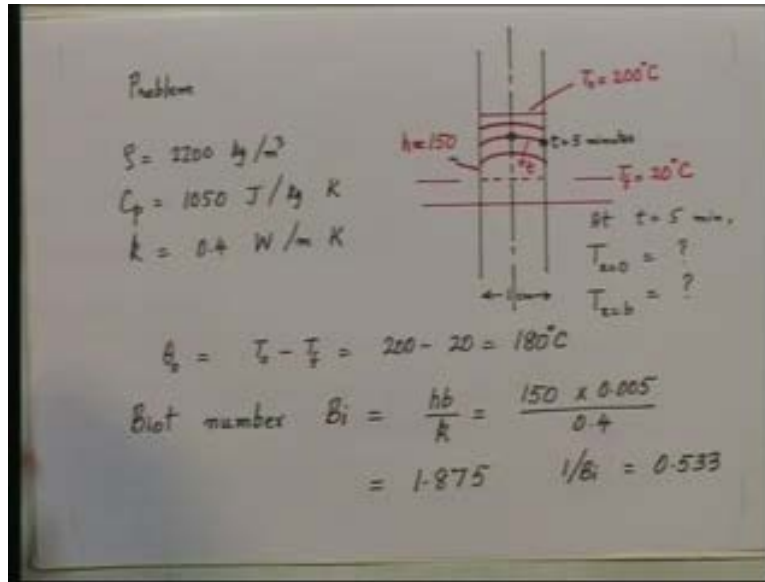
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And the results are presented graphically as θ by θ_0 equal to θ at r by R equal to 0 upon θ_0 . This is the temperature in the case of cylinder; this is the temperature on the axis of the cylinder into θ upon θ_0 at r by R equal to θ_0 . So you see for the slab where x by b equal to 0, now for the cylinder where r by R equal to θ_0 . If it is the sphere, this would be the center of this sphere r by R equal to θ_0 . So if the problem of the slab, the cylinder and this sphere are all similar. We have talked in detail about the problem of the slab but identical problems can be set up for the cylinder - long solid cylinder - and the sphere. They have been solved; series solutions are available and like this slab, we have the solution available in the form of 2 charts for the cylinder and 2 charts for the sphere.

The only difference is instead of b being the characteristic dimension in the Fourier number or in the Biot number, we now have capital R which is the radius of the cylinder or the radius of this sphere as the characteristic dimension. That is the only difference so whatever we say for the slab has also been done for the cylinder; its sphere solutions are available in the form of an infinite series and also in the form of charts. Now let us illustrate these ideas with the help of the numerical problem. We will do a numerical problem for an infinite slab; let me say I am going to do a numerical problem now.

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Let us say the problem is as follows - we have an infinite slab. Let me draw one; we have an infinite slab of width, we have an infinite slab of width 1 centimeter, whose width is 1 centimeter. It is initially at a temperature - let me draw the temperature distribution. Initially, it is initially at a temperature of 200 degrees centigrade so let us say this is the 0 line for temperature and initially it is at a temperature of 200 degrees centigrade, uniform temperature of 200 degrees centigrade. So t_0 equal to 200 degree centigrade; certainly this slab is put into surroundings which is, the surrounding temperature is 20; so let me draw 20 here. Let say this is t_f equal to 20; surroundings are at 20 degree centigrade.

So, the slab is going to cool and with time if it is immersed and we are going to get temperature distributions like this, etcetera. Eventually if you wait long enough at time t equal to infinity, the temperature distribution in the slab will be tending almost to t_f - the true value t_f that is 20 degrees centigrade. So with time, the temperature is going to, temperature distribution inside the slab is going to change like this.

Now, we would like to find the temperature in the slab after a time t equal to 5 minutes. Let us say this corresponds, this temperature distribution corresponds to t equal to 5

minutes. You are asked to find the temperature in the center of the slab after t equal to 5 minutes and you are asked to find the temperature at the face of the slab after t equal to 5 minutes. These are the 2 points which I have marked with the circle. You are asked to find t at x equal to 0 and after 5 minutes, at t equal to 5 minute; we want to know t at x is equal to 0 and we want to know t at x is equal to b - these are the 2 values we want.

We want to use the charts; we are given, the properties of course are needed so we are told that row, the properties are as follows. row is equal to 200 kilograms per meter cubed, C_p is equal to 1050 joules per kilogram Kelvin, specific heat and thermal conductivity k is equal to .4 watts per meter Kelvin; these are the values given, property values given.

You are also told that the heat transfer coefficient h at the surface is 150; the value of h is equal to 150 in the usual units - watts per meters squared Kelvin. Find the temperature at the 2 points at, that is at the center line, at the center plane and at the face after 5 minutes have elapsed; that is the problem. It is a straight forward problem in which we want to use the, make use of the charts nothing else. Now first of all, what is θ_0 ? θ_0 is equal to t_0 minus t_f ; it is equal to 200 minus 20 equal to 180 degree centigrade. Biot number Bi is equal hb by k which is equal to, h is 150 watts per meter square Kelvin. B is equal to half a centimeter; B is half the width of the slab, half centimeter so it will be point 0.05 meters divided by k which is .4 so the value of Bi comes out to be 1.875 and one by Bi , the inverse is what we need will come out be .533.

Now, let us go to the next part; we need the Fourier number.

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Fourier number $F_0 = \frac{\alpha t}{b^2} = \frac{0.4}{2200 \times 1050} \times \frac{300}{0.005^2}$
 $= 2.0779$

From the charts $\theta_{x/b=0} / \theta_0 = 0.12$
 $\theta / \theta_{x/b=0} = 0.48$

$\theta_{x/b=0} = 0.12 \times 180 = 21.6^\circ\text{C}$
 $T_{x/b=0} = 21.6 + 20 = 41.6^\circ\text{C} \leftarrow$
 $\theta_{x/b=1} = 0.48 \times 21.6 = 10.4^\circ\text{C}$
 $T_{x/b=1} = 10.4 + 20 = 30.4^\circ\text{C} \leftarrow$

The Fourier number will be given by, Fourier number will be equal to alpha t by b squared which is equal to, alpha is k by rho C_p so alpha is, k by rho is 2200 into C_p is 1050 so that's alpha multiplied by the time in seconds which will be 5 minutes to 300 seconds and b squared is point .005, the whole squared which comes out to be the Fourier number, comes out to be 2.0779.

Now you go to the charts. So we will go to the first chart which I showed you moments ago; let me just show it again. The first chart gives the temperature at the center line so let us look at that - our Fourier number is 2.7 which is somewhere here on the chart 2.07. 1 upon the Biot number is .5 so if you go up like this vertically and this is .5 here, if you go up vertically you will go somewhere here roughly. I can do it more exactly and one has to take account of the fact that this logarithmic scale on the y axis and the x axis and so on but if you can somewhere at this point which is about .12, that is what I am reading it.

So, theta at x by b equal to 0 upon theta naught from the first chart is .12; so let us write down that from the chart. From the charts, the first chart gives me theta at x by b equal to 0 divided by theta naught. Corresponding to the value of the Fourier number and the 1 upon Biot number, the first chart gives me about .12 all right. The second chart gives me

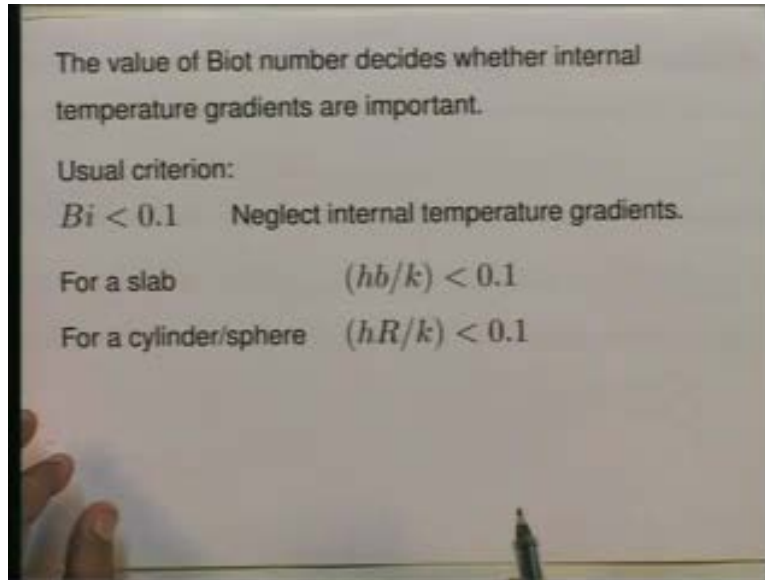
the ratio of the temperature at any point to the temperature at the center line - this is the second chart. Now x/b , we want the temperature at x/b equal to 1. So we are going to go on this line x/b equal to 1, go up till you get 1 upon hb/k equal to .533 which is approximately somewhere here; go horizontally and you will get θ/θ_0 , x/b equal to naught equal to .48 θ/θ_0 at x/b x/b equal to naught equal to .48. So these are the 2 values which we got for this problem from the charts.

Now, we want the temperature so let us get the temperature; from the first value we get θ/θ_0 at x/b equal to naught is equal to .12 into θ_0 which is 180 that gives us 21.6 degrees centigrade. Therefore t at x/b equal to naught equal to 21.6 plus 20 which is equal to 41.6 degrees centigrade. So this is the first values if we are looking for and the second one that we are looking for is θ/θ_0 at x/b is equal to 1 is equal to .48 multiplied by θ_0 at x/b equal to naught that is 21.6. So that comes out to be 10.4 therefore t at x/b equal to naught is equal to t at x/b equal to 1 not x/b equal to naught. t at x/b equal to 1 is equal to 10.4 plus 20 which is equal to 30.4 degrees centigrade so this is the second value that we get, the second number that we are looking for from the charts.

So, you see how to use the charts given for a given situation. Calculate the Fourier number, calculate 1 upon the Biot number, use the first chart to get the temperature at the center line, use the second chart to get the ratio of the temperature at any point to the temperature at the center line. Once you have done that, you got the temperature at any point inside this slab at any given specified time – that is what the charts are for. One could also use the charts in the reverse manner; one could ask the question how much time does it take for a particular point in the slab to reach a particular temperature – that is the reverse problem. In which case what we are saying is the value on the y axis is known; you need the value on the x axis that is you need to find out the Fourier number. Now try doing that on your own it is not very difficult. You will have to think a little, try doing the reverse problem. That is for a given slab, given properties, a given situation find the time taken for a particular point in this slab to reach a given temperature during the cooling process. I want you to do that type of problem on your own.

Now I talked about significance of the Biot number; let me again come back to it.

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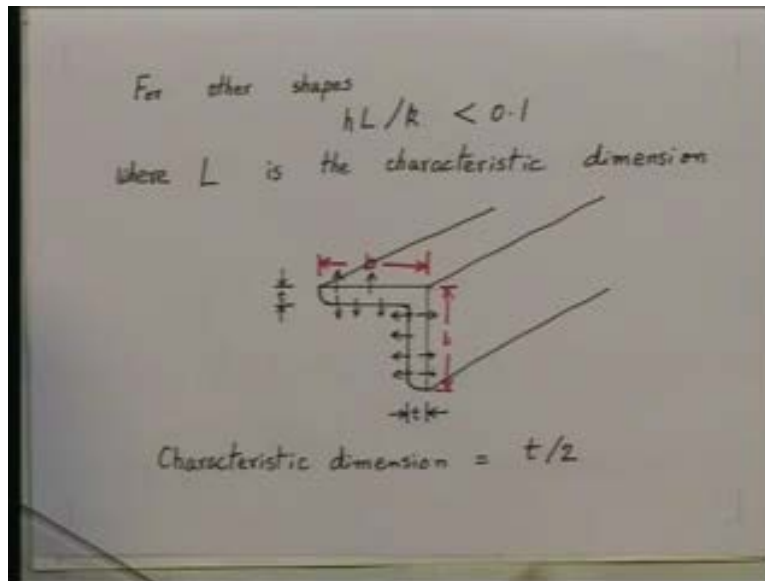


The value of the Biot number decides whether internal temperature gradients are important. I have said that to you earlier; as the Biot number tends to 0, internal temperature gradients are negligible but where as an engineer we must have some criteria with. There is no such thing as a 0 Biot number because that would require a situation which is absolutely ideal, a k equal infinity which doesn't exist. The usual criterion as engineers which we use is that if the Biot number is less than .1, we neglect internal temperature gradients. We find that then temperature gradients are small enough that we can neglect them. For a slab therefore, what we are saying is hb by k is less than .1. For a cylinder or a sphere what we are saying is hR by k is less than .1; that is how we are putting down the criterion.

.1 is somewhat arbitrary; it has been picked by people based on looking at the solutions in the charts that is from Biot number 0 to .1. If you consider the changes in temperature across the whole slab or across the whole cylinder they are small enough not to worry about for most situations. So this is a criterion that we use - we calculate the Biot number based on the characteristic dimension of that of the body and if that Biot number

is .1 or less we say we will neglect internal temperature gradients and use the simpler solution which I got for you earlier. If not, then we must obviously have more detailed solution of the partial differential equation. Now what about some other shapes? Suppose I take some shapes and ask what we will have, what you do with them for other shapes.

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For other shape means shapes other than infinite slab or an infinite cylinder or a sphere. For other shapes the criterion that we use is hL by k must be less than .1 where L is the characteristic dimension of the solid body, it is a characteristic dimension. L is measure of the distance - the maximum distance heat has to flow within the body. Why is it this? So therefore for a slab, it is half the weight because heat flows from the center line outward towards to 2 faces. For the cylinder why is it the radius? Because from the center axis, heat flowing outwards towards the faces or inwards from, if the bodies heating up inward from the circular face toward a center line.

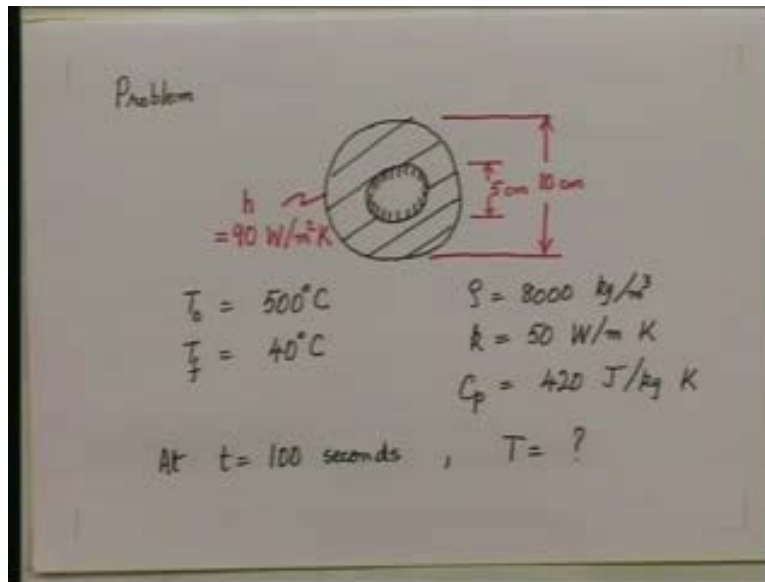
Now, I suppose I have some other shape, let us take an example. Let me take a shape which is a structural, a shape which we use so often in practice. I am going to draw here an angle, a shape which is used in so much structural work. Let us say I have an angle like this. Its width is, its thickness rather is t in this direction; this direction also it's t like

this and it's a long angle like this and in this direction the width is b . Let us say like this. It is b like this direction, b in this direction then angle b and b are the legs and t is the thickness of the angle, it is a long angle.

What will I take as characteristic dimension if I have, if this is going through some transient heat conduction process and I want to know whether to neglect internal temperature gradients or not neglect internal temperature gradients? Well, ask yourself now how will heat flow if this angle is going through a cooling process. If you ask yourself that question, you will see that the heat from this angle will primarily flow like this. These are the dominating faces through which heat is flowing like this, isn't it? If undergoing an unsteady state like this or like this. So the distance which the heat flows will be half the thickness t and therefore the characteristic dimension which I should take for an angle would be t divided by 2 - that would be the characteristic dimension to take.

If I use this characteristic dimension and if I find that the Biot number is less than .1, well then I would neglect internal temperature gradients. Now let us move on a little and do one more numerical problem. Problem I am going to do is the following now, we are given a hollow metal sphere. Let me draw that for you; we are given a hollow metal sphere.

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Given a hollow metal sphere like this - a hollow metal sphere 10 centimeters in diameter od and 5 centimeters id. Its inner diameter, it's a hollow sphere this is 5 centimeters and the od is 10 centimeters. It is a solid sphere but hollow in the middle initially at a temperature t_0 , initially at a temperature of 500 degrees centigrade. Certainly it is cooled in oil bath; that oil bath, it is at a temperature t_f equal to 40 degrees centigrade, cooled in an oil bath temperature of 40 and the surface heat transfer coefficient is 90 watts per meter squared Kelvin.

So, the values of h at this outer surface is 90 watts per meter squared Kelvin; you are told that there is negligible heat transfer at the inner surface. So at this inner surface, this is the inner surface in the hollow sphere, there is negligible heat transfer h equal to 0, no heat transfer or negligible heat transfer. So all the heat transfer is taking place from the outer surface; the properties of the material are given. You are told that ρ equal to 8000; it is a metal, ρ is 8000 kilograms per meter cubed. k in the thermal conductivity - it is a metal - is about 50 watts per meter kelvin and C_p is equal to 420 per kilogram Kelvin. These are the properties of the metal.

Find the temperature of the sphere after 100 seconds have elapsed. Find the temperature at any point in the sphere after 100 seconds have elapsed; at t equal to 100 seconds T is equal to what? Find the temperature in the hollow sphere at any point after 100 seconds have elapsed. Now the first thing we have to do is to find the Biot number for this metal sphere; so first let us now calculate the Biot number. The Biot number is given by

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Handwritten calculations on a whiteboard:

$$Bi = \frac{hL}{k} \quad \text{Take } L = 2.5 \text{ cm}$$

$$= \frac{90 \times 0.025}{50} = 0.045 < 0.1$$

\therefore internal temperature gradients are negligible

$$\frac{\theta}{\theta_0} = e^{-\frac{hAt}{\rho C_p v}}$$

$$\frac{hAt}{\rho C_p v} = \frac{90 \times 4\pi \times 0.05^2 \times 100}{8000 \times 420 \times \frac{4}{3} \pi (0.05^3 - 0.025^3)} = 0.7694$$

For the hollow sphere hL by k , now what will we take as a characteristic dimension? You see this is the hollow sphere; heat is flowing from inside out, the maximum distance that the heat will flow will be from here outwards because there is no heat flow on the inner, inside in the hollow sphere inside. So the maximum distance the heat is going to flow is from here to here which is 2.5 centimeters. So let us take L equal to 2.5 centimeters, put that it so what will we get? We will get B_i is equal h which is 90 multiplied by the characteristic dimension .025 divided by the conductivity 50 and that comes out to be .045 which is much less than .1. Therefore this is a case where we can neglect internal temperature gradients; therefore because of this value of Biot number, therefore internal temperature gradients are negligible.

So, we have taken a characteristic dimension and then using that characteristic - taking a reasonable characteristic dimension - using that value we have shown that the Biot number is less than .1 and concluded that internal temperature gradients are negligible. Once you arrive at that conclusion, then the problem is one of only substituting into the expression which we have derived earlier with negligible internal temperature gradients; that is θ/θ_0 is equal to e to the power of minus hAt divided by $\rho C_p v$, that is all we have to do.

So first, let us calculate hAt by $\rho C_p v$. hAt divided by $\rho C_p v$ is equal to: h is 90, A is $4\pi r^2$, 4π the area on the outer surface .05 squared, into the time t 100 seconds divided by ρ 8000 into C_p which is 420 multiplied by the volume of the sphere $\frac{4}{3}\pi r^3$ into .05 cubed minus .025 cubed, that is the volume. And if you calculate that you will get .7694 that is the value of hAt upon $\rho C_p v$.

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$$\frac{\theta}{\theta_0} = e^{-0.7694} = 0.4633$$

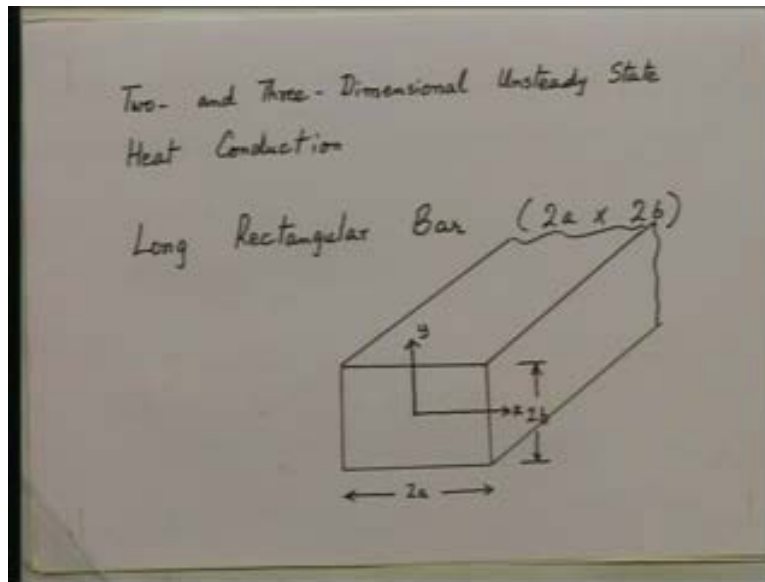
$$T = 0.4633(500 - 40) + 40 = 253.1^\circ\text{C} \leftarrow$$

Now let us plug into our relation so we get θ/θ_0 is equal to e to the power of minus .7694 which is equal to .4633. Therefore θ , rather T , the temperature T is equal to .4633 multiplied by θ_0 minus θ_f that is 500 minus 40 plus θ_f that is 40 and that comes out to be 253.1 degrees centigrade. So that is the answer to the problem now. So

this is the answer to the problem because we could neglect internal temperature gradients. If you, if for instance in this particular case, the data were such that the Biot number were much, were more than .1 and you find yourself unable to neglect internal temperature gradients, then you can formulate the problem in terms of the partial differential equation in spherical coordinates. You will get boundary condition, you will have initial condition and you will have to solve that partial differential equation with those given conditions - get a series solution. It can probably be done but it will be far more complicated than what we have done by in this method when we have been able to neglect internal temperature gradients.

Now, we have now looked at one dimensional problems but I want to tell you also that the one dimensional unsteady state solutions that we have got are also useful for two and three dimensional unsteady state situations, two and three dimensional unsteady state heat conduction problems.

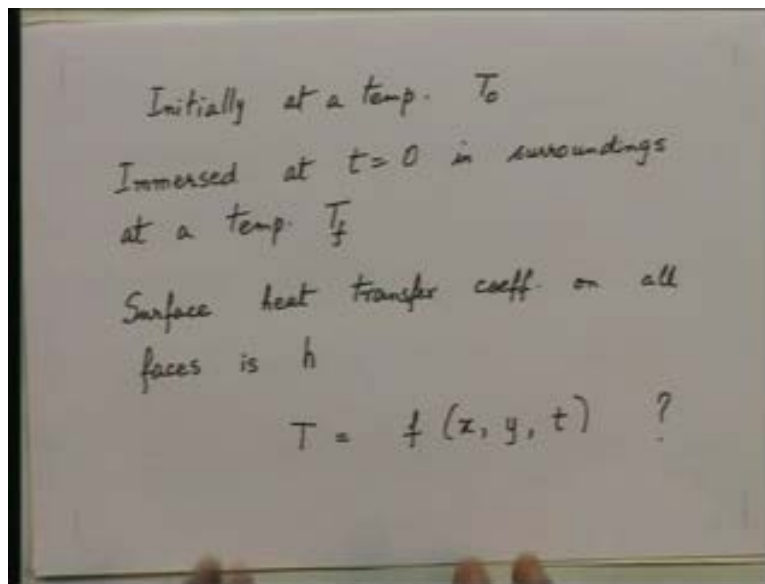
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We want to briefly look at such solutions; we are not going to solve the differential equations ourselves but I am going to give you the solutions which have been obtained by others. Let us look at the following problem; let us say I have a long rectangular bar, a

long rectangular bar whose dimensions are $2a$ multiplied by $2b$. Let us say I have long rectangular bar dimensions $2a$ by $2b$ something like this. Let us say I have a cross section which is like this $2a$ by $2b$. This is $2a$ and this is $2b$ - a long rectangular bar whose cross section is $2a$ by $2b$; it long in the z direction like this, long in the z direction like this. And let us say the center line is the center of the bar; it is here so this is our center line, the origin of the xy coordinate axis so this is x this is y . So long rectangular bar like this initially at temperature t_0 , initially at a temperature t_0 the bar is initially at t_0 .

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Initially immersed at time t equal to 0 in surroundings at a temperature T_f , surface heat transfer coefficient on all faces, surface heat transfer coefficient on all faces is h . Find the temperature in this rectangular bar as a function of x , y and time - that is the problem. The temperature in this in the bar will be a function of x , y and times vary in 2 dimension; x and y it will also vary in time. Find the temperature distribution in this bar as a function of x , y and time; that is the problem. Now next time, we look at this solution. We are not going to solve the problem; we are going give you the solution which has been obtained for this situation and show how to use the charts for one dimensional situation, for this two dimensional problem.